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BAYESIAN MULTIUSER DETECTION BASED ON A NETWORK OF NLMS FILTERS

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ABSTRACT

The Network of Kalman Filters structure was proposed, recently, to perform an optimal Bayesian symbol-by-symbol estimation in the multiuser detection context. By approximating the prediction error covariance matrix on each branch by a constant diagonal one, we show in this paper that the NKF structure can be expressed into a particular Network of normalized LMS filters exhibiting less computational complexity. The choice of the value of the step-size is also discussed. In order to overcome its heuristic choice, we, here, propose a new adaptive step size based on the second order moment of the estimated symbols. The form of the step-size still contains an information on the *a priori* state estimation. The performance of the resulted receiver structure is evaluated by means of computer simulations for very high asynchronous system load, in multipath fading channel, and compared to MAP, NKF, MMSE and Rake receivers.

1. INTRODUCTION

One of the main drawbacks of the Code Division Multiple Access communication systems can be identified in their vulnerability to multiple access interference (MAP). In addition, the existence of dispersive effects of the channel, such as multipath, destroys the orthogonality of the spreading codes in CDMA systems leading to the inter-symbols interference (ISI). As a result, the conventional RAKE receiver reaches a noise floor at a high frame error rate [1]. Hence, during the 90s, great attention has been devoted to multiuser detection strategies which exploit the interference as an additional information source. These strategies can be divided into sequence detection and single symbol detection under a constrained and an unconstrained alphabet. These receivers optimize a Maximum A Posteriori criterion, a Maximum Likelihood criterion or a Minimum Mean Square Error criterion. These approaches depend on the input-output modelling of the channel.

In recent years, with the advent of powerful computers, much attention has been given to Bayesian multiuser detectors. From Bayesian point of view, all the information that can be extracted form data about signal unknowns is contained in the *a posteriori* distribution of the unknowns. The analytical computation of the *a posteriori* distribution is infeasible due to its prohibitively high computational complexity.

Recently, in [2], the multiuser detection problem is viewed as a Bayesian estimation of the transmitted symbol based on the following symbol-level state-space model:

$$\mathbf{r}(k) = \mathbf{A}(k)\mathbf{d}(k) + \mathbf{b}(k)$$

where $\mathbf{A}(k)$ is the measurement matrix containing both the spreading codes and the channel coefficients, $\mathbf{d}(k)$ is a $\widetilde{k}K$ vector containing $\widetilde{k}K$ symbols contributing to $\mathbf{r}(k)$ and $\mathbf{b}(k)$ is an additive white Gaussian vector. $\widetilde{k}$ denotes the number of interfering symbols.

The conditional probability density function (pdf), $p(\mathbf{d}(k)|\mathbf{R}^k)$, where $\mathbf{R}^k = [\mathbf{r}(k), \mathbf{r}(k-1), ..., \mathbf{r}(0)]$, is needed to derive the optimal Maximum A Posteriori or MMSE detection. In the case of the MMSE, the solution is delivered recursively via the Kalman filter approach. In fact, it is known that the Kalman filter is an optimal linear estimation method in the mean squared error (MSE) sense. However, in [2, 3], it is shown that the optimality of the Kalman filter in no longer valid because of the non-gaussianity of the state noise.

By approximating the *a posteriori* pdf by a weighted sum of Gaussian density functions, it is shown in [2, 3] that the infinite horizon *a posteriori* symbol state pdf is propagated through the non-Gaussian state-model resulting in a MMSE state estimate which is a linear combination of the outputs of parallel Kalman filters. The resulting algorithm is computationally intensive, and requires matrix inversion.

In order to reduce the corresponding complexity and having a comparable performance, we propose in this paper to constraint each filtering error covariance matrix on each branch of the network to a diagonal matrix. The resulting structure is a network of Normalized LMS Filters (NLMSF). This structure is justified since the covariance matrix update is no further needed if we suppose that the last derived MMSE state estimate is consistent.

It is known that the convergence and the steady behavior of the NLMS algorithm is step size value dependent [4]. Therefore, we discuss in the second part of this paper the choice of the step size. We here propose a variable step-size such that it still includes an information about the *a priori* state estimation error. The simulations results show the better performance of the variable step-size NLMSF structure compared to the fixed value step-size NLMSF structure.

The next section presents the symbol-level state-space model for the multiuser CDMA system. Section 3 presents the Network of Kalman Filters (NKF) detector structure. Section 4 presents the new resulting structure based on a network of NLMS filters. Section 5 presents the new variable step-size of the normalized LMS algorithm. Section 6 gives some simulation results. Finally, section 7 draws our conclusion.

2. SYMBOL RATE STATE-SPACE MODEL

We consider K asynchronous users transmitting over K different frequency selective channels. We denote by $d_i(m)$ the symbol of the $i-$th user transmitted in the time interval

$[mT_s,(m+1)T_s[$ where T_s represents the symbol period. We introduce $\mathbf{c}_i=[c_i^0,\dots,c_i^{L-1}]^T$ as the spreading code of user i. L is the processing gain. So, the transmitted signal due to the i-th user can be written as $s_i(t)=\sum_n d_i(n)c_i(t-nT_s)$, where $c_i(t)=\sum_{q=0}^{L-1}c_i^q\psi(t-qT_c)$ and $1/T_c$ denotes the chip rate. $\psi(t)$ is a normalized chip waveform of duration T_c. The baseband received signal containing the contribution of all the users over the frequency selective channels denoted by $h^{(i)}(t)$, $i=1,...,K$, is given by:

$$r(t)=\sum_{i=1}^{K}\sum_{n}\sum_{q=0}^{L-1}d_i(n)c_i^q h^{(i)}(t-qT_c-nT_s)+b(t) \quad (1)$$

where $b(t)$ is an additive noise and $h^{(i)}(t)=\widetilde{h}^{(i)}\star\psi(t)$ including the equipment filtering (chip pulse waveform, transmitted filter and its matched filter in the receiver) and propagation effects (multipath, time delay).
The baseband received signal sampled at the chip rate $1/T_c$ leads to a chip-rate discrete-time model which can be written in $[kT_c,(k+1)T_c[$ as,

$$r(k)=r(t=kT_c)=\sum_{i=1}^{K}\sum_{j}\tilde{g}_i(k,k-jL)d_i(j)+b(k) \quad (2)$$

where $\tilde{g}_i(k,l)=\sum_{q=0}^{L-1}c_i^q h^{(i)}(k,(l-q)T_c)$, is the global channel function including spreading and convolution by the channel. It is convenient to combine the signature modulation process with the effects of the channel in order to obtain an equivalent model in which the symbol'streams of the individual users are time-division multiplexed before their transmission over a multiuser channel.
By concatenating the elements of $r(k)$ in a vector $\mathbf{r}(k)$, and according to equation (2), we have,

$$\mathbf{r}(k)=[r(kL),\dots,r(kL+L-1)]^T \quad (3)$$
$$=\sum_p \mathbf{B}(k,p)\mathbf{x}(k-p)+\mathbf{b}(k) \quad (4)$$

where the matrix $\mathbf{B}(k,p)$ is of size (L,K) and is obtained as follows:

$$\mathbf{B}(k,p) = [\mathbf{g}_1(k,p),\dots,\mathbf{g}_K(k,p)]$$
$$\mathbf{g}_i(k,p) = [\tilde{g}_i(k,PL),\dots,\tilde{g}_i(k,pL+L-1)]^T, i=1,\dots,K$$

$\mathbf{x}(k)=[d_1(k),\dots,d_K(k)]^T$ is a vector of size $(K,1)$ containing the symbols of K users and $\mathbf{b}(k)=[b(nL),\dots,b(nL+L-1)]^T$ is a vector of size $(L,1)$ containing the noise samples on a symbol period.
By denoting $\tilde{k}=\lceil\frac{P+L-1}{L}\rceil$, where P represents the maximum delay introduced by the multipath channels, as the number of the symbols interfering in the transmission channel, the received signal can be expressed as a block transmission CDMA model,

$$\mathbf{r}(k)=\mathbf{A}(k)_{L\times\tilde{k}K}\mathbf{d}(k)_{\tilde{k}K\times 1}+\mathbf{b}(k)_{L\times 1} \quad (5)$$
$$\mathbf{A}(k)=[\mathbf{B}(k,0),\dots,\mathbf{B}(k,\tilde{k}-1)]$$
$$\mathbf{d}(k)=[\mathbf{x}(k)^T,\dots,\mathbf{x}(k-\tilde{k}+1)^T]^T$$

Matrix $\mathbf{A}(k)$ is of size $(L,\tilde{k}K)$. We note that in the case of a time-invariant channel case, the observation matrix $\mathbf{A}(k)$ is a constant matrix $\mathbf{A}$.
Equation (5) represents the measurement equation required in the state-space model of the DS-CDMA system. $\mathbf{d}(k)$ represents the $(\tilde{k}K\times 1)$ state vector containing all the symbols contributing to $\mathbf{r}(k)$. The state vector $\mathbf{d}(k)$ is time dependent and its first order transition equation is described as follows:

$$\mathbf{d}(k+1)=\mathbf{F}\mathbf{d}(k)+\mathbf{G}\mathbf{x}(k+1) \quad (6)$$

where:

$$\mathbf{F}=\begin{pmatrix} \mathbf{0}_{K\times K} & \mathbf{0}_{K\times K} & \cdots & \cdots & \mathbf{0}_{K\times K}\\ \mathbf{I}_{K\times K} & \mathbf{0}_{K\times K} & \ddots & \ddots & \vdots\\ \mathbf{0}_{K\times K} & \ddots & \ddots & \ddots & \vdots\\ \vdots & \ddots & \ddots & \ddots & \mathbf{0}_{K\times K}\\ \mathbf{0}_{K\times K} & \cdots & \cdots & \mathbf{I}_{K\times K} & \mathbf{0}_{K\times K}\end{pmatrix}_{\tilde{k}K\times\tilde{k}K}$$
$$\mathbf{G}=\begin{pmatrix}\mathbf{I}_{K\times K}\\ \mathbf{0}_{K\times K}\\ \vdots\\ \mathbf{0}_{K\times K}\end{pmatrix}_{\tilde{k}K\times K}$$

$\mathbf{0}$ is the $(K\times K)$ null matrix and $\mathbf{I}$ is the $(K\times K)$ identity matrix. We assume that the users are uncorrelated and transmit white symbol streams, i.e. $E\left[\mathbf{x}(k)\mathbf{x}(j)^T\right]=\sigma_d^2\mathbf{I}_{K\times K}\delta(k-j)$ where $\delta(.)$ denotes the Kronecker symbol and σ_d^2 denotes the symbol variance. This state formulation corresponds to a linear system (linear transition and observation equation) corrupted with a Gaussian additive observation noise.

3. NETWORK OF KALMAN FILTERS BASED MULTIUSER DETECTOR

3.1 Non-Gaussian state noise problem

The Kalman filter derivation makes use of the Gaussian hypothesis of the observation noise $\mathbf{b}(k)$ and the state noise $\mathbf{G}\mathbf{x}(k)$ [5]. This is not valid in our case for the plant noise $(\mathbf{G}\mathbf{x}(k))$ which is by definition formed by a set of a discrete transmitted symbols. So, the *a posteriori* pdf, $p(\mathbf{d}(k)|\mathbf{R}^k)$, can be considered as a multiomodal pdf (a set of impulses centered on the possible states). The Kalman filter approximates the first and the second order of the exact pdf [5]. The Kalman filter ignores the binary character of the state noise and loses its optimality.

3.2 The symbol-by-symbol NKF multiuser detector algorithm

In order to enhance the Kalman solution optimality, the *a posteriori* pdf $p(\mathbf{d}(k)|\mathbf{R}^k)$ is approximated by a Weighted Sum of Gaussian terms (see reference [6]), where $\mathbf{R}^k=[\mathbf{r}(k),\mathbf{r}(k-1),...,\mathbf{r}(0)]$. Using the Bayesian filtering equation, it is shown in [2, 3] that the WSG pdf approximation can be propagated through the Bayesian filtering equations,

$$p(\mathbf{d}(k)|\mathbf{R}^k) = \theta_k p(\mathbf{d}(k)|\mathbf{R}^{k-1})p(\mathbf{r}(k)|\mathbf{d}(k)) \quad (7)$$
$$p(\mathbf{d}(k)|\mathbf{R}^{k-1}) = \int p(\mathbf{d}(k)|\mathbf{d}(k-1))\, p(\mathbf{d}(k-1)|\mathbf{R}^{k-1})d\mathbf{d}(k-1) \quad (8)$$

where the normalizing constant θ_k is given by

$$\frac{1}{\theta_k} = \int p(\mathbf{r}(k)|\mathbf{d}(k))p(\mathbf{d}(k)|\mathbf{R}^{k-1})d\mathbf{d}(k)$$

The densities $p(\mathbf{r}(k)|\mathbf{d}(k))$ and $p(\mathbf{d}(k)|\mathbf{d}(k-1))$ are determined from (5) and (6) and the *a priori* distributions of $\mathbf{d}(k)$ and $\mathbf{b}(k)$.
We can show, see reference [2], that if at iteration $(k-1)$, we approximate the pdf $p(\mathbf{d}(k-1)|\mathbf{R}^{k-1})$ by a WSG as follows: $\sum_{i=1}^{\xi(k-1)} \alpha_i(n)\mathcal{N}(\mathbf{d}(k-1)-\mathbf{d}_i(k-1|k-1),\mathbf{P}_i(k-1|k-1))$, where $\mathbf{d}(k|l)$ and $\mathbf{P}(k|l)$ denotes the state vector estimate and the corresponding error covariance matrix, respectively, at time k assuming the knowledge of all the observations collected up to time l ($i.e.\mathbf{R}^l$), the predicted pdf $p(\mathbf{d}(k)|\mathbf{R}^{k-1})$ can be written as a WSG using the equation (8) and the approximation of the plant noise as follows: $p(\mathbf{d}(k)|\mathbf{d}(k-1)) = p(\mathbf{G}\mathbf{x}(k)) = \sum_{q=1}^{2^K} p_q\mathcal{N}(\mathbf{G}\mathbf{x}(k)-\mathbf{G}\mathbf{x}_q,\mathbf{\Delta}_q)$ [1] where $\mathbf{x}_q$ is a possible value of $\mathbf{x}(k)$ and where $p_q = \frac{1}{2^K}$ (BPSK symbols are i.i.d) and $\mathbf{\Delta}_q = \varepsilon_0\mathbf{I}_{L\times L}(\varepsilon_0 \ll 1)$.
Using the equation (7), the estimated pdf $p(\mathbf{d}(k)|\mathbf{R}^k)$ can be written, also, as a WSG with a number of terms equal to $\xi(k) = 2^K.\xi(k-1)$. Each Gaussian parameters (mean and covariance matrix) in the expression of the predicted pdf $p(\mathbf{d}(k)|\mathbf{R}^{k-1})$ is updated by a Kalman filter. In order to stabilize the number of terms in the WSG, we reinject the MMSE estimate, $E(\mathbf{d}(k)|\mathbf{R}^k)$, which is assumed to be Gaussian: $\mathcal{N}(\widehat{\mathbf{d}}_{MMSE}(k),\overline{\mathbf{P}}(k))$, *.i.e.* we force $\xi(k)$ to 1. The obtained MMSE estimate is a convex combination of the outputs of all the Kalman filters. The obtained NKF used for optimal symbol by symbol estimation is given by the following algorithm:
Prediction step: $p(d(k)|R^{k-1})$ computation

$$\begin{aligned}\mathbf{d}_i(k|k-1) &= \mathbf{F}\widehat{\mathbf{d}}_{MMSE}(k-1)+\mathbf{G}\mathbf{x}_i && (9)\\ \mathbf{e}_i(k|k-1) &= \mathbf{r}(k)-\mathbf{A}\mathbf{d}_i(k|k-1) && (10)\\ \mathbf{P}_i(k|k-1) &= \mathbf{F}\overline{\mathbf{P}}(k-1)\mathbf{F}^T+\mathbf{\Delta}_i && (11)\end{aligned}$$

Filtering step: $p(d(k)|R^k)$ computation

$$\begin{aligned}\mathbf{d}_i(k|k) &= \mathbf{d}_i(k|k-1)+\mathbf{K}_i(k)\mathbf{e}_i(k|k-1) && (12)\\ \mathbf{P}_i(k|k) &= \left(\mathbf{I}_{\widetilde{k}K\times\widetilde{k}K}-\mathbf{K}_i(k)\mathbf{A}\right)\mathbf{P}_i(k|k-1) && (13)\\ \mathbf{K}_i(k) &= \frac{\mathbf{P}_i(k|k-1)\mathbf{A}^T}{\sigma^2\mathbf{I}_{L\times L}+\mathbf{A}\mathbf{P}_i(k|k-1)\mathbf{A}^T} && (14)\end{aligned}$$

MMSE state estimation

$$\begin{aligned}\beta_i(k) &= \mathcal{N}(\mathbf{e}_i(k|k-1),\sigma^2\mathbf{I}_{L\times L}+\mathbf{A}\mathbf{P}_i(k|k-1)\mathbf{A}^T)\\ \alpha_i(k) &= \frac{\beta_i(k)}{\sum_{q=1}^{2^K}\beta_q(k)} && (15)\\ \widehat{\mathbf{d}}_{MMSE}(k) &= \sum_{i=1}^{2^K}\alpha_i(k)\mathbf{d}_i(k|k) && (16)\end{aligned}$$

$$\overline{\mathbf{P}}(k) = \sum_{i=1}^{2^K}\alpha_{i,q}(\mathbf{P}_i(k|k)+ \left(\mathbf{d}_i(k|k)-\widehat{\mathbf{d}}_{MMSE}(k)\right)\left(\mathbf{d}_i(k|k)-\widehat{\mathbf{d}}_{MMSE}(k)\right)^T)$$

[1] $\mathcal{N}(\mathbf{a},\mathbf{B}) = exp\{-\frac{1}{2}\mathbf{a}^T\mathbf{B}^{-1}\mathbf{a}\}/(2\pi)^{n/2}|\mathbf{B}|^{1/2}$ where $\mathbf{a}$ is a random vector with a covariance matrix $\mathbf{B}$.

The NKF detector presents a computational complexity about $(O(2^K(K^2\widetilde{k}^2+3K^2\widetilde{k}+K\widetilde{k})+K^3))$ which is less than the complexity of the MAP detector, $O(2^{\widetilde{k}K})$, for $\widetilde{k} >= 2$.

4. THE NETWORK OF NORMALIZED LMS FILTERS STRUCTURE

In order to reduce the complexity of the NKF multiuser detector, which rises from the joint decoding of all the K users, we propose to approximate the prediction error covariance matrix on each branch, $\mathbf{P}_i(k|k-1)$, by a constant diagonal one. This is equivalent to use a stochastic steepest-descent minimization of the mean square error function $E[|\mathbf{r}(k)-\mathbf{A}\widehat{\mathbf{d}}_i(k)|]^2$ by assuming the knowledge of the spreading codes and the channels of all users.
Thus, the NKF algorithm can be rewritten by approximating the prediction error covariance matrices $\mathbf{P}_i(k+1|k)$ with a scaled version of the identity matrix, as follows, where η is a positive constant,

$$\mathbf{P}_i(k|k-1) = \eta\mathbf{I}_{\widetilde{k}K}, \text{ for } i = 1..2^K$$

The matrices $\mathbf{\Delta}_{i=1..2^K}$ are set to the null matrices. In this manner, we reduce the complexity of NKF multiuser detector about $O(2^K(\widetilde{k}K)^2)$.

In the next, we adopt a slightly different notation for clarity. The predicted estimates $\{\mathbf{d}_i(k|k-1)\}$ are replaced by $\widehat{\mathbf{d}}_i(k)$ and the filtered estimates $\{\mathbf{d}_i(k|k)\}$ become $\widehat{\widehat{\mathbf{d}}}_i(k)$. The NKF algorithm reduces to the following where σ_b^2 denotes the spectral density of the observation noise,
A priori update

$$\begin{aligned}\hat{\mathbf{d}}_i(k) &= \mathbf{F}\hat{\mathbf{d}}_{NLMS}(k-1)+\mathbf{G}\mathbf{x}_i && (17)\\ \mathbf{e}_i(k) &= \mathbf{r}(k)-\mathbf{A}(k)\hat{\mathbf{d}}_i(k) && (18)\\ \mathbf{\Gamma}_i(k) &= \eta\mathbf{A}(k)\mathbf{A}(k)^T+\sigma_b^2\,\mathbf{I}_{L\times L} && (19)\end{aligned}$$

A posteriori update

$$\widehat{\widehat{\mathbf{d}}}_i(k) = \widehat{\mathbf{d}}_i(k)+\eta\frac{\mathbf{A}(k)^T}{\mathbf{\Gamma}_i(k)}\mathbf{e}_i(k) \quad (20)$$

MMSE Estimation

$$\begin{aligned}\beta_i(k) &= \mathcal{N}(\mathbf{e}_i(k),\mathbf{\Gamma}_i(k)) && (21)\\ \alpha_i(k) &= \frac{\beta_i(k)}{\sum_{j=1}^{2^K}\beta_j(k)} && (22)\\ \hat{\mathbf{d}}_{LMS}(k) &= \sum_{i=1}^{2^K}\alpha_i(k)\widehat{\mathbf{d}}_i(k) && (23)\end{aligned}$$

The filter outputs, $\mathbf{A}(k)\hat{\mathbf{d}}_i(k)$, are compared to the received sample $\mathbf{r}(k)$ to generate a set of innovations or error signals, $\mathbf{e}_i(k) = \mathbf{r}(k)-\mathbf{A}(k)\hat{\mathbf{d}}_i(k) = \mathbf{r}(k)-\widehat{\mathbf{r}}(k), i = 1..2^K$. The innovation covariance matrix becomes $\mathbf{\Gamma}_i(k)$, given by equation (19). The measurement update is reduced to equation (20).
In fact, equation (20) is an update of a normalized LMS filter minimizing the MSE of the predicted output assuming that $\mathbf{x}(k) = \mathbf{x}_i$ has been transmitted. The parameter η is identified as the step size in [7], where its value is chosen fixed.
In the sequel, and in order to overcome the problem of the choice of the parameter η, we build an adaptive step-size value denoted by $\eta(k)$.

5. PROPOSED ADAPTIVE STEP-SIZE: $\eta(K)$

The convergence and the steady behavior of the NLMS algorithm is step size value dependent [4]. Here, we choose the value of the step size, $\eta(k)$, such that it still includes an information about the *a priori* state estimation error.
The matrices $\mathbf{\Delta}_{i=1..2^K}$ are set to the null matrices. Therefore, from the equation (11), we can write:

$$Tr(\hat{\mathbf{P}}_i(k|k-1)) = \sum_{j=1}^{(\widetilde{k}-1)K} \overline{\mathbf{P}}(k-1)[j,j] = (\widetilde{k}-1)K\eta(k) \tag{24}$$

where Tr denoted the trace operator. $\hat{\mathbf{P}}(k-1)[j,j]$ [2] $= E\{(d(k-j)-\hat{d}(k-j|k-1))^2|\mathbf{R}^{k-1}\}$, for $j=1,\ldots,\widetilde{k}K$. Thus, we can choose the value of η at iteration k as follows:

$$\eta(k) \simeq \frac{1}{(\widetilde{k}-1)K} \sum_{j=1}^{(\widetilde{k}-1)K} (d(k-j)-\hat{d}(k-j|k-1))^2 \tag{25}$$

where $d(k-j)$ is replaced by the hard estimation of $d(k-j|k-1))$ [3, 8].
The value of $\eta(k)$ is valid for $\widetilde{k}>1$. This condition implies that we have an important Inter-Symbol Interference. In the case of $\widetilde{k}=1$, the state vector can be zero padded in order to force $\widetilde{k}$ to be greater than 1. However, in this case, the classical receiver such us the RAKE receiver presents good performance. In fact, from the study done in [3], we have shown that the NKF-detector outperforms the classical receivers such a: MMSE, Decision Feedback Equalizer (DFE) and Kalman, and performs close the optimal MAP symbol by symbol detector with less complexity, in the case of severe multiple access and inter-symbol interferences, i.e. $\widetilde{k}\geq 2$.
Finally, a receiver structured into a network of normalized LMS (NLMS) filters is obtained, where, each NLMS filter is weighted, as in the NKF multiuser detector, by the coefficient $\alpha_i(k)$ which informs us on the severity of the estimation on each branch. The complexity of the obtained network of NLMS filters is about $O(2^K(2K^2\widetilde{k}+K\widetilde{k})+K^3)$ which is less than the Maximum A Posteriori detector ($O(2^{2\widetilde{k}K})$) and the NKF detector ($O(2^K(K^2\widetilde{k}^2+3K^2\widetilde{k}+K\widetilde{k})+K^3)$), for $\widetilde{k}\geq 2$.

6. SIMULATION RESULTS

In this section, we give some simulation results of the resulted network of NLMS filters. We assume a symbol asynchronous multiuser CDMA channel. The multipath channel is given by: $H(z)=0.802+0.535\times z^{-1}+0.267\times z^{-2}$. We set $K=3$, and the time delays for the users are arbitrarily set to zero, two and four chips, respectively. We assume that every user has equal power in each receiving path. We employ the Gold sequences as spreading codes with $L=7$. We note that this situation is a severe case since we have, during the transmission, two interfered symbols.
Figure 1 gives the bit error rate of user 2 versus signal to noise ratio for the derived network of NLMS with a fixed step size and a the proposed variable step size and the Network of Kalman Filters detector. For comparison purposes, bit error rate (BER) of the Rake detector (see [9]), the Minimum Mean Square Error detector computed on a one symbol period (see reference [10]), the optimal Maximum A Posteriori (MAP) symbol by symbol detector (derived in [3]) and the single user bound on a AWGN is also plotted.

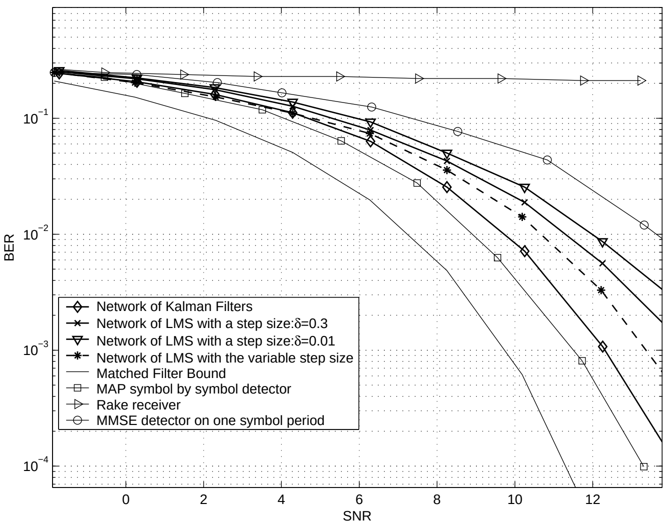

Figure 1: BER performance of the derived network of NLMS detector.

We observe that the network of NLMS filters with a variable step size circumvents the problem of the heuristic choice of the value of the step size. The fixed step size of 0.3 improves the performance compared to the step size of 0.01 for the high SNR. Therefore, the step size parameter measures the severity of the prediction error. It plays the role of the Kalman gain. The performance of the NKF and the network of LMS filters are better than the Rake and the MMSE detectors.

7. CONCLUSION

In this paper, a network of Normalized LMS Filters (NLMSF) is derived. The resulted structure does not require any covariance matrix updates. In the second part, a new variable step-size, $\eta(k)$, is described. It includes an information about the *a priori* state estimation error. Monte-Carlo simulations have shown that the performance of the NLMS structure are close to those of the NKF with less complexity and, still, are better than those of MMSE and Rake receivers, in a heavy system load and an important inter-symbol interference (ISI) term.

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[2] $\mathbf{R}[i,j]$ denotes the element of matrix $\mathbf{R}$ in the i-th row and j-th column

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