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A NOVEL SIGNAL SUBSPACE APPROACH FOR MOBILE POSITIONING WITH TIME-OF-ARRIVAL MEASUREMENTS

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ABSTRACT

The problem of locating mobile terminals has recently received considerable attention particularly in the field of wireless communications. In this paper, a simple signal subspace based algorithm is devised for mobile positioning with the use of time-of-arrival (TOA) measurements received at three or more reference base stations. Computer simulations are included to contrast the estimator performance with Cramér-Rao lower bound and computationally attractive TOA-based localization methods in the literature.

1. INTRODUCTION

Mobile terminal (MT) positioning has been receiving considerable interest, especially after the Federal Communications Commission in the United States has adopted rules to improve the Emergency 911 (E-911) services by mandating the accuracy of locating a E-911 caller to be within a specified range, even for a wireless phone user [1]. Apart from emergency assistance, mobile position information is also the key enabler for a large number of innovative applications such as personal localization and monitoring, fleet management, asset tracking, travel services, location-based advertising and billing [2].

Common positioning approaches [3] are based on time-of-arrival (TOA), received signal strength, time-difference-of-arrival and/or angle-of-arrival measurements determined from the MT signal received at several reference base stations (BSs) with known locations. In this paper, we focus on two-dimensional MT localization given the TOA information. In the TOA method, the one-way propagation time of the signal travelling between the MT and each of the BSs is measured. Each TOA measurement then provides a circle centered at the BS on which the MT must lie. With three or more BSs, the measurements are converted into a set of circular equations, from which the MT position can be determined with the knowledge of the BS geometry.

The optimum TOA-based localization approach involves solving the nonlinear circular equations in an iterative manner and commonly used techniques [4]-[7] include linearization via Taylor-series expansion, steepest descent method and Newton-type iteration. However, this approach is computationally intensive and sufficiently precise initial estimates are required to obtain the global solution. On the other hand, computationally efficient but suboptimum position estimators which allow real-time realization as well as ensure global convergence, have also been proposed in the literature [8]-[12]. In the least squares calibration method [8], the nonlinear equations are reorganized into a set of linear equations via introduction of an extra variable which is a function of the source position, and these linear equations are then solved straightforwardly by using least squares (LS). Alternatively, the common variable in the linear equations can be eliminated via subtraction of each equation from all others, and this technique is referred to as the linear least squares estimator [9]. Based on a new geometrical formulation, Caffery has proposed the straight lines of position (SLOP) method [10] where a different set of linear equations is constructed. Instead of forming linear equations, computationally simple positioning algorithms [11]-[12] have also been derived using the squared TOA measurements or equivalently the squared distance measurements. In [11], modified multidimensional scaling (MDS) is utilized while a noise subspace based algorithm with a linear constraint for 3-BS case is proposed in [12] but only 3 MTs can be dealt with. In this paper we will devise a signal subspace based localization approach which allows any number of BSs and thus can be treated as a generalization of [12].

The rest of the paper is organized as follows. The development of the signal subspace localization algorithm is presented in Section 2. Simulation results are included in Section 3 to evaluate the estimator performance of the proposed position estimator. Finally, conclusions and future works are provided in Section 4.

2. ALGORITHM DEVELOPMENT

Let (x_0, y_0) be the MT position to be determined and the known coordinates of the ith BS be (x_i, y_i), $i = 1, \cdots, M$ where $M \geq 3$ is the total number of receiving BSs. The distance between the ith BS and jth BS, which is denoted by $d_{i,j}$, is given by

$$d_{i,j} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \quad i, j = 1, \cdots, M \tag{1}$$

Similarly, the distance between the MT and the ith BS is defined as

$$d_{0,i} = d_{i,0} = \sqrt{(x_0 - x_i)^2 + (y_0 - y_i)^2} \quad i = 1, \cdots, M \tag{2}$$

Since TOA is the one-way propagation time taken for the signal to travel from the MT to a BS, we have the following relationship:

$$t_i = \frac{d_i}{c}, \quad i = 1, \cdots, M \tag{3}$$

where t_i denotes the noise-free TOA at the ith BS and c is the speed of light. The range measurement based on t_i, in the presence of measurement errors, denoted by $r_{0,i}$, is modeled as

$$r_{0,i} = d_{0,i} + n_{0,i} + \mathscr{U}(\alpha - p)\, q_{0,i} \quad i = 1, \cdots, M \tag{4}$$

where the second and the third components represent the line-of-sight (LOS) error and possible non-line-of-sight (NLOS) error, respectively. The $n_{0i} \sim \mathscr{N}(0, \sigma_{0i}^2)$ where $\mathscr{N}(\boldsymbol{\mu}, \boldsymbol{\Gamma})$ means the Gaussian distribution with mean $\boldsymbol{\mu}$ and covariance $\boldsymbol{\Gamma}$. The $q_{0,i} \sim \mathbb{U}(0,R)$, R is the maximum NLOS distance, $p \sim \mathbb{U}(0,1)$, $\alpha \in [0,1]$ is the probability of obtaining NLOS distance measurement, $\mathbb{U}(a,b)$ stands for the uniform distribution with a and b respectively the starting and ending points and $\mathscr{U}(p)$ denotes the unit step function. Define a $M \times 2$ matrix $\mathbf{X}$ of the form:

$$\mathbf{X} = [\ \mathbf{x} - \mathbf{1}_M x_0 \quad \mathbf{y} - \mathbf{1}_M y_0\ ] \tag{5}$$

where $\mathbf{x} = [\ x_1 \ \cdots \ x_M\ ]^T$, $\mathbf{y} = [\ y_1 \ \cdots \ y_M\ ]^T$ and $\mathbf{1}_M$ is a $M \times 1$ vector with all elements equal 1. We further define $\mathbf{D} = \mathbf{X}\mathbf{X}^T$ which is a rank-2 symmetric matrix and its (m,n) entry is given by

$$[\mathbf{D}]_{m,n} = 0.5(d_{0m}^2 + d_{0n}^2 - d_{mn}^2) \tag{6}$$

where $d_{mn} = d_{nm}$ is of known value. We notice that the diagonal elements of $\mathbf{D}$ are

$$[\mathbf{D}]_{m,m} = d_{0m}^2, \qquad m = 1, 2, \cdots, M$$

Although $\mathbf{D}$ is unknown, we are able to construct its approximate version at sufficiently small noise conditions, denoted by $\hat{\mathbf{D}}$, with the use of the noisy $\{r_{0m}\}$ and noise-free $\{d_{mn}\}$. Apparently, the (m,n) entry of $\hat{\mathbf{D}}$ is

$$[\hat{\mathbf{D}}]_{m,n} = 0.5(r_{0m}^2 + r_{0n}^2 - d_{mn}^2) \tag{7}$$

Decomposing the symmetric $\hat{\mathbf{D}}$ by eigenvalue factorization yields

$$\hat{\mathbf{D}} = \mathbf{U}_s \boldsymbol{\Lambda}_s \mathbf{U}_s^T + \mathbf{U}_n \boldsymbol{\Lambda}_n \mathbf{U}_n^T \tag{8}$$

where $\boldsymbol{\Lambda}_s = \mathrm{diag}(\lambda_1, \lambda_2)$ and $\boldsymbol{\Lambda}_n = \mathrm{diag}(\lambda_3, \cdots, \lambda_M)$ are the diagonal matrices of eigenvalues of $\hat{\mathbf{D}}$ with $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_M \geq 0$, $\mathbf{U}_s = [\mathbf{u}_1\ \mathbf{u}_2]$ and $\mathbf{U}_n = [\mathbf{u}_3 \ \cdots \ \mathbf{u}_M]$ are orthonormal matrices whose columns are the corresponding eigenvectors. Since the rank of the ideal $\mathbf{D}$ is 2, an LS estimate of $\mathbf{X}$ up to a rotation, denoted by $\hat{\mathbf{X}}^{\mathbf{r}}$, can be computed as [11]:

$$\begin{aligned} \hat{\mathbf{X}}^{\mathbf{r}} &= \arg\min_{\mathbf{X}} ||\hat{\mathbf{D}} - \mathbf{X}\mathbf{X}^T||_F^2 \\ &= \mathbf{U}_s \boldsymbol{\Lambda}_s^{\frac{1}{2}} \end{aligned} \tag{9}$$

where $||\ ||_F$ represents the Frobenius norm, and $\boldsymbol{\Lambda}_2^{\frac{1}{2}} = \mathrm{diag}(\lambda_1^{\frac{1}{2}}, \lambda_2^{\frac{1}{2}})$. The relationship between $\hat{\mathbf{X}}^{\mathbf{r}}$ and $\mathbf{X}$ is then:

$$\mathbf{X} \approx \hat{\mathbf{X}}^{\mathbf{r}} \boldsymbol{\Omega} \tag{10}$$

where $\boldsymbol{\Omega}$ is an unknown rotation matrix to be determined. From (10), an optimal estimate of $\boldsymbol{\Omega}$ in the LS sense is easily shown to be

$$\begin{aligned} \hat{\boldsymbol{\Omega}} &= (\hat{\mathbf{X}}^{\mathbf{r}\,T} \hat{\mathbf{X}}^{\mathbf{r}})^{-1} \hat{\mathbf{X}}^{\mathbf{r}\,T} \mathbf{X} \\ &= \boldsymbol{\Lambda}_s^{-\frac{1}{2}} \mathbf{U}_s^T \mathbf{X} \end{aligned} \tag{11}$$

Substituting (11) into (10), we have

$$\mathbf{X} \approx \mathbf{T}\mathbf{X} \tag{12}$$

where $\mathbf{T} = \hat{\mathbf{X}}^{\mathbf{r}} (\hat{\mathbf{X}}^{\mathbf{r}\,T} \hat{\mathbf{X}}^{\mathbf{r}})^{-1} \hat{\mathbf{X}}^{\mathbf{r}\,T} = \mathbf{U}_s \mathbf{U}_s^T$. From (5) and (12), we can construct two sets of linear equations in x_0 and y_0, respectively:

$$\mathbf{x} - \mathbf{1}_M x_0 \approx \mathbf{T}(\mathbf{x} - \mathbf{1}_M x_0) \tag{13}$$

and

$$\mathbf{y} - \mathbf{1}_M y_0 \approx \mathbf{T}(\mathbf{y} - \mathbf{1}_M x_0) \tag{14}$$

In this study, we propose to use the standard LS technique to solve the overdetermined system of (13) and (14) which can be grouped together as

$$(\mathbf{I}_M - \mathbf{T})[\ \mathbf{x} \quad \mathbf{y}\ ] \approx (\mathbf{I}_M - \mathbf{T})\mathbf{1}_M [\ x_0 \quad y_0\ ] \tag{15}$$

The LS estimate is easily shown to be

$$\begin{aligned} [\ x_0 \quad y_0\ ] &\approx ((\mathbf{I}_M - \mathbf{T})\mathbf{1}_M)^{\dagger} (\mathbf{I}_M - \mathbf{T})[\ \mathbf{x} \quad \mathbf{y}\ ] \\ &= \frac{\mathbf{1}_M^T \mathbf{U}_n \mathbf{U}_n^T}{\mathbf{1}_M^T \mathbf{U}_n \mathbf{U}_n^T \mathbf{1}_M} [\ \mathbf{x} \quad \mathbf{y}\ ] \end{aligned} \tag{16}$$

where $\mathbf{A}^{\dagger}$ denotes the pseudo-inverse of $\mathbf{A}$ and note that $\mathbf{U}_n^T \mathbf{U}_n = \mathbf{I}_{M-2}$ and $\mathbf{I}_M - \mathbf{T} = \mathbf{U}_n \mathbf{U}_n^T$. In particular, when there are only 3 BSs, $\mathbf{U}_n = \mathbf{u}_3$, (16) can be simplified to

$$[\ x_0 \quad y_0\ ] = \frac{\mathbf{u}_3^T}{\mathbf{u}_3^T \mathbf{1}_M} [\ \mathbf{x} \quad \mathbf{y}\ ] \tag{17}$$

which is exactly the solution given by [12]. It is noteworthy that unlike [12] which utilizes the noise subspace component subject to a linear constraint, we work on the signal subspace. The proposed method generalizes the method given by [12] in the sense that any number of BSs is allowed as long as $M \geq 3$ while the latter only operates for $M = 3$.

3. NUMERICAL EXAMPLES

Computer simulation had been conducted to evaluate the performance of the proposed TOA-based positioning approach. We compared the mean square position errors (MSPEs) of the signal subspace estimator with the modified MDS [11], the SLOP method [10] as well as the Cramér-Rao lower bound (CRLB) [11] in MT localization. The LOS range errors $\{n_{ij}\}$ were zero-mean white Gaussian processes with variance $d_{i,j}^2/\mathrm{SNR}$ where SNR is the signal-to-noise ratio. All results were averages of 10000 independent runs.

In the first scenario, all the distance measurements were considered as LOS paths and thus $\alpha = 0$. The position of the MT in each run was uniformly distributed within the circle with origin $(0,0)$m and radius 3000m. We started with 3 BSs with coordinates $(0,0)$m, $(0,6000)$m and $(6000,6000)$m. The BSs with coordinates $(6000,0)$m, $(6000,-6000)$m, $(-6000,0)$m, $(-6000,-6000)$m, $(-6000,0)$m and $(-6000,6000)$m were then added successively. Figure 1 shows the MSPEs versus number of BSs when the range error variance was kept at 30 dB. It is seen that the proposed algorithm had similar MSPEs with the modified MDS but it outperformed the SLOP method for $3 \leq M \leq 6$ and all had comparable performance for $7 \leq M \leq 9$. Figure 2 shows the MSPEs versus SNR when $M = 6$ with the BS geometry equalled to the previous test. We observe that the MSPEs of the proposed method were more or less the same as the modified MDS but were smaller than those of the SLOP method and decreased with the CRLB.

In the second scenario, the effects of NLOS were investigated. The parameters α and R of the signal model (4) were set to 0.1 and 100 respectively while the other settings were the same as the first scenario. It can be seen from Figure 3 that the proposed method performed better than the modified MDS and SLOP methods when $M \leq 7$ except the modified MDS had the best performance at $M = 5$. When $8 \leq M \leq 9$, the SLOP method outperformed the proposed method by about 0.5 dB. From Figure 4, we see that the MSPEs of the proposed method were comparable to those of the modified MDS method but less than those of the SLOP method in all SNR. It indicates that the proposed method has superiority over the SLOP method when there are NLOS distance measurements.

Finally, the computational complexity of all methods was compared in a computer with Pentium 4 3.0G Hz CPU and 512MB RAM. The simulation settings were the same as the first scenario. For each number of BS, all methods were run 10000 time and the average was plotted. It is shown from Figure 5 that the computational requirement of the proposed method was less than that of the modified MDS and SLOP methods, especially when M was large. It demonstrates that the proposed method is more computationally attractive.

4. CONCLUSIONS AND FUTURE WORKS

A novel signal subspace based approach has been devised for mobile terminal (MT) localization using distance measurements. It can be observed from the simulation results that the subspace method can attain suboptimum performance in both scenarios and performs well especially when the number of BSs is small and the SNR is high, even in NLOS scenarios. Furthermore, the proposed method is computationally simple and thus is suitable for environments in which processing time is critical.

One of our future works is to improve the estimation performance of the proposed technique. This can be achieved by only changing the values of $r_{0,i}$ but keeping the values of $d_{i,j}$ unaltered in the rank reduction process, or finding a better estimate of $\mathbf{X}$ from $\hat{\mathbf{D}}$ via introduction of an appropriate weighting matrix. On the other hand, it is expected that the process of finding $\mathbf{U}_s$ can be speeded up by utilizing fast algorithms in the scientific computing literature. It is also a challenging task to derive the theoretical performance of the subspace method.

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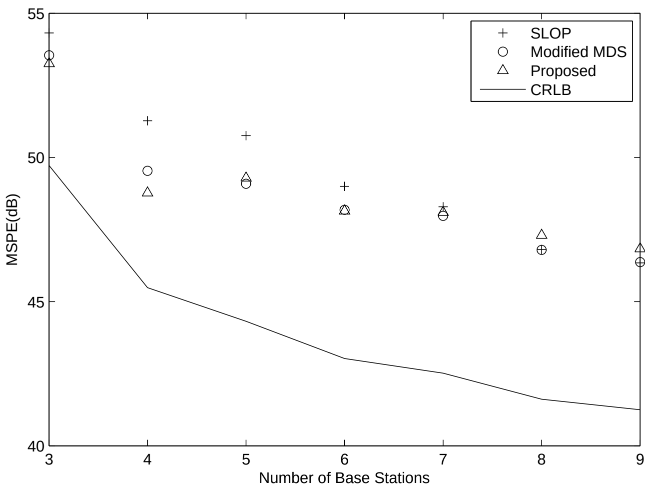

Figure 1: Mean square position error versus number of BSs at LOS environment

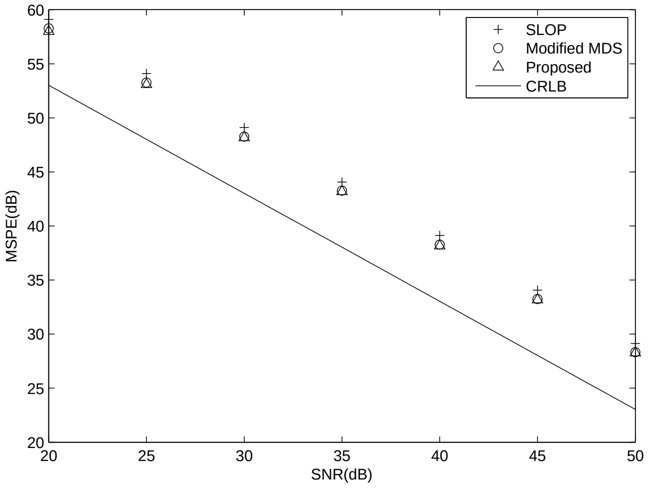

Figure 2: Mean square position error versus SNR at LOS environment

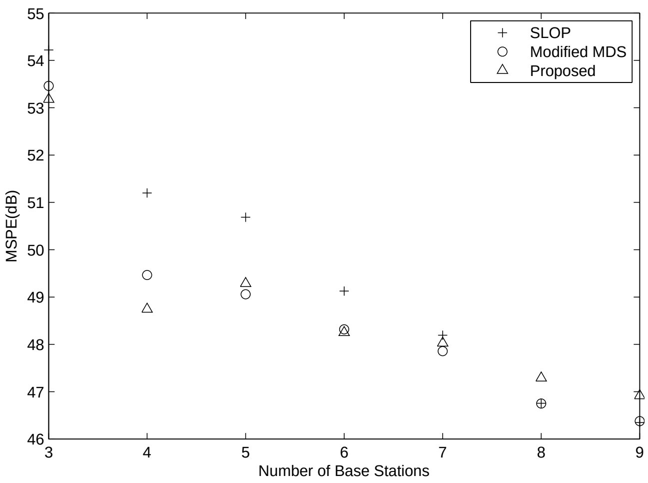

Figure 3: Mean square position error versus number of BSs at NLOS environment

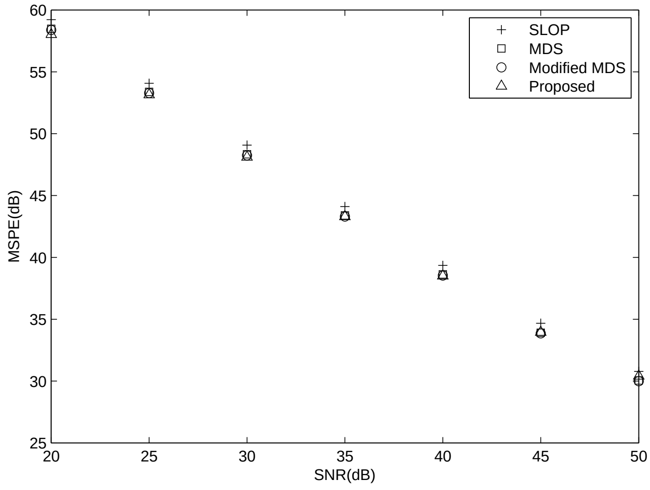

Figure 4: Mean square position error versus SNR at NLOS environment

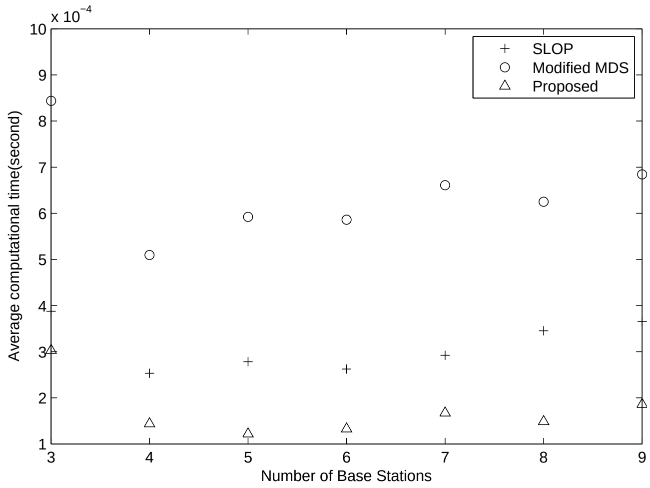

Figure 5: Average computational time