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ANALYSIS OF A MULTICHANNEL FILTERED-X PARTIAL-ERROR AFFINE PROJECTION ALGORITHM

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ABSTRACT

The paper provides an analysis of the transient and the steady-state behavior of a filtered-x partial error affine projection algorithm suitable for multichannel active noise control. The analysis relies on energy conservation arguments, it does not apply the independence theory nor does it impose any restriction to the signal distributions. The paper shows that the partial error filtered-x affine projection algorithm in presence of stationary input signals converges to a cyclostationary process, i.e., the mean value of the coefficient vector, the mean-square-error and the mean-square-deviation tend to periodic functions of the sample time.

1. INTRODUCTION

Active noise controllers are based on the destructive interference in given locations of the noise produced by some primary sources and the interfering signals generated by some secondary sources driven by an adaptive controller [1]. In the multichannel approach, in order to spatially extend the silenced region multiple reference sensors, actuators and error sensors are used. Due to the multiplicity of the signals involved, to the strong correlations between them and to the long impulse response of the acoustic paths, multichannel active noise controllers suffer the complexity of the coefficient updates, the data storage requirements, and the slow convergence of the adaptive algorithms [2]. To improve the convergence speed, different filtered-*x* affine projection (FX-AP) algorithms have been used [3], [4] in place of the usual filtered-*x* LMS algorithms, but at the expense of a further, even though limited, increment of the complexity of updates. Various techniques have been proposed in the literature to reduce the implementation complexity of adaptive FIR filters having long impulse responses. Interpolated FIR filters [5], selected partial updates [6], [7] and set-membership filters [8] have attracted the interest of researchers. Among the partial update strategies, a simple yet effective approach is provided by the *Partial Error* (PE) technique, which has been first applied in [6] for reducing the complexity of linear multichannel controllers equipped with the filtered-*x* LMS algorithm. The PE technique consists in using sequentially at each iteration only one of the K error sensor signals in place of their combination and it is capable to reduce the adaptation complexity with a factor K. In [9], the PE technique was applied, together with other methods, for reducing the computational load of multichannel active noise controllers equipped with filtered-*x* affine projection algorithms. When dealing with novel adaptive filters, it is important to assess their performance not only through extensive simulations but also with theoretical analysis results. In the literature, very few results deal with the analysis of filtered-*x*, affine projection or partial-update algorithms. The convergence analysis results for these algorithms are often based on the independence theory (IT) and they constrain the probability distribution of the input signal to be Gaussian or spherically invariant [10]. The IT hypothesis assumes statistical independence of time-lagged input data vectors. As it is too strong for filtered-*x* [11] and AP algorithms [12], different approaches have been studied in the literature in order to overcome this hypothesis. In [11], an analysis of the mean weight behavior of the filtered-*x* LMS algorithm, based only on neglecting the correlation between coefficient and signal vectors, is presented. Moreover, the analysis of [11] does not impose any restriction on the signal distributions. Another analysis approach that avoids IT is applied in [12] for the mean-square performance analysis of AP algorithms. This relies on energy conservation arguments, and no restriction is imposed on the signal distributions. In [4], we applied and adapted the approach of [12] for analyzing the convergence behavior of multichannel FX-AP algorithms. In this paper, we extend the analysis approach of [4] and study the transient and steady-state behavior of a filtered-*x* partial error affine projection (FX-PE-AP) algorithm. The paper shows that the FX-PE-AP algorithm in presence of stationary input signals converges to a cyclostationary process, i.e., that the mean value of the coefficient vector, the mean-square-error and the mean-square-deviation tend to periodic functions of the sample time. In the experimental results, we also show the FX-PE-AP algorithm is capable to reduce the adaptation complexity with a factor K with respect to an approximate FX-AP algorithm introduced in [4], but it also reduces the convergence speed by the same factor.

The paper is organized as follows. Section 2 reviews the multichannel feedforward active noise controller structure and introduces the FX-PE-AP algorithm. Section 3 discusses the asymptotic solution of the FX-PE-AP algorithm and compares it with that of FX-AP algorithms and with the minimum-mean-square solution of the ANC problem. Section 4 presents the analysis of the transient and steady-state behavior of the FX-PE-AP algorithm. Section 5 provides some experimental results. Conclusions follow in Section 6.

Throughout this paper small boldface letters are used to denote vectors and bold capital letters are used to denote matrices, e.g., $\mathbf{x}$ and $\mathbf{X}$, all vectors are column vectors, the boldface symbol $\mathbf{I}$ indicates an identity matrix of appropriate dimensions, the symbol $\odot$ denotes linear convolution, $\text{diag}\{\dots\}$ is a block-diagonal matrix of the entries,

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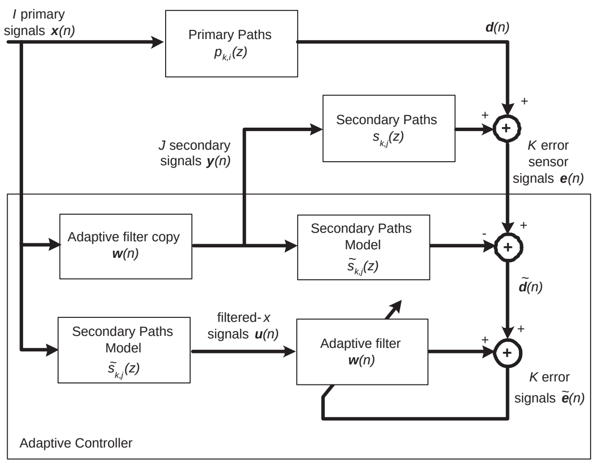

Figure 1: Delay-compensated filtered-x structure for active noise control.

$E[\cdot]$ denotes mathematical expectation, $\|\cdot\|_{\mathbf{\Sigma}}$ is the weighted Euclidean norm, e.g., $\|\mathbf{w}\|_{\mathbf{\Sigma}} = \mathbf{w}^T\mathbf{\Sigma}\mathbf{w}$ with $\mathbf{\Sigma}$ a symmetric positive definite matrix, $\text{vec}\{\cdot\}$ indicates the vector operator and $\text{vec}^{-1}\{\cdot\}$ the inverse vector operator that returns a square matrix from an input vector of appropriate dimensions, $\otimes$ denotes the Kronecker product, $a\%b$ is the remainder of the division of a by b.

2. THE PARTIAL ERROR FILTERED-X AP ALGORITHM

Fig. 1 shows the block diagram of a multichannel delay-compensated filtered-x active noise control system. As usual, the primary and secondary paths, which propagate the primary and secondary source signals, respectively, are modelled with linear FIR filters. In order to compensate for the propagation delay introduced by the secondary paths, the outputs $\mathbf{d}(n)$ of the primary paths are estimated by subtracting the outputs of the secondary path models from the error sensors signals $\mathbf{e}(n)$. In this paper we assume perfect modelling of the secondary paths [we consider $\tilde{s}_{k,j}(z) = s_{k,j}(z)$ for any choice of j and k], but this limitation can be easily removed by following the same methodology of [4].

For simplicity, we assume that any input i of the adaptive controller is connected to any output j with an FIR filter. It is worth noting that the theory we present in Sections 3 and 4 can be applied to any linear or nonlinear filter whose output depends linearly on the filter coefficients [4].

The following notation is used throughout the paper:
I, J, and K are the number of primary source signals, secondary source signals, and error sensors, respectively,
L is the affine projection order,
$s_{k,j}(n)$ is the impulse response of the secondary path that connects the jth secondary source to the kth error sensor,
$\mathbf{w}_{j,i}(n)$ is the coefficient vector of the FIR filter that connects the input i to the output j of the adaptive controller,
$\mathbf{x}_i(n)$ is the ith primary source input signal vector,
$\mathbf{x}(n) = \left[\mathbf{x}_0^T(n), \dots, \mathbf{x}_{I-1}^T(n)\right]^T$,
$\mathbf{w}_j(n) = \left[\mathbf{w}_{j,0}^T(n), \dots, \mathbf{w}_{j,I-1}^T(n)\right]^T$,
$y_j(n) = \mathbf{w}_j^T(n)\mathbf{x}(n)$ is the jth secondary source signal,
$d_k(n)$ is the output of the kth primary path,
$\mathbf{w}(n) = \left[\mathbf{w}_0^T(n), \dots, \mathbf{w}_{J-1}^T(n)\right]^T$,
M is the total number of coefficients of $\mathbf{w}(n)$,
$\mathbf{u}_k(n) = \left[s_{k,0}(n) \odot \mathbf{x}^T(n), \dots, s_{k,J-1}(n) \odot \mathbf{x}^T(n)\right]^T$,
$\mathbf{d}_k(n) = \left[d_k(n), \dots, d_k(n-L+1)\right]^T$,
$\mathbf{U}_k(n) = \left[\mathbf{u}_k(n), \dots, \mathbf{u}_k(n-L+1)\right]$,
$e_k(n) = d_k(n) + \mathbf{u}_k^T(n)\mathbf{w}(n)$,
$\mathbf{e}_k(n) = \mathbf{d}_k(n) + \mathbf{U}_k^T(n)\mathbf{w}(n)$.

The FX-PE-AP algorithm considered in this paper is characterized by the adaptation rule of (1),

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \mu\mathbf{U}_{n\%K}(n)\mathbf{R}_{n\%K}^{-1}(n)\mathbf{e}_{n\%K}(n), \qquad (1)$$

where $\mathbf{R}_k(n) = \mathbf{U}_k^T(n)\mathbf{U}_k(n) + \delta\mathbf{I}$.

By manipulating (1), the adaptation rule can also be written in the compact form of (2),

$$\mathbf{w}(n+1) = \mathbf{V}_{n\%K}(n)\mathbf{w}(n) - \mathbf{v}_{n\%K}(n), \qquad (2)$$

with $\mathbf{V}_k(n) = \mathbf{I} - \mu\mathbf{U}_k(n)\mathbf{R}_k^{-1}(n)\mathbf{U}_k^T(n)$ and $\mathbf{v}_k(n) = \mu\mathbf{U}_k(n)\mathbf{R}_k^{-1}(n)\mathbf{d}_k(n)$.

By iterating K times equation (2) from $n = mK + i$ till $n = mK + i + K - 1$, with $m \in \mathbb{N}$ and $0 \le i < K$, we obtain the expression of (3), which will be used for the algorithm analysis,

$$\mathbf{w}(mK+i+K) = \mathbf{M}_i(mK+i)\mathbf{w}(mK+i) - \mathbf{m}_i(mK+i), \qquad (3)$$

where:

$$\mathbf{M}_i(n) = \mathbf{V}_{(i+K-1)\%K}(n+K-1)\mathbf{V}_{(i+K-2)\%K}(n+K-2)\dots\mathbf{V}_i(n),$$

$$\mathbf{m}_i(n) = \mathbf{V}_{(i+K-1)\%K}(n+K-1)\dots\mathbf{V}_{(i+1)\%K}(n+1)\mathbf{v}_{i\%K}(n) + \mathbf{V}_{(i+K-1)\%K}(n+K-1)\dots\mathbf{V}_{(i+2)\%K}(n+2)\mathbf{v}_{(i+1)\%K}(n+1) + \dots + \mathbf{v}_{(i+K-1)\%K}(n+K-1).$$

3. THE ASYMPTOTIC SOLUTION

For i ranging from 0 to $K-1$, (3) provides a set of K independent equations, that can be separately studied. The system matrix $\mathbf{M}_i(n)$ and excitation matrix $\mathbf{m}_i(n)$ have different statistical properties for different indexes i. For every i the recursion in (3) converges to a different asymptotic coefficient vector and it provides different values of the steady-state mean-square-error and the mean-square-deviation. If the input signals are stationary and if the recursion in (3) is convergent for every i, it can be shown that the algorithm converges to a cyclostationary process of periodicity K.

For every index i, the coefficient vector $\mathbf{w}(mK+i)$ tends for $m \to +\infty$ to a asymptotic vector $\mathbf{w}_{\infty,i}$. As we already observed for FX-AP algorithms [4], this asymptotic solution differs from the minimum-mean-square (MMS) solution of the active noise control problem and it depends on the statistical properties of the input signals. In fact, by taking the expectation of (3) and considering the fixed-point of this equation, it can be easily deduced that

$$\mathbf{w}_{\infty,i} = \left(E\left[\mathbf{M}_i(n)\right] - \mathbf{I}\right)^{-1} E\left[\mathbf{m}_i(n)\right]. \qquad (4)$$

Since the matrices $E[\mathbf{M}_i(n)]$ and $[\mathbf{m}_i(n)]$ vary with i, so do the asymptotic coefficient vectors $\mathbf{w}_{\infty,i}$. Thus, the vector $\mathbf{w}(n)$ for $n \to +\infty$ tends to the periodic sequence formed by the repetition of the K vectors $\mathbf{w}_{\infty,i}$ with $i = 0, 1, \dots, K-1$. The asymptotic sequence depends from the step-size μ and differs from the asymptotic solution of FX-AP algorithms reported in [4].

4. TRANSIENT ANALYSIS AND STEADY-STATE ANALYSIS

The aim of the transient and steady-state analysis is to study the time evolution of the expectation of the weighted Euclidean norm of the coefficient vector $E\left[\|\mathbf{w}(n)\|^2_{\mathbf{\Sigma}}\right] = \mathbf{w}(n)^T\mathbf{\Sigma}\mathbf{w}(n)$ for some choices of the symmetric positive definite matrix $\mathbf{\Sigma}$ [12].

According to equation 3, we can separately analyze the evolution of $E\left[\|\mathbf{w}(mK+i)\|^2_{\mathbf{\Sigma}}\right]$ for the different indexes i. By applying the approach of [12], the following result, which describes the transient behavior of the FX-PE-AP algorithm can be proven, as done in [4] for FX-AP algorithms.

Theorem 1 *For every i with $0 \le i < K$ and for $n = mK+i$ with $m \in \mathbb{N}$, under the assumption that $\mathbf{w}(n)$ is uncorrelated with $\mathbf{M}_i(n)$ and with $\mathbf{q}_{\mathbf{\Sigma},i}(n) = \mathbf{M}_i^T(n)\mathbf{\Sigma}\,\mathbf{m}_i(n)$, the transient behavior of the FX-PE-AP algorithm with updating rule given by (3) is described by the state recursions*

$$E[\mathbf{w}(n+K)] = \mathbf{M}_i E[\mathbf{w}(n)] - \mathbf{m}_i, \tag{5}$$

and

$$\mathbf{W}(n+K) = \mathbf{G}_i\mathbf{W}(n) + \mathbf{y}_i(n), \tag{6}$$

where $\mathbf{M}_i = E[\mathbf{M}_i(n)]$, $\mathbf{m}_i = E[\mathbf{m}_i(n)]$,

$$\mathbf{G}_i = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -p_{0,i} & -p_{1,i} & -p_{2,i} & \dots & -p_{M^2-1,i} \end{bmatrix},$$

$$\mathbf{W}(n) = \begin{bmatrix} E[\|\mathbf{w}(n)\|_{\mathrm{vec}^{-1}\{\sigma\}}] \\ E[\|\mathbf{w}(n)\|_{\mathrm{vec}^{-1}\{\mathbf{F_i}\sigma\}}] \\ \vdots \\ E[\|\mathbf{w}(n)\|_{\mathrm{vec}^{-1}\{\mathbf{F_i}^{M^2-1}\sigma\}}] \end{bmatrix},$$

$$\mathbf{y}_i(n) = \begin{bmatrix} \left(\mathbf{g}_i^T - 2E[\mathbf{w}^T(n)]\mathbf{Q}_i\right)\sigma \\ \left(\mathbf{g}_i^T - 2E[\mathbf{w}^T(n)]\mathbf{Q}_i\right)\mathbf{F}_i\sigma \\ \vdots \\ \left(\mathbf{g}_i^T - 2E[\mathbf{w}^T(n)]\mathbf{Q}_i\right)\mathbf{F}_i^{M^2-1}\sigma \end{bmatrix},$$

the $M^2 \times M^2$ matrix $\mathbf{F}_i = E\left[\mathbf{M}_i^T(n) \otimes \mathbf{M}_i^T(n)\right]$,
the $M \times M^2$ matrix $\mathbf{Q}_i = E\left[\mathbf{m}_i^T(n) \otimes \mathbf{M}_i^T(n)\right]$,
the $M^2 \times 1$ vector $\mathbf{g}_i = \mathrm{vec}\{E[\mathbf{m}_i(n)\mathbf{m}_i^T(n)]\}$
the $p_{j,i}$ are the coefficients of the characteristic polynomial of $\mathbf{F}_i$, i.e., $p_i(x) = x^{M^2} + p_{M^2-1,i}x^{M^2-1} + \ldots + p_{1,i}x + p_{0,i} = \det(x\mathbf{I} - \mathbf{F}_i)$, and $\sigma = \mathrm{vec}\{\mathbf{\Sigma}\}$.

According to Theorem 1, for every index i the transient behavior of the FX-PE-AP algorithm is described by the cascade of two linear systems, with system matrices $\mathbf{M}_i$ and $\mathbf{F}_i$, respectively. The stability in the mean sense and in the mean-square sense can be deduced by the stability properties of these two linear systems. Indeed, the FX-PE-AP algorithm will converge in the mean for any step-size matrix μ such that, for every i, $|\lambda_{\max}(\mathbf{M}_i)| < 1$. The algorithm will converge in the mean-square sense if, in addition, for every i $|\lambda_{\max}(\mathbf{F}_i)| < 1$.

With the steady-state analysis we are interested in evaluating the mean-square-deviation (MSD) and the mean-square-error (MSE) at steady-state. The adaptation rule of (3) provides different values of MSE and MSD for the different indexes i. Therefore, in what follows we define:

$$\begin{aligned}\mathrm{MSD}_i &= \lim_{m\to+\infty} E\left[\|\mathbf{w}(mK+i) - \mathbf{w}_{\infty,i}\|^2\right] \\ &= \lim_{m\to+\infty} E\left[\mathbf{w}^T(mK+i)\mathbf{w}(mK+i)\right] - \|\mathbf{w}_{\infty,i}\|^2, \end{aligned}\tag{7}$$

$$\mathrm{MSE}_i = \lim_{m\to+\infty} E\left[\sum_{k=1}^{K} e_k^2(mK+i)\right]. \tag{8}$$

In the hypothesis that, for every n, $\mathbf{w}(n)$ is independent from $\sum_{k=1}^{K}\mathbf{u}_k(n)\mathbf{u}_k^T(n)$ and from $\sum_{k=1}^{K} d_k(n)\mathbf{u}_k(n)$, the MSE can be expressed as

$$\begin{aligned}\mathrm{MSE}_i \;=\; & S_d + 2\mathbf{R}_{ud}^T\mathbf{w}_{\infty,i} + \\ & \lim_{m\to+\infty} E\left[\mathbf{w}^T(mK+i)\mathbf{R}_{uu}\mathbf{w}(mK+i)\right], \end{aligned}\tag{9}$$

where: $S_d = E\left[\sum_{k=1}^{K} d_k^2(n)\right]$, $\mathbf{R}_{uu} = E\left[\sum_{k=1}^{K}\mathbf{u}_k(n)\mathbf{u}_k^T(n)\right]$, and $\mathbf{R}_{ud} = E\left[\sum_{k=1}^{K}\mathbf{u}_k(n)d_k(n)\right]$.

The computations in (7) and (9) require the evaluation of $\lim_{m\to+\infty} E\left[\|\mathbf{w}(mK+i)\|_{\mathbf{\Sigma}}\right]$, where $\mathbf{\Sigma} = \mathbf{I}$ in (7) and $\mathbf{\Sigma} = \mathbf{R}_{uu}$ in (9). This limit can be estimated with the same methodology of [12] and thus the following expressions for the MSD_i and MSE_i are obtained

$$\mathrm{MSD}_i = \left(\mathbf{g}_i^T - 2\mathbf{w}_{\infty,i}^T\mathbf{Q}_i\right)(\mathbf{I} - \mathbf{F}_i)^{-1}\mathrm{vec}\{\mathbf{I}\} - \|\mathbf{w}_{\infty,i}\|^2; \tag{10}$$

$$\begin{aligned}\mathrm{MSE}_i = & S_d + 2\mathbf{R}_{ud}^T\mathbf{w}_{\infty,i} + \\ & \left(\mathbf{g}_i^T - 2\mathbf{w}_{\infty,i}^T\mathbf{Q}_i\right)(\mathbf{I} - \mathbf{F}_i)^{-1}\mathrm{vec}\{\mathbf{R}_{uu}\}. \end{aligned}\tag{11}$$

5. EXPERIMENTAL RESULTS

In this section, we provide a few experimental results that compare theoretically predicted values with values obtain from simulations. We also compare the performance of the FX-PE-AP algorithm with that of the approximate FX-AP algorithm with adaptation rule given by (12),

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \mu\sum_{k=0}^{K}\mathbf{U}_k(n)\mathbf{R}_k^{-1}(n)\mathbf{e}_k(n). \tag{12}$$

whose convergence properties where analyzed in [4]. Indeed, the FX-PE-AP algorithm in (1) has been obtained by applying the PE methodology to the FX-AP algorithm in (12). Compared with (12), the FX-PE-AP adaptation in (1) reduces the computational load by a factor K.

We considered a multichannel active noise controller with $I = 1$, $J = 2$, $K = 2$. The transfer functions of the primary paths are given by

$$\begin{aligned} p_{1,1}(z) &= 1.0z^{-2} - 0.3z^{-3} + 0.2z^{-4}, \\ p_{2,1}(z) &= 1.0z^{-2} - 0.2z^{-3} + 0.1z^{-4}. \end{aligned}$$

and the transfer functions of the secondary paths are

$$\begin{aligned} s_{1,1}(z) &= 2.0z^{-1} - 0.5z^{-2} + 0.1z^{-3}, \\ s_{1,2}(z) &= 2.0z^{-1} - 0.3z^{-2} - 0.1z^{-3}, \\ s_{2,1}(z) &= 1.0z^{-1} - 0.7z^{-2} - 0.2z^{-3}, \\ s_{2,2}(z) &= 1.0z^{-1} - 0.2z^{-2} + 0.2z^{-3}. \end{aligned}$$

The input signal is a zero-mean, unit-variance colored Gaussian noise with $E[x(n)x(n-m)] = 0.9^{|m|}$ and a zero-mean, white Gaussian noise is added to $d_k(n)$ to get a 40 dB signal-to-noise ratio. The controller is a two-channel linear filter with memory length 5, i.e. with $M = 10$, and the parameter δ is set to 0.001.

Table 1 provides with three-digits precision the first five coefficients of the MMS solution, $\mathbf{w}_o$, and the asymptotic solutions of the FX-PE-AP algorithm at even samples, $\mathbf{w}_{\infty,0}$, and odd samples, $\mathbf{w}_{\infty,1}$, and of the approximate FX-AP algorithm, $\mathbf{w}_\infty$, for $\mu = 0.5$ and for the affine projection orders $L = 1, 2$, and 3. From Table 1 it is evident that the bias varies with the affine projection order and that the asymptotic solutions $\mathbf{w}_{\infty,0}$, $\mathbf{w}_{\infty,1}$, and $\mathbf{w}_\infty$ are different. However, we must point out that their differences reduce with the step-size and for smaller step-sizes they can be difficulty appreciated.

Figure 2 diagrams the steady-state MSE, estimated with (11) or obtained from simulations with time averages over half billion samples, versus step-size μ and for AP order L = 1, 2 and 3. Figures 2-(a) and 2-(b) plot the steady-state MSE of the FX-PE-AP algorithm for the even and odd samples, respectively, and Figure 2-(c) plots the steady-state MSE of the approximate FX-AP algorithm. While for large values of the step-size the FX-PE-AP algorithm provides steady-state mean-square-errors higher than those of the FX-AP, the MSE of the two algorithms tends to the same value when μ tends to zero. Moreover, from Figure 2 we see that the expression in (11) provides reasonable estimates of the steady-state MSE of the adaptive algorithm, with errors that can be both positive or negative depending on the AP order, the step-size and the odd or even sample times. The same observation applies to the expression of the MSD in (10). For example, Figure 3 diagrams the steady-state MSD of the FX-PE-AP algorithm, estimated with (10) or obtained from simulations, versus step-size μ at even samples and for AP order L = 1, 2 and 3. While an error of one order of magnitude is observed for $L = 1$, the estimation for $L = 2$ and $L = 3$ are very accurate.

Eventually, Figure 4 diagrams the learning curves of the residual error for the FX-PE-AP algorithm and the approximate FX-AP algorithm with a step-size equal to 0.015. Each point of Figure 4 represents the ensemble average, estimated over 100 runs of the algorithm, of the mean value of the residual error computed on 100 successive samples. In the figure the asymptotic values (dashed lines) of the residual errors estimated with (11) are also shown. As already observed for the filtered-*x* PE LMS algorithm [2], from Figure 4 it is apparent that for this step-size the FX-PE-AP algorithm has a convergence speed that is the half (i.e., $1/K$) of the approximate FX-AP algorithm. In fact, the two diagrams can be overlapped but the time scale of the FX-PE-AP algorithm is the double of the FX-AP algorithm. We must point out that, for larger value of the step-size, the reduction of convergence FX-PE-AP algorithm can be even larger than a factor K.

6. CONCLUSION

In this paper, we have provided an analysis of the transient and the steady-state behavior of a FX-PE-AP algorithm. We have shown that the algorithm in presence of stationary input signals converges to a cyclostationary process, i.e., the mean value of the coefficient vector, the mean-square-error and the mean-square-deviation tend to periodic functions of the sample time. Moreover, we have compared the FX-PE-AP with the approximate FX-AP algorithm introduced in [4]. Compared with the approximate FX-AP algorithm, the FX-PE-AP algorithm is capable of reducing the adaptation complexity of a factor K. Nevertheless, also the convergence speed of the algorithm reduces of the same value.

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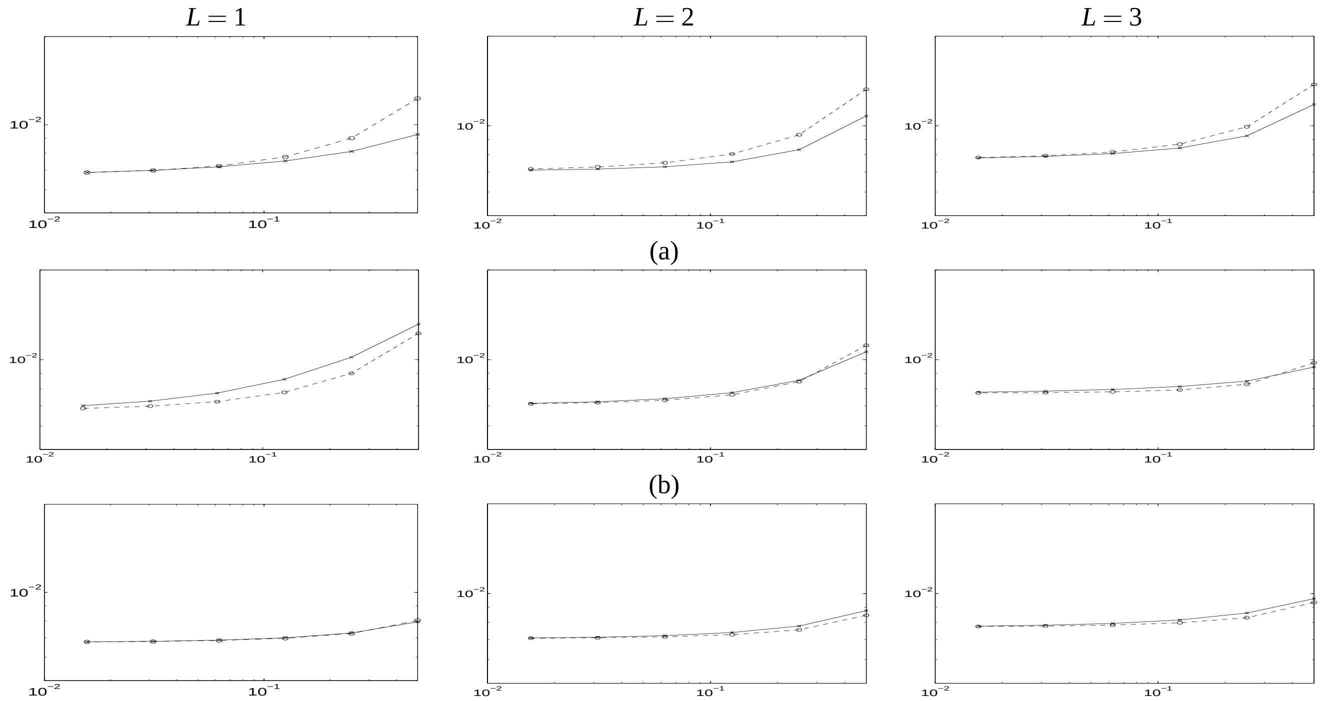
Table 1: First five coefficients of the MMS solution ($\mathbf{w}_o$) and of the asymptotic solutions of FX-PE-AP ($\mathbf{w}_{\infty,0}$, $\mathbf{w}_{\infty,1}$) and of FX-AP algorithm ($\mathbf{w}_\infty$).

	$L=1$			$L=2$			$L=3$		
$\mathbf{w}_o$	$\mathbf{w}_{\infty,0}$	$\mathbf{w}_{\infty,1}$	$\mathbf{w}_\infty$	$\mathbf{w}_{\infty,0}$	$\mathbf{w}_{\infty,1}$	$\mathbf{w}_\infty$	$\mathbf{w}_{\infty,0}$	$\mathbf{w}_{\infty,1}$	$\mathbf{w}_\infty$
0.958	0.951	0.960	0.944	0.876	0.884	0.885	0.876	0.878	0.877
-0.787	-0.789	-0.800	-0.783	-0.716	-0.713	-0.729	-0.752	-0.737	-0.745
0.363	0.344	0.351	0.343	0.300	0.295	0.309	0.332	0.319	0.322
-0.212	-0.181	-0.186	-0.180	-0.148	-0.141	-0.148	-0.158	-0.147	-0.147
0.047	0.040	0.041	0.039	0.036	0.027	0.031	0.036	0.026	0.029



(c)

Figure 2: Theoretical (- -) and simulation values (–) of steady-state MSE versus step-size of the FX-PE-AP algorithm (a) at even samples and (b) at odd samples, and (c) of the approximate FX-AP algorithm for $L=1,2$, and 3.

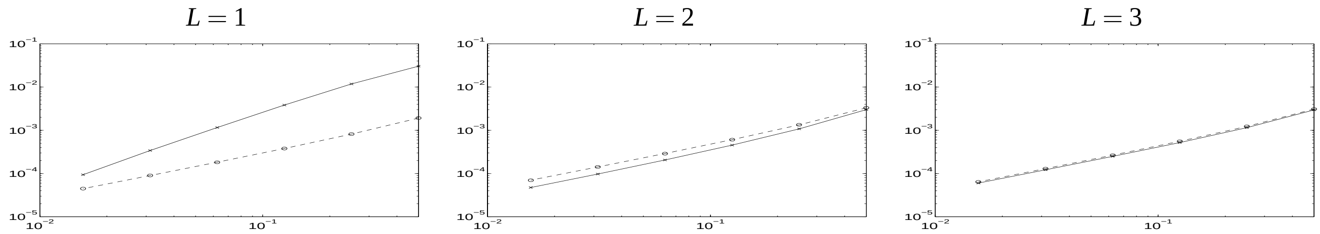
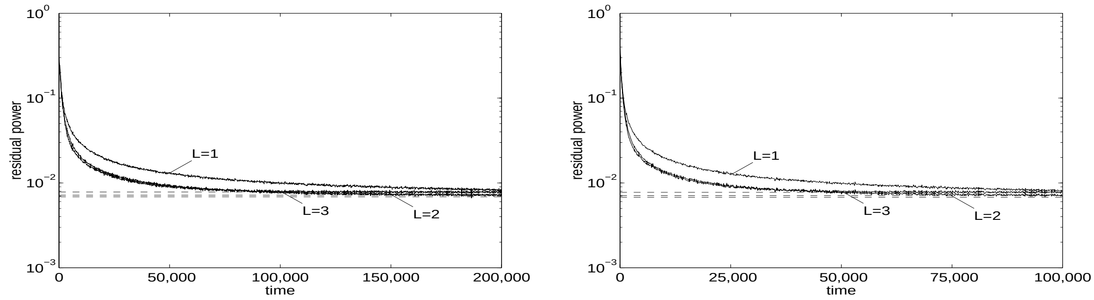


Figure 3: Theoretical (- -) and simulation values (–) of steady-state MSD versus step-size of the FX-PE-AP algorithm at even samples for $L=1,2$, and 3.



(a) (b)

Figure 4: Evolution of residual error with (a) FX-PE-AP and (b) FX-AP algorithm for $L=1,2$, and 3.