14th European Signal Processing Conference (EUSIPCO 2006), Florence, Italy, September 4-8, 2006, copyright by EURASIP

THE DEMODULATION OF M-PSK AND M-QAM SIGNALS USING PARTICLE FILTERING

Maoge Xu, Yaoliang Song

Department of Electronic Engineering
Nanjing University of Science & Technology

Nanjing, 210094, R.P.China
email: {xumaoge, yaoliangsong}@njust.edu.cn

ABSTRACT

In this paper, the problem of particle filter to demodulate uncoded M-PSK and M-QAM signals over Rayleigh flat fading channels is investigated. Based on the Jakes' model, the channel state is modelled as a first order autoregressive (AR) process. The observation noise is assumed complex Gaussian. It is shown that, in the demodulation of uncoded PSK signals, particle filter doesn't have superiority compared to the decision-directed Kalman filter due to the M-ary phase ambiguity of the PSK signals， but it is not the case to detect uncoded QAM signals. As can be seen, while using the same pilot symbol rate in the demodulation of uncoded M-QAM signals particle filter outperforms the decision-directed Kalman filter and it performs well even in the low pilot symbol rate.

1. INTRODUCATION

In recent years, much attention has been devoted to a group of techniques known as particle filtering methods (also referred as sequential Monte Carlo (SMC) algorithms) [1]. All of these techniques are aimed at building a recursive Bayesian filter, which estimates the posterior probability density function (pdf) based on Monte Carlo simulations. Particle filter is an important alternative for predicting and estimation unknown parameters of interest in real-time applications, especially in systems with nonlinearities and non-Gaussianties where classical approaches based on the well-known Kalman filter [2] provide solutions that may be far from optimal.

Specifically, a main stream of research in the application of particle filtering to communications is currently under way. A considerable amount of research has recently been devoted to PSK signal detection using particle filter in frequency-nonselective (flat) Rayleigh fading channels [3, 8]. The basic idea is to make use of the structure of the state space model, and incorporate efficient variance reduction strategies, so that we can sequential impute multiple samples of the transmitted symbols based on the current observation, and obtain the MMAP (marginal maximum a posteriori) estimates of the transmitted symbols. This kind of particle fitler has been called Mixture Kalman filter [3] or particle filter with Rao-Blackwellistion variance reduction strategies [8].

This article may be the supplement to [3], as we pointed out that particle filtering doesn't have superiority compared to decision-directed Kalman filter in the demodulation of uncoded PSK signals due to the phase ambiguity, but it is not the case to detect uncoded QAM signals and coded PSK signals. In the demodulation of QAM signals, particle filter outperforms the decision-directed Kalman filter while using the same pilot symbol rate and it performs well even in the low pilot symbol rate. We don't consider the coded PSK signals here.

The rest of the paper is organized as follow. In section 2 the communication system under study is described. In section 3 we introduce particle filter and decision-directed Kalman filter for this communication system. Simulation results and our analysis are provided in section 4. A brief summary is given in section 5.

2. SYSTEM DISCRIPTION

Consider a sequence of uncoded M-PSK or M-QAM symbols transmitted through a Rayleigh flat fading channel with additive Gaussian noise. The symbol transmitted during the t th signaling interval is s_t, for $t = 1,\cdots T$. Assuming symboling rate sampling, the received signal sample in the t th signaling interval is

$$y_t = s_t \cdot c_t + n_t \qquad n_t \sim N_c(0,\sigma^2) \tag{1}$$

Where c_t and n_t are complex, zero mean Gaussian random variable representing the sampled fading and the additive noise processes. Among various channel models, the information theoretic results in [7] have shown that the first-order Gaussian-Markov process provides an accurate model for Rayleigh flat fading channels and therefore, will be adopted henceforth. The dynamic of the channel state c_t are modeled by

$$c_t = \alpha \cdot c_{t-1} + \nu_t \tag{2}$$

Where the ν_t is the white complex Gaussian with zero-mean and covariance $\sigma_v^2/2$ per dimension and is statistically independent of c_t , Parameter α is the fading correlation coefficient that characterizes the degree of time variations, depends on the channel Doppler spread and can be accurately obtained in [7].

3. ALGORTHIM

3.1 Particle Filter

Consider the flat-fading channel with additive observation Gaussian noise model, given by (1) and (2). Denote $Y_t \triangleq (y_0 \cdots y_t)$ and $S_t = (s_0, \cdots s_t)$ .We considers the case of uncoded system, where the transmitted symbols are assumed to be independent from a finite alphabet set $\mathcal{A} = \{a_1, \cdots a_M\}$. After received $y_{1:t} \triangleq (y_1 \quad y_2 \quad \cdots \quad y_t)$, our aim is to recursively estimate the posterior probability density function $p(c_{1:t}, s_{1:t} \mid y_{1:t})$. In fact the joint posterior density $p(c_{1:t}, s_{1:t} \mid y_{1:t})$ can be factorized as $p(c_{1:t}, s_{1:t} \mid y_{1:t}) = p(c_{1:t} \mid y_{1:t}, s_{1:t}) p(s_{1:t} \mid y_{1:t})$. Given $s_{1:t}$, the probability density $p(c_{1:t} \mid y_{1:t}, s_{1:t})$ is a Gaussian distribution whose parameters may be computed using the Kalman filter. It is possible to reduce the problem of estimating $p(c_{1:t}, s_{1:t} \mid y_{1:t})$ to one of sampling from a lower-dimensional distribution $p(s_{1:t} \mid y_{1:t})$, which intuitively requires a reduced number of samples N in order to reach a given precision. This is proved in [5] where it is shown that the variance of the estimates is lower when $c_{1:t}$ can be integrated out analytically. The detail description of above idea can be found in [3, 8].

The brief algorithm is as follow:

1) *Initialization:* Each Kalman filter is initialized as $\kappa_0^{(j)} = (\mu_0^{(j)}, \Sigma_0^{(j)})$, with $\mu_0^{(j)} = 0$, $\Sigma_0^{(j)} = 2\Sigma$, $j = 1, \cdots N$, where Σ is the stationary covariance of x_t and is computed analytically from (1)(The factor 2 is to accommodate the initial uncertainty). All importance weights are initialized as $\omega_0^{(j)} = 1$, $j = 1, \cdots N$

Based on the state-space model (1), (2), the following steps are implemented at time t to update each weighted sample. For $j = 1, \cdots N$.

2) *Compute the one-step predictive update of each Kalman filter*

$$K_t^{(j)} = \alpha \cdot \Sigma_{t-1}^{(j)} \cdot a' + \sigma_v^2$$
$$\gamma_t^{(j)} = K_t^{(j)} + \sigma^2$$

3) *Compute the trial sampling density:* For each $a_i \in \mathcal{A}$ compute

$$\begin{aligned}\rho_{t,i}^{j} &\triangleq P(s_t = a_i \mid S_{t-1}^{(j)}, Y_t)\\ &\propto p(y_t, Y_{t-1}, s_t = a_i, S_{t-1}^{(j)})\\ &= p(y_t \mid s_t = a_i, S_{t-1}^{(j)}, Y_{t-1}) \bullet p(s_t = a_i \mid S_{t-1}^{(j)}, Y_{t-1})\\ &= p(y_t \mid s_t = a_i, S_{t-1}^{(j)}, Y_{t-1}) \bullet p(s_t = a_i)\end{aligned}$$

Furthermore, we observe that

$$p(y_t \mid s_t = a_i, S_{t-1}^{(j)}, Y_{t-1}) \sim N_c(a_i \mu_{t-1}^{(j)}, \gamma_t^{(j)})$$

4) *Impute the symbol* s_t: Draw $s_t^{(j)}$ from the set with probability

$$P(s_t^{(j)} = a_i) \propto \rho_{t,i}^{(j)},$$

Append $s_t^{(j)}$ to $S_{t-1}^{(j)}$ and obtain $S_t^{(j)}$.

5) *Compute the importance weight:*

$$\begin{aligned}w_t^{(j)} &= w_{t-1}^{(j)} \cdot p(y_t \mid S_{t-1}^{(j)}, Y_{t-1})\\ &= w_{t-1}^{(j)} \cdot \sum p(y_t \mid s_t = a_i, S_{t-1}^{(j)}, Y_{t-1}) p(s_t = a_i)\\ &\propto w_{t-1}^{(j)} \cdot \sum \rho_{t,i}^{(j)}\end{aligned}$$

6) *Compute the one-step filtering update of the Kalman filter*:
Based on the imputed symbol $s_t^{(j)}$ and the observation y_t , complete the Kalman filter update to obtain $\mu_t^{(j)}, \Sigma_t^{(j)}$, as follows:

$$\mu_t^{(j)} = \alpha \cdot \mu_{t-1}^{(j)} + \frac{K_t^{(j)} \cdot s_t^{(j)H}}{\gamma_t^{(j)}} (y_t - s_t^{(j)} \cdot \alpha \cdot \mu_{t-1}^{(j)})$$

$$\Sigma_t^{(j)} = K_t^{(j)} - \frac{1}{\gamma_t^{(j)}} K_t^{(j)} \cdot s_t^{(j)H} \cdot s_t^{(j)} \cdot K_t^{(j)}$$

Moreover, the *a posterior* symbol probability can be estimated as

$$\begin{aligned}P(s_t = a_i \mid Y_t)\\ &= E\{1(s_t = a_i) \mid Y_t\}\\ &\cong \sum_{j=1}^{N} 1(s_t^{(j)} = a_i) w_t^{(j)}, \ i = 1 \ldots M.\end{aligned}$$

Note a hard decision on the symbol s_t is obtained by max the *a posterior* probability.

It is easy to see that the channel state c_t and the user data s_t are all unknown and needed to be estimated. It is well known that M-PSK signals have phase ambiguity, so there may be multiple pairs of (c_t, s_t) which are equally likely solution for an observation. For the proposed particle filtering above which has multiple samples of the transmitted symbols based on the current observation, it is hard for us to distinguish these multiple samples using different weights. There will be more detail explanation in the section 4.

The way to avoid the situation may be to use error correcting coding such as convolutional coding or trellis coding which will produce not i.i.d. symbols. There are some simulation results in [3].

3.2 Decision-Directed Kalman Filter

The state space model of (1) and (2) allow us to use Kalman filter to adaptively track the channel gain c_t. Noted that the algorithm needs the information symbol s_t, so is working in the decision-directed mode. That is just to use the decision symbols replace the information symbol. The algorithm is standard and is given below.

1) *Initialization:* Kalman filter is initialized as $\kappa_0 = (\mu_0, \Sigma_0)$, with $\mu_0 = 0$, $\Sigma_0 = \Sigma$ where Σ is the stationary covariance of x_t and is computed analytically from (1).

2) *Compute the one-step predictive update of each Kalman filter:*

$$K_t = \alpha \cdot \Sigma_{t-1} \cdot \alpha^H + \sigma_v^2$$

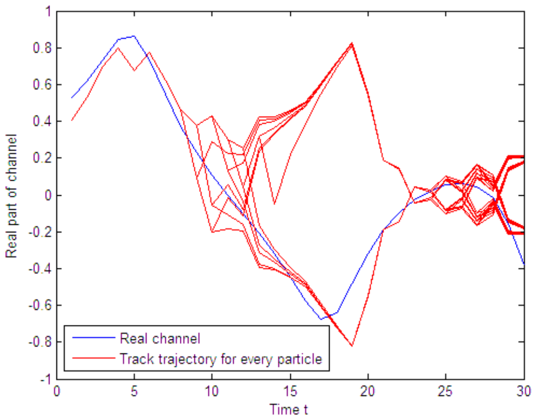

Figure 1 – Real channel and Particle filtering track trajectory for every particle.

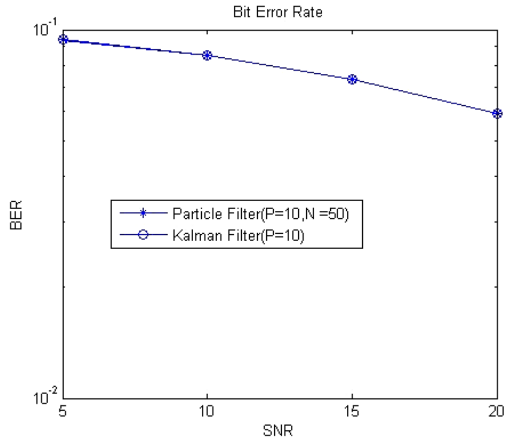

Figure 2 – The comparison of Particle filtering and decision-directed Kalman filter in the demodulation of uncoded 8-PSK using the same pilot symbol rate.

$$\gamma_t = K_t + \sigma^2$$

4) *Compute the one-step filtering update of the Kalman filter* :

$$\mu_t = \alpha \cdot \mu_{t-1} + \frac{K_t \cdot s_t^H}{\gamma_t}(y_t - s_t \cdot \alpha \cdot \mu_{t-1})$$

$$\Sigma_t = K_t - \frac{1}{\gamma_t} K_t \cdot s_t \cdot s_t{}^H \cdot K_t \cdot$$

4. SIMULATION

In this section, we provide computer simulation examples to demonstrate the performance of the proposed particle filter and decision-directed Kalman filter for uncoded PSK and QAM signal detection in flat fading channels. The normalised Doppler frequency is 0.05, which is a fast fading scenario. In order to avoid the degeneracy we send one pilot symbol every P symbols.

In the first simulation we detect the 8PSK signals. As can be seen that the bit error is less dependent different levels of signal to noise and much more dependent on whether a fade occurs (the phase of the channel may be lost during a fade) and how fast one may recover from it. Because of the phase ambiguity, even particle filter has multiple samples of the transmitted symbols based on the current observation, it only has the same ability to recover from a deep fade as decision-directed Kalman filter. That is because due to the different multiple samples $s_{t-1}^{(j)}$, the estimated channel of these samples $\mu_{t-1}^{(j)}$ recover from a different phase (i.e. the opposite sign of the channel for BPSK for example, see figure 1), these different $\mu_{t-1}^{(j)}$ which almost only have different phase get almost the same likehood $p(y_t \mid S_{t-1}^{(j)}, Y_{t-1})$ during the next observation. So the weights $w_t^{(j)}$ of the particle which have correct decision $s_{t-1}^{(j)}$ may be not large enough to let particle filter have correct decision during a fade.

As one can see in figure 2, there is no appreciable difference in the BER performance between particle filter and the decision-directed Kalman filter; there is also no difference if we have more particles.

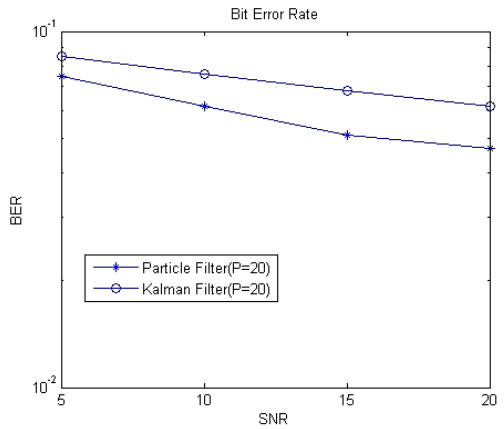

Figure 3 – The comparison of Particle filtering and decision-directed Kalman filter in the demodulation of uncoded 8-QAM signals using the same pilot symbol rate.

In the next simulation we detect the 8QAM signals. There is no phase ambiguity to QAM signals, so the multiple samples of the transmitted symbols based on the current observation let particle filter have the ability to recover from a deep fade. The proposed algorithm performs well, and it outperforms the decision-directed Kalman filter.

Figure 3 illustrates the BER performance of particle filter (the number of particle N = 50) and decision-directed Kalman filter with the same pilot symbol rate (P = 20). Figure 4 illustrates the BER performance of particle filter for P = 20 and decision-directed kalman filter for P = 10.

We must recommend that the complexity of the proposed particle filter is much bigger than the decision-directed Kalman filter. M-detector or stochastic M-detector can be incorporated to decrease the complexity [9].

5. CONCLUTION

In this article we compare the performance of particle filtering and decision-directed Kalman filter in the demodulation

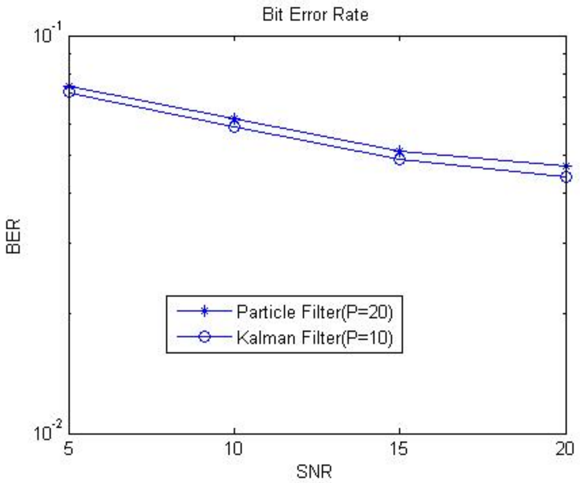

Figure 4 – The comparison of Particle filtering and decision-directed Kalman filter in the demodulation of uncoded 8-QAM signals using the different pilot symbol rate, P = 20 for particle filtering and P = 10 for decision-directed Kalman filter.

of uncoded M-PSK and M-QAM signals. As can be seen the phase ambiguity is a major obstacle of particle filtering in the demodulation of uncoded M-PSK signals transmitted through the Rayleigh flat-fading channels. We can avoid the phase ambiguity in the demodulation of M-QAM signals or may avoid the phase ambiguity in the demodulation of coded M-PSK signals (it can be seen in [3]). The complexity of the proposed particle filter and future modified methods are also recommended.

REFERENCES

[1]A. Doucet, N. de Freitas, and N. Gordon,"*Sequential Monte Carlo Methods in Practice*." New York : Springer,2001

[2] S. Haykin, *Adaptive Filter Theory*, 3rd ed. Upper Saddle River, NJ: Prentice-Hall, 1996.

[3] E.Punskaya, "*Particle Filtering for Optimal Detection in Fading Channels*, CUED/F-INFENG/TR 384, 2000. Tech.rep.

[4] A. Doucet, S. Godsill and C. Andrieu, "*On Sequential Monte Carlo Sampling Methods for Bayesian Filtering, Statistics and Computing*, " vol. 10, no. 3, pp. 197-208,2000.

[5] P. Y. Kam and C. H. The, "*An Adaptive Receiver with Memory for Slowly Fading Channels*", IEEE Trans. Commun, vol. COM-32, pp.654-659, June 1984.

[7] H.Wang and P.Chang, "*On the Verifying the First-Order Markovian Assumption For a Rayleigh Fading Channel Model*,"IEEE Trans. Veh.Technol, vol. 45, pp.353-357, May 1996.

[8] R. Chen, X. Wang, and J.S.Liu, "*Adaptive Joint Detection and Decoding in Flat-Fading Channels via Mixture Kalman Filtering*," IEEE Trans.Inf.Theory, vol. 46 no. 6,pp. 2079-2094, Sep. 2000

[9] Y. Huang, J. zhang, and P.M.Djuric, "*Bayesian Detection for BLAST*," IEEE Trans.Signal Process, vol. 53 no. 3,pp. 1086-1096, Sep. 2005