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TWO-STAGE BLIND IDENTIFICATION OF SIMO SYSTEMS WITH COMMON ZEROS

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ABSTRACT

Blind identification of SIMO systems is dependent on identifiability conditions which include the requirement that there are no common zeros between multiple channels. We demonstrate that common zeros are likely to exist for long channels, such as acoustic impulse responses and we illustrate the performance degradation due to common zeros in blind system identification. Subsequently, we propose a new scheme where the characteristic zeros components of the transfer functions are identified using a blind multichannel method and the common zeros component is obtained with a single channel approach.

1. INTRODUCTION

The problem of Blind System Identification (BSI) is of great importance in a variety of signal processing and communications applications, such as acoustic dereverberation, speech source separation and image deblurring [1]. The objective is to recover the source signal only through multiple observations by estimating the channels. The sufficient and necessary conditions for multichannel identifiability are [2]: (i) the channels must be co-prime, i.e., they do not share any common zeros, and (ii) the auto-correlation matrix of the source signal must be of full rank. Several algorithms, such as [3, 4, 5, 6] rely on these conditions.

Since current BSI algorithms do not operate successfully in the presence of common zeros, it is necessary to identify the common roots among multiple channels to avoid the performance degradation caused by them. Tugnait [7, 8] investigated the blind identification of multichannel systems over communication channels with common zeros, where the common zeros are assumed to be minimum phase. Non-blind methods to estimate the common roots of polynomials without factorization have been proposed in [9, 10]. Work has also been carried out in [11] to gain some insight of the common zeros problems in the context of adaptive blind multichannel identification algorithm.

In this paper, we investigate the effect of common zeros on blind multichannel system identification. We demonstrate that the zeros of long channels tend to cluster near the unit circle and therefore common or near common zeros are likely to occur. Correspondingly, we propose a two-stage method for blind identification of multichannel systems with common zeros, which is based on a channel decomposition scheme. First, a subspace multichannel technique is used to identify the distinct components of two channels. Second, the common zeros of the multiple channels are identified using a single channel approach.

The remainder of this paper is organized as follows. In Section 2, the problem of blind identification of Single-Input-Multiple-Output SIMO systems with common zeros is formulated where the effect of common zeros on the performance of blind multichannel identification algorithms is demonstrated. The proposed two-stage method is described in Section 3. The performance gains of the new method are demonstrated with simulation results in Section 4 and conclusions are drawn in Section 5.

2. PROBLEM FORMULATION

Consider a linear time-invariant SIMO system where the relationship between the input and mth output is given by

$$x_m(n) = h_m(n) * s(n) + \nu_m(n), \qquad m = 1, 2, \ldots, M \quad (1)$$

where '$*$' denotes linear convolution, $s(n)$ is the source signal, $h_m(n)$ is the L-tap channel impulse response of the mth channel and $\nu_m(n)$ is additive noise. Equation (1) can be written equivalently

$$X_m(z) = H_m(z)S(z) + \mathcal{N}_m(z), \qquad m = 1, 2, \ldots, M \quad (2)$$

where $X_m(z)$, $H_m(z)$, $S(z)$ and $\mathcal{N}_m(z)$ are the z-transforms of $x_m(n)$, $h_m(n)$, $s(n)$ and $\nu_m(n)$, respectively.

It was shown in [11] that the performance of BSI algorithms is degraded when the zeros between different channels are close to each other, but not necessarily exactly common. To illustrate this, the Multi-Channel Least Mean Square algorithm (MCLMS) proposed in [3] was deployed over a short two-channel SIMO system specified by the position of channel roots, where the distance between the zeros of the two channels is defined as $\Delta z = 2\cos\theta,\ 0 \leq \theta \leq \pi/2$, as shown in Fig. 1. The zeros of the two channels get closer as the value of θ is increased and are identical for $\theta = \pi/2$. We then study the convergence of the MCLMS under different channel conditions of Δz. The number of iterations needed for the cost function of the MCLMS to reach -60 dB versus the channel zero separation Δz is plotted in Fig. 2. The asymptotic curve shows that the time for the MCLMS to converge increases exponentially as Δz reduces. It can be further expected that the MCLMS will not converge at all if the zeros are exactly common, as will be seen in Section 4.

In a multichannel system with M channels the likelihood of having common zeros decreases as M increases. Therefore, most current techniques avoid the common zeros problem by assuming a large number of channels. However, in many applications, M is limited. Typical impulse responses of acoustic channels contain several thousand taps. Recent results by Hughes and Nikeghbali [12] show that as the order of a polynomial increases, its roots tend to cluster near the unit circle and their angles become uniformly distributed. This indicates that the mean distance between adjacent zeros of different channels will reduce as the order increases. Simulation on different channels with long impulse

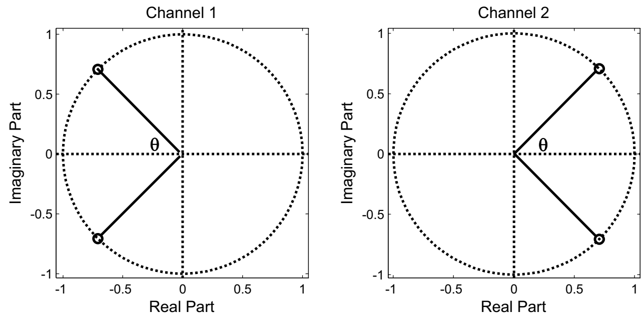

Figure 1: Position of the channel zeros for the example two zeros, two channel system.

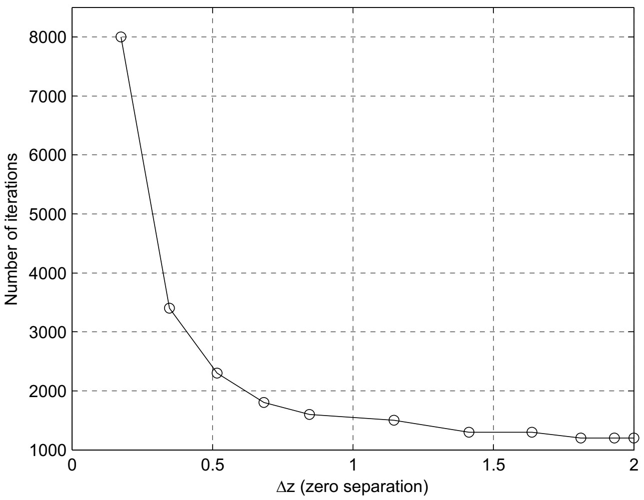

Figure 2: Number of samples needed for the cost function of MCLMS to reach −60 dB for different zero separation.

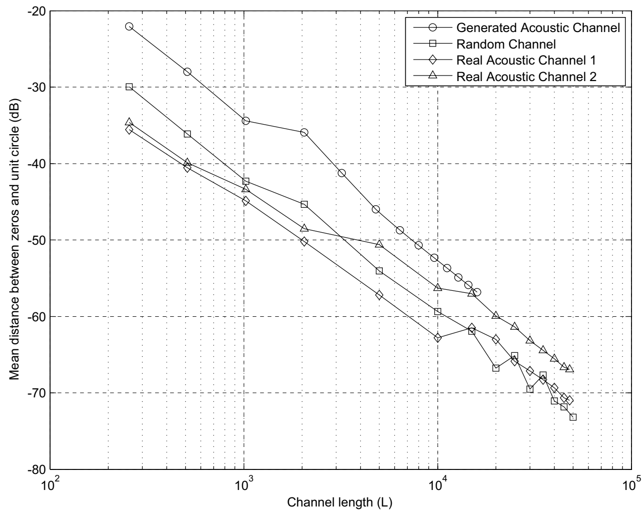

Figure 3: Mean distance between zeros and unit circle *vs.* the length of channel.

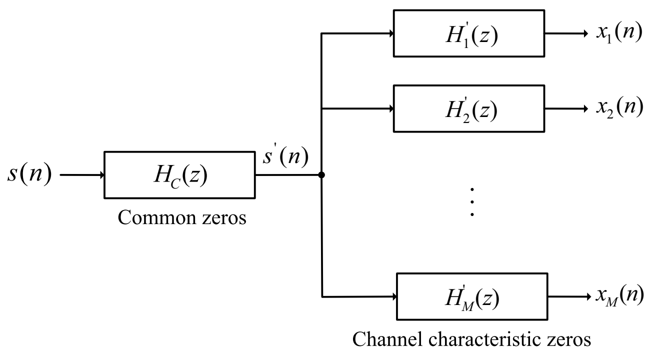

Figure 4: SIMO FIR system with common zeros.

responses confirms this. Fig. 3 shows a plot of the mean distance between the zeros and the unit circle versus an increasing channel length for four different channels: two real acoustic channels measured in a reverberant room, a random channel with normally distributed coefficients, and an acoustic channel generated with the method of image [13].

Therefore, the problem is to identify $H_m(z)$ using only the observations $X_m(z)$ without performance degradation in the presence of common zeros.

3. TWO-STAGE BLIND CHANNEL IDENTIFICATION

We next introduce a new approach for identification of multichannel systems with common zeros. This is done in two stages. First, components associated with the characteristic zeros are blindly identified and inverse filtered. Next, the component associated with the common zeros is estimated. For clarity of presentation, a two-channel system is used as an example, however, it is straightforward to extend this method to the general M-channel case.

In the presence of common zeros, the conventional multichannel system $H_m(z)$ can be decomposed into one component containing the common zeros of all channels, $H_C(z)$, and one component containing the characteristic zeros of each channel, $H'_m(z)$, i.e.,

$$H_m(z) = H_C(z)H'_m(z), \qquad m = 1,2 \tag{3}$$

where $\deg[H_m(z)] = L-1$, $\deg[H_C(z)] = L_C - 1$, $\deg[H'_m(z)] = L' - 1$ and where $L' \leq L$.

Note that we do not explicitly consider the noise robustness of the multichannel identification algorithms since we are primarily interested in the common zeros problem in this paper. Therefore, our objective is to identify $H_C(z)$ and $H'_m(z)$ from only the observations $X_m(z)$,

$$X_m(z) = S(z)H_C(z)H'_m(z), \;\; m = 1,2 \tag{4}$$

The system is depicted in the system diagram in Fig. 4.

3.1 Stage 1: Characteristic zeros identification

From (4), the outputs of the system can be written

$$X_m(z) = H'_m(z)S'(z), \tag{5}$$

where $S'(z) = S(z)H_C(z)$.

Since the transfer functions $H'_m(z)$ do not share any common zeros, we can identify them blindly using, for example, the subspace method [5], which is derived from the cross-relation [2] between two channels:

$$\begin{aligned} x_1(n) * h_2'(n) &= s'(n) * h_1'(n) * h_2'(n) \\ &= x_2(n) * h_1'(n). \end{aligned} \quad (6)$$

This results in a system of equations [3]

$$\mathbf{R}\mathbf{h}' = \mathbf{0}, \quad (7)$$

with

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}_{x_2x_2} & -\mathbf{R}_{x_2x_1} \\ -\mathbf{R}_{x_1x_2} & \mathbf{R}_{x_1x_1} \end{bmatrix}, \quad (8)$$

where $\mathbf{R}_{x_ix_j} = E\{\mathbf{x}_i(n)\mathbf{x}_j(n)\}$, $\mathbf{x}_i(n) = [x_i(n)\ \ x_i(n-1)\ \dots\ x_i(n-L+1)]$ and $\mathbf{h}' = [\mathbf{h}_1'^T\ \mathbf{h}_2'^T]^T$ is the composite channel response vector composed of two channel coefficient vectors $\mathbf{h}_m' = [h_m'(0)\ h_m'(1)\ \dots\ h_m'(L'-1)]^T$. The channel estimates are the eigenvectors corresponding to the smallest eigenvalue of $\mathbf{R}$ and are determined up to an arbitrary scale factor. Alternatively, adaptive estimation algorithms, such as [3, 4] can also be used.

The signal, $s'(n)$ is then formed using the estimated impulse responses $\hat{\mathbf{h}}_m'$. Since the impulse responses may be non-minimum phase and have no common zeros, MINT [6] can be used for inversion and $s'(n)$ can be obtained as

$$s'(n) = x_1(n) * g_1'(n) + x_2(n) * g_2'(n), \quad (9)$$

where $g_m'(n), m = 1,2$ are the inverse filters obtained from the MINT method.

3.2 Stage 2: Common zeros identification

Using the results obtained from Section 3.1, the single channel associated with common zeros is to be identified. For a static or a slowly time-varying channel, this can be achieved blindly by locating the position of corresponding zeros following the technique proposed in [14].

Representing $S'(z)$ and $H_C(z)$ in factored form, we have

$$S'(z) = A\prod_{k=1}^{N_1}(1-a_kz^{-1})\prod_{k=1}^{N_2}(1-s_k'z^{-1})(1-s_k'^{*}z^{-1}), \quad (10)$$

$$H_C(z) = B\prod_{k=1}^{N_3}(1-b_kz^{-1})\prod_{k=1}^{N_4}(1-h_{C,k}z^{-1})(1-h_{C,k}^{*}z^{-1}), \quad (11)$$

where A and B denote gains, a_k and b_k represent the real roots and $\{s_k, s_k^*\}$ and $\{h_{C,k}, h_{C,k}^*\}$ represent the complex-conjugate pairs of roots. N_1, N_2 and N_3, N_4 are defined as the numbers of real and complex-conjugate pairs of roots for $S'(z)$ and $H_C(z)$, respectively.

In the duration for which the channel is considered static, there is a fixed pattern of those zeros that belong to the channel transfer function, , while the zeros of the input vary with time [14]. Therefore, by observing the output signal over several input frames at the microphone, we may identify those zeros which are stationary from those which are dynamic.

The roots of $H_C(z)$ can be written $z_i = \alpha_i e^{j\theta_i}$. In practice, the length of each frame can be very large, in which case the Lindsey-Fox factorizing method [15] can be used for efficient factorization. In the technique used for the identification of the fixed-zero pattern with common zeros component $H_C(z)$, the z-plane is represented by a grid [14] in terms of magnitude and phase in order to capture the position of the zeros. For each cell, an integer counter is defined and is initialized to zero. For each input frame, we map the location of zeros of corresponding output into one of these cells, whose counter is incremented by one. Therefore, after evaluating N frames, the cells with counter values equal to N identify the zeros that have fixed-patterns. Also, the number of common zeros $\deg[H_C(z)]$ is estimated based on static roots. Two or three frames are usually sufficient for identification.

As discussed in Section 2, for very long channels, the length of $H_C(z)$ is also large, in which case the zeros tend to cluster around the unit circle and their angles are uniformly distributed [12]. Therefore a non-linear magnitude quantization is used in order to provide more accuracy. Let B_m denote the number of bits used to linearly quantize the magnitude of roots ranging from 0 to 1, a nonlinear equation [14] is used to map these linearly quantized cells to nonlinear ones:

$$\tilde{\alpha}_{iq} = \frac{1}{\tan(\pi/D)}\left[\tan(\frac{\pi}{D}) + \tan(\frac{\pi}{D}\alpha_{iq} - \frac{\pi}{D})\right], \quad (12)$$

where D is a control parameter and α_{iq} is the linearly quantized magnitude value α_i. We also quantize the phase of roots by using B_p bits. Those zeros that are outside the unit circle are converted to their reciprocals before being evaluated, and they will be converted back when reconstructing the estimated impulse response $\hat{h}_C(n)$ after identification.

Finally, $\hat{h}_C(n)$ and $\hat{h}_m'(n)$ are utilized to generate the channel estimate as

$$\hat{h}_m(n) = \hat{h}_C(n) * \hat{h}_m'(n). \quad (13)$$

4. SIMULATIONS

We present simulation results to demonstrate the performance of the proposed approach.

We first evaluate the performance of the proposed method using a system comprising two random channels of length $L = 33$. We have chosen to illustrate our method for a relatively low order system so that results can be presented in the following plots showing clearly the individual zeros. There are eight common zeros between them, i.e., $L_C = 9$. The system is plotted in Fig. 5, where the common roots for both channels are marked with triangles, the characteristic roots for channel one are marked with circles and those for channel two with squares. The performance was evaluated with the Normalized Projection Misalignment (NPM) defined as [4]

$$\mathrm{NPM} = 20\log_{10}\left(\frac{1}{\|\mathbf{h}\|}\left\|\mathbf{h} - \frac{\mathbf{h}^T\hat{\mathbf{h}}}{\hat{\mathbf{h}}^T\hat{\mathbf{h}}}\hat{\mathbf{h}}\right\|\right)\mathrm{dB}, \quad (14)$$

where $\mathbf{h} = [\mathbf{h}_1^T\ \mathbf{h}_2^T]^T$ is the true channel vector and $\hat{\mathbf{h}} = [\hat{\mathbf{h}}_1^T\ \hat{\mathbf{h}}_2^T]^T$ is the vector of channel estimates.

Figure 6 shows the result for the blind multichannel identification where the true channel zeros are indicated with circles and the identified channel with crosses.

Next, in Fig. 7 the result of the blind identification of the common component is shown where the true zeros are represented by triangles and the estimated zeros by crosses.

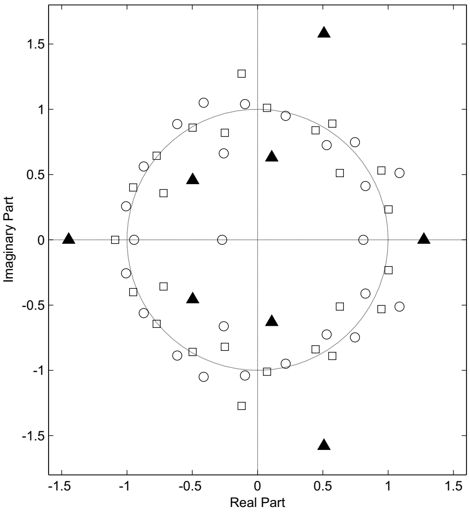

Figure 5: Channel zeros for channel 1 (circles), channel 2 (squares) and the common zeros (triangles).

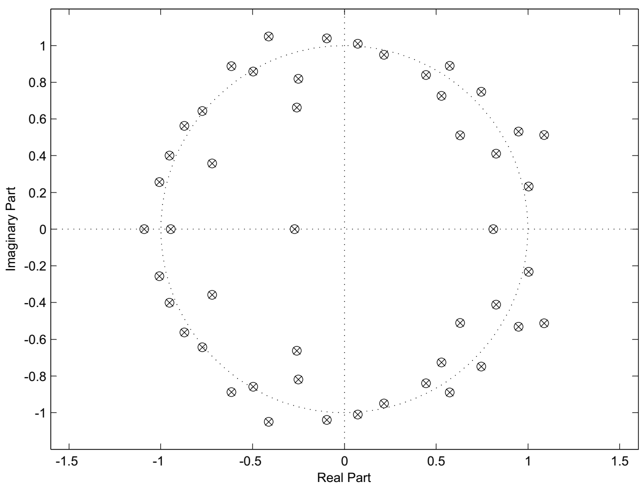

Figure 6: Characteristic (non-common) channel zeros for the true channels (circles) and the estimated channels (crosses).

The algorithm was run using $B_m = 10$ bits, $B_p = 12$ bits and $D = 2.05$. The estimated channel impulse response is reconstructed in Fig. 8, which shows a) the true channel, $\mathbf{h}_1$ and b) the estimated channel, $\hat{\mathbf{h}}_1$. It can be seen from the figures that the proposed approach successfully identifies the channel.

We also performed an experiment to compare the performance of the proposed approach with the MCLMS and the subspace method using random channels of varying length where the number of common zeros is not controlled explicitly. A random input sequence was used and the SNR was set to 60 dB to effectively remove the influence of noise. For the proposed approach, any pair of channel zeros with $\Delta z \leq 0.0001$ were considered to be common and this approach reduces to the subspace approach when there are no common zeros between multiple channels. For adaptive approaches, such as MCLMS, the minimum reachable NPM value for the input sequence was used and all the algorithms were also run for 100 Monte Carlo runs. The results are presented in Fig. 9 where the NPM is plotted versus system size for the MCLMS (diamonds), subspace approach (circles) and

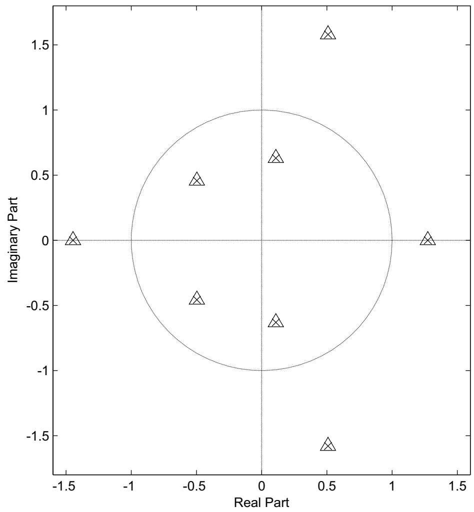

Figure 7: Channel zeros for true common zeros (circles) and identified ones (crosses).

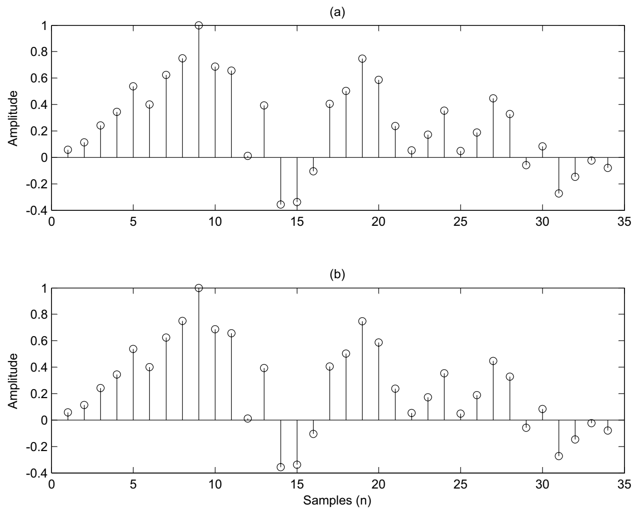

Figure 8: Impulse response of (a) the true channel $\mathbf{h}_1$ and (b) estimated channel $\hat{\mathbf{h}}_1$ using the proposed approach.

the proposed method (squares). It can be seen from that, the proposed method provides an averaging 5 dB improvement of NPM over the subspace method, while the MCLMS method does not converge at all in the presence of common zeros.

In Fig. 10, we compared the NPM performance of the proposed approach and subspace approach [5] in the presence of noise, using the SIMO system in Fig. 5. It is seen that the proposed method is much more robust robust to common zeros and appears to perform better than the subspace method for SNR > 30 dB, , where more than 10 dB improvement of NPM is observed at SNR = 40 dB.

5. CONCLUSIONS

We have considered the problem of common zeros in blind system identification algorithms. It has been demonstrated that as channel zeros become closer, the performance of MCLMS algorithms is degraded. We have also demonstrated that zeros of large order polynomials tend to be close to

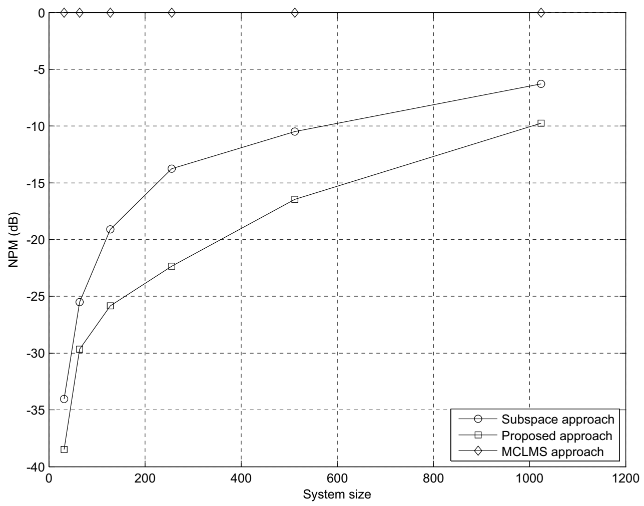

Figure 9: NPM vs. system size for channels identified with the MCLMS (diamonds), the subspace method (circles) and the proposed approach (squares).

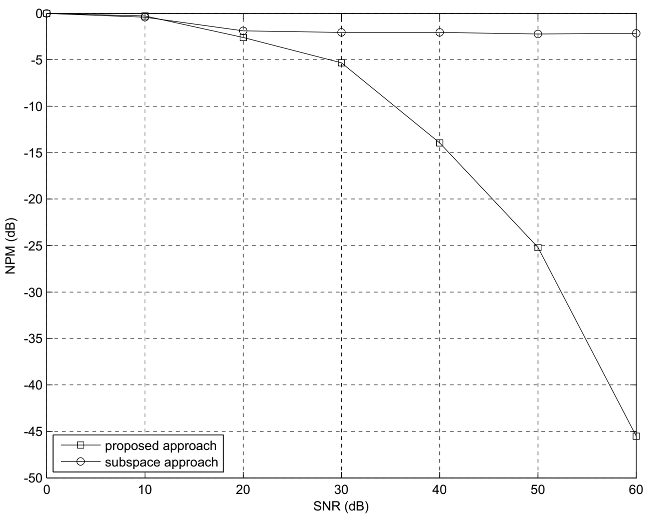

Figure 10: NPM vs. SNR for the subspace method (circles) and the proposed approach (squares).

the unit circle and hence the likelihood of common zeros is increased. Consequently, we proposed a new two-stage approach where characteristic (non-common) zeros components are identified using a multichannel subspace approach and the common zeros component is obtained with a single channel method. Simulation results for various channels confirmed the efficiency of our method demonstrating up to 40dB improvement in NPM over existing methods.

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