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INTERFERENCE SUPPRESSION IN MULTIUSER DOWNLINK MIMO BEAMFORMING USING AN ITERATIVE OPTIMIZATION APPROACH

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ABSTRACT

Multiple antennas at the transmitter and the receiver have the potential to either increase the data rate through spatial multiplexing or enhance the quality of transmission through exploitation of diversity. In this paper, we address the problem of multi-user multiplexing using spatial diversity techniques so that a base station could serve multiple users in the same frequency band making huge saving in bandwidth utilization. In particular, we have proposed various techniques to improve substantially the performance of a recently proposed signal-to-leakage maximization based algorithm. Our simulation results reveal a lower error floor and more than 10 dB improvement in BER performance.

1. INTRODUCTION

Multiuser MIMO systems have gained a considerable amount of interest in the recent years due to their potential for high capacity, increased diversity and interference suppression. Recent research in multiuser MIMO systems is aimed at suppression of interference so that the per user capacity will be closer to the capacity of a single user MIMO system. The focus of this paper is on spatial diversity techniques in a downlink wireless communication system, where a basestation (BS) could simultaneously serve multiple users without compromising available radio spectrum, see [2] for an introduction to multiuser downlink beamforming. This requires the BS to pre-compensate interference so that a particular user in the cell will not see the signals that are meant to be transmitted to other users. It is also possible for the BS to perform beamforming to suppress multi-user interference (MUI) to end users and to maximize overall capacity.

In an attempt to suppress MUI, several techniques have been proposed [2, 3, 4, 5, 6, 7]. One technique is to pre-process the signal at the BS so that MUI will be completely cancelled at the receiver for each user. Two such methods known as ”block-diagonalization” and ”successive optimization” have been proposed in [3]. However both these methods require the number of transmitting antennas to be greater than the sum of all receiving antennas of all users. Another approach proposed in [5] makes use of space-time block codes (STBC) to design a unitary precoder to cancel the co-channel interference. Once again this method requires a large number of antennas. A closed form solution is presented in [6] which is based on maximizing a lower bound for the product of signal-to-interference plus noise ratio (SINR). The algorithm achieves good performance but again it requires the number of transmitting antennas to be greater than the number of receiving antennas. All these schemes provide superior performance however they impose a restriction on the number of transmit antennas to be greater than the number of antennas of all users combined.

An iterative algorithm based on uplink-downlink duality was presented in [7], where the global optimum for the downlink beamforming is obtained for the case of single antenna at the receivers. However, in this paper, we adopt a signal to leakage criterion proposed in [1], but propose various techniques to improve the performance further. Even though, this family of algorithms is not supported by any known optimality criteria such as SINR and minimum mean square error (MMSE), we considered this criterion for its simplicity.

According to the approach in [1], the transmit weight vector for the i^{th} user will be determined by maximizing the transmit power to the i^{th} user while minimizing the interference (leakage) caused to all other users. However, instead of considering the interference at the output of the array of antennas of each user, we considered the interference at the output of the beamformer of each user. The rational behind this method is that the BS knows the set of beamformers that each user will eventually use, hence it can take advantage of this in the design process. We demonstrated the performance of the proposed method could be further improved by designing the transmit weight vectors using an iterative optimization approach.

The remaining sections are organized as follows. In section 2, some notational conventions are defined. In section 3, we define the MU-MIMO system model. In section 4, we propose the algorithm for designing beamformer weight vectors. In section 5, simulation results are presented. Finally, conclusion is drawn in section 6.

2. NOTATION

Some notational conventions are:

- Lower case letters denote scalars, upper case letters denote matrices and boldface denote vectors.
- Subscript $(\cdot)^H$ denotes Hermitian transpose.
- The operator $\mathrm{E}(\cdot)$ denotes expectation.
- The operator $\|\cdot\|$ denotes matrix norm.
- The operator $\lambda_{max}(\cdot,\cdot)$ denotes the largest generalised eigenvalue of a pair of matrices.
- Let $\mathbf{w}$ be a beamforming weight vector. Then $\mathbf{w}_i^l$ denotes the beamforming weight vector of the i^{th} user in the l^{th} iteration.
- The operator $\mathscr{P}(\cdot)$ returns the principal eigenvector of a matrix, that is the eigenvector corresponding to its maximal eigenvalue.

- The operator null$(\cdot)$ returns the eigenvectors, that are in the null space of a matrix.

3. SYSTEM MODEL

Consider a downlink MU-MIMO system consisting of one BS with N transmit antennas communicating with K users each having M_i receive antennas. A block diagram is shown in Figure 1, where $s_i(n)$ denotes the signal for i^{th} user at time n. The signal $s_i(n)$ is then multiplied by a beamformer weight vector $\mathbf{w}_i(n)$ before being transmitted over a multi-user channel. Hence, the N×1 transmitted signal vector at time n is given by

$$\mathbf{x}(n) = \sum_{k=1}^{K} \mathbf{w}_k s_k(n) \tag{1}$$

It is assumed that the data $s_k(n)$ and the beamformer weights $\mathbf{w}_k$ are normalized so that

$$E|s_k(n)|^2 = 1, \|\mathbf{w}_k\|^2 = 1 \quad for \;\; k = \{1, ..., K\}$$

The N×1 signal vector $\mathbf{x}(n)$ is then transmitted over a multi-user channel. Assuming the channel is frequency non-selective, the received signal vector $\mathbf{y}_i(n)$ for the i^{th} user at time n is written as

$$\mathbf{y}_i(n) = \mathbf{H}_i \sum_{k=1}^{K} \mathbf{w}_k s_k(n) + \mathbf{v}_i(n) \tag{2}$$

where, $\mathbf{v}_i(n)$ is the additive white Gaussian (AGW) noise vector. The channel $\mathbf{H}_i$ is assumed to be block fading. Assuming the i^{th} user employs M_i antennas, the $\mathrm{M}_i \times \mathrm{N}$ channel matrix can be written as

$$\mathbf{H_i} = \begin{bmatrix} h_i^{(1,1)} & \dots & h_i^{(1,\mathrm{N})} \\ \vdots & \ddots & \vdots \\ h_i^{(\mathrm{M}_i,1)} & \dots & h_i^{(\mathrm{M}_i,\mathrm{N})} \end{bmatrix} \tag{3}$$

where, $h_i^{(k,l)}$ denote the channel coefficient between the l^{th} transmit and k^{th} receive antennas for user i. We assume that the receiver for user i feedbacks the channel state information (CSI) $\mathbf{H}_i$ to the BS without any error.

4. ALGORITHMS

In the remaining sections, we will drop the time index n for notational simplicity. Hence, we can rewrite equation (2) as

$$\mathbf{y}_i = \mathbf{H}_i \mathbf{w}_i s_i + \sum_{k=1, k\neq i}^{K} \mathbf{H}_i \mathbf{w}_k s_k + \mathbf{v}_i \tag{4}$$

where the second term quantifies the interference caused to user i from all other users. The aim is to mitigate this interference for all users. Assume that the estimate of s_i for the i^{th} user is based on a max-ratio combining technique [1], i.e.,

$$s_i \triangleq \frac{\mathbf{w}_i^H \mathbf{H}_i^H \mathbf{y}_i}{\|\mathbf{H}_i \mathbf{w}_i\|^2} \tag{5}$$

where, “ H ” denotes Hermitian transpose. Then

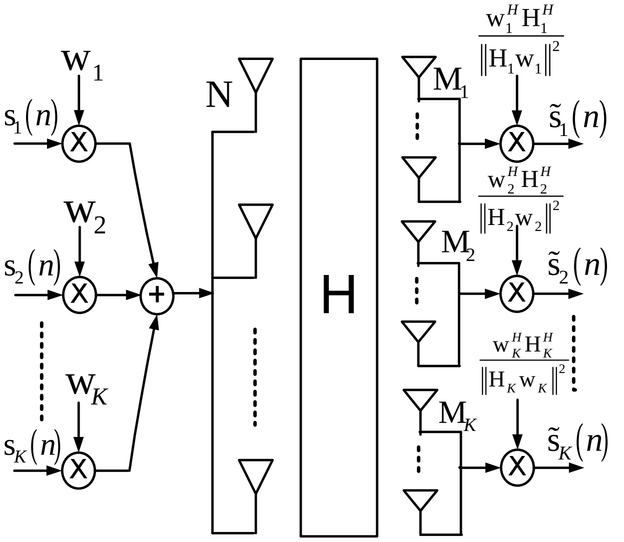

Figure 1: The block diagram for a multiuser MIMO system with N transmit antennas and K users, each equipped with M_i receive antennas

$$\tilde{s}_i = s_i + \frac{\mathbf{w}_i^H \mathbf{H}_i^H \sum_{k=1,k\neq i}^{K} \mathbf{H}_i \mathbf{w}_k s_k}{\|\mathbf{H}_i \mathbf{w}_i\|^2} + \frac{\mathbf{w}_i^H \mathbf{H}_i^H \mathbf{v}_i}{\|\mathbf{H}_i \mathbf{w}_i\|^2} \tag{6}$$

and the output signal-to-interference-noise ratio (SINR) for user i would be given by [7]

$$SINR_i = \frac{\|\mathbf{H}_i \mathbf{w}_i\|^2}{\sigma_i^2 + \frac{\sum_{k=1,k\neq i}^{K} \|\mathbf{w}_i^H \mathbf{H}_i^H \mathbf{H}_i \mathbf{w}_k\|^2}{\|\mathbf{H}_i \mathbf{w}_i\|^2}} \tag{7}$$

The power of the desired signal in (4) is given by $\|\mathbf{H}_i \mathbf{w}_i\|^2$. Similarly the interference caused by the i^{th} user to the k^{th} user is given by $\|\mathbf{H}_k \mathbf{w}_i\|^2$. The quantity, called leakage for user i, as the total power leaked from this user to all other users is defined in [1] as:

$$\sum_{k=1,k\neq i}^{K} \|\mathbf{H}_k \mathbf{w}_i\|^2 \tag{8}$$

4.1 The signal to Leakage Ration Algorithm [1]

Given a fixed transmit power for each user, the weight vectors $\mathbf{w}_i$, $i = 1, ..., K$, are designed such that the signal-to-leakage ratio (SLR) is maximized for every user [1]

$$\mathbf{w}_i^o = \arg\max_{w_i} \underbrace{\frac{\|\mathbf{H}_i \mathbf{w}_i\|^2}{\sum_{k=1,k\neq i}^{K} \|\mathbf{H}_k \mathbf{w}_i\|^2}}_{SLR \; for \; user \; i} \quad s.t. \; \|\mathbf{w}_i\|^2 = 1, i = \{1, ...K\} \tag{9}$$

By denoting $\tilde{\mathbf{H}}_i = [\mathbf{H}_i^H ... \mathbf{H}_{i-1}^H \mathbf{H}_{i+1}^H ... \mathbf{H}_K^H]^H$ as an extended channel matrix that excludes the channel $\mathbf{H}_i$, the SLR for user i can be written as

$$SLR_i = \frac{\|\mathbf{H}_i \mathbf{w}_i\|^2}{\|\tilde{\mathbf{H}}_i \mathbf{w}_i\|^2} = \frac{\mathbf{w}_i^H \mathbf{H}_i^H \mathbf{H}_i \mathbf{w}_i}{\mathbf{w}_i^H \tilde{\mathbf{H}}_i^H \tilde{\mathbf{H}}_i \mathbf{w}_i} \tag{10}$$

The above equation can be solved using the Rayleigh-Ritz quotient result [8],

$$\frac{\mathbf{w}_i^H\mathbf{H}_i^H\mathbf{H}_i\mathbf{w}_i}{\mathbf{w}_i^H\tilde{\mathbf{H}}_i^H\tilde{\mathbf{H}}_i\mathbf{w}_i} \le \lambda_{max}(\mathbf{H}_i^H\mathbf{H}_i, \tilde{\mathbf{H}}_i^H\tilde{\mathbf{H}}_i) \tag{11}$$

where λ_{max} is the largest generalized eigenvalue of the matrix pair $\mathbf{H}_i^H\mathbf{H}_i$ and $\tilde{\mathbf{H}}_i^H\tilde{\mathbf{H}}_i$. The equality holds only if $\mathbf{w}_i$ is proportional to the generalized eigenvector corresponding to the largest generalized eigenvalue, i.e.,

$$\mathbf{w}_i^o \propto \mathscr{P}(\mathbf{H}_i^H\mathbf{H}_i, \tilde{\mathbf{H}}_i^H\tilde{\mathbf{H}}_i) \tag{12}$$

where $\mathscr{P}\{\cdot\}$ is the principal eigenvector of the matrix, that is the eigenvector corresponding to its maximal eigenvalue. The proportionality constant is chosen to normalize the norm of $\mathbf{w}_i^o$ to unity. If $\tilde{\mathbf{H}}_i^H\tilde{\mathbf{H}}_i$ is invertible, then the generalized eigenvalue problem reduces to:

$$\lambda_{max}(\mathbf{H}_i^H\mathbf{H}_i, \tilde{\mathbf{H}}_i^H\tilde{\mathbf{H}}_i) = \lambda_{max}((\tilde{\mathbf{H}}_i^H\tilde{\mathbf{H}}_i)^{-1}(\mathbf{H}_i^H\mathbf{H}_i)) \tag{13}$$

and $\mathbf{w}_i^o$ is the eigenvector corresponding to the largest eigenvalue of $(\tilde{\mathbf{H}}_i^H\tilde{\mathbf{H}}_i)^{-1}(\mathbf{H}_i^H\mathbf{H}_i)$.

4.2 The proposed Algorithm

The method proposed in [1] considers the interference present at the output of the array of antennas of each user in the design process. However, we observed consideration of the interference present at the beamformer output instead of the output of the array of antennas substantially improves the overall BER performance. This is possible in the design, as the BS knows the beamformer vectors that will be eventually used by all users, as it knows the forward channel of all users. The proposed design is based on an iterative optimization approach.

4.2.1 First Iteration

In the first iteration the SLR considered for user 1 will be same that in [1]

$$SLR_1 = \frac{\mathbf{w}_1^H\mathbf{H}_1^H\mathbf{H}_1\mathbf{w}_1}{\mathbf{w}_1^H\tilde{\mathbf{H}}_1^H\tilde{\mathbf{H}}_1\mathbf{w}_1} \tag{14}$$

Similarly to (12) the solution to maximizing (14) is given by the Rayleigh-Ritz quotient result. Then the beamformer weight for user 1 is given by

$$\mathbf{w}_1^1 = \mathscr{P}(\mathbf{H}_1^H\mathbf{H}_1, \tilde{\mathbf{H}}_1^H\tilde{\mathbf{H}}_1) \tag{15}$$

In order to compute weight vector for user 2 we use the fact that the BS knows the beamformer weight vector for user 1. Hence the channel from the signal $s_2(n)$ at the BS to the output of the beamformer of user 1 can be written as $\theta_1^{\mathbf{H}}\mathbf{H}_1\mathbf{w}_2$, where $\theta_1 = \mathbf{H}_1\mathbf{w}_1$ is the required beamformer for user 1. Therefore the interference power caused by user 2 to user 1 can be written as $\mathbf{w}_2^{\mathbf{H}}\mathbf{H}_1^{\mathbf{H}}\theta_1\theta_1^{\mathbf{H}}\mathbf{H}_1\mathbf{w}_2$ instead of $\mathbf{w}_2^{\mathbf{H}}\mathbf{H}_1^{\mathbf{H}}\mathbf{H}_1\mathbf{w}_2$. We therefore replace the term $\mathbf{H}_1^H\mathbf{H}_1$ in the denominator of (10) with a rank 1 matrix $\mathbf{H}_1^{\mathbf{H}}\theta_1\theta_1^{\mathbf{H}}\mathbf{H}_1 = \mathbf{H}_1^{\mathbf{H}}\mathbf{R}_1\mathbf{H}_1$. We can now define the SLR for user 2 as

$$SLR_2 = \frac{\mathbf{w}_2^H\mathbf{H}_2^H\mathbf{H}_2\mathbf{w}_2}{\mathbf{w}_2^H(\mathbf{H}_1^H\mathbf{R}_1\mathbf{H}_1 + \sum_{j=3}^K\mathbf{H}_j^H\mathbf{H}_j)\mathbf{w}_2} \tag{16}$$

Similarly we can generalize the above equation for the i^{th} user in the first iteration as

$$SLR_i = \frac{\mathbf{w}_i^H\mathbf{H}_i^H\mathbf{H}_i\mathbf{w}_i}{\mathbf{w}_i^H\underbrace{\left(\sum_{j=1}^{i-1}\mathbf{H}_j^H\mathbf{R}_j\mathbf{H}_j + \sum_{j=i+1}^{K}\mathbf{H}_j^H\mathbf{H}_j\right)}_{\mathbf{P}}\mathbf{w}_i} \tag{17}$$

Therefore in the first iteration, the beamforming weight vector for users excluding user 1 are computed according to the following pseudocode,

```
if
  rank $(\mathbf{P}) \ge N$;
then
  $\mathbf{w}_i^1$ = $\mathscr{P}(\mathbf{P})$;
else
  $\mathbf{w}_i^1$ = null $(\mathbf{P})$;
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where null$(\mathbf{P})$ denotes the eigenvectors, that are in the null space of $\mathbf{P}$.

4.2.2 Other Iterations

After the first iteration we have a set of beamformer weight vectors for all users. We can now use these weight vectors to carry out further iterations. Carrying out iterations will force the beamformer weight vectors to converge to a set of weight vectors which will result in further reduction of CCI. In the l^{th} iteration, the SLR for user i is determined as

$$SLR_i^l = \frac{\mathbf{w}_i^{(l)H}\mathbf{H}_i^H\mathbf{H}_i\mathbf{w}_i^{(l)}}{\mathbf{w}_i^{(l)H}\underbrace{\left(\sum_{j=1,j\neq i}^{K}\mathbf{H}_j^H\mathbf{H}_j\mathbf{w}_j^{(l-1)}\mathbf{w}_j^{(l-1)H}\mathbf{H}_j^H\mathbf{H}_j\right)}_{\mathbf{Q}}\mathbf{w}_i^l} \tag{18}$$

where $\mathbf{w}^{(l-1)}$ is the weight vector obtained in the previous iteration i.e. $l-1$. If the total number of antennas N at the BS is not less than the total number of users K, the beamformer weight vector for the i^{th} user will be computed according to

$$\mathbf{w}_i^l = null(\mathbf{Q}) \tag{19}$$

For $\mathbf{P}$ in (17) and $\mathbf{Q}$ in (18), it is possible to have a null space of dimension greater than one. In this case, the beamformer weight vector should be chosen as a linear combination of all null vectors. The determination of the optimum combination in the sense of maximizing power transferred to the desired user is also an eigenvector problem. The linear combination coefficients are given by the eigenvector (denoted by $\mathbf{g}_k$) corresponding to the largest eigenvalue of $(\mathbf{A}_k^H\mathbf{H}_k^H\mathbf{H}_k\mathbf{A}_k)$

where $\mathbf{A}_k$ is a matrix containing all the null vectors of $\mathbf{P}$ or $\mathbf{Q}$ for user k, and $\mathbf{H}_k$ is the channel matrix of user k. The weight vector for user k is obtained as

$$\mathbf{w}_i^l = \mathbf{A}_k\mathbf{g}_{\mathbf{k}} \tag{20}$$

5. SIMULATION RESULTS

We considered a multi-user MIMO system with one BS equipped with N antennas and K users each equipped with M_i antennas. The data symbols are generated using QPSK modulation. The total transmitted power per symbol period across all transmit antennas is normalized to unity. The entries of channel $\mathbf{H}$ are zero mean iid Gaussian random variables with unity variance and generated independently for each transmission symbol. The noise is zero mean and spatially and temporally uncorrelated, i.e:

$$\mathbf{E}\{v_i v_i^H\} = \sigma_i^2 \mathbf{I}_{M_i}$$

$$\mathbf{E}\left\{\mathbf{tr}(\mathbf{H}_i\mathbf{H}_i^H)\right\} = M_i N$$

The Figure 2, depicts the difference between SLR and the proposed algorithm for the first iteration only. The result shows the BER performance of all the users using both SLR and the proposed algorithm. We have considered the case with 6 transmitting antennas and 5 users each with 3 receiving antennas. It can be seen that the SLR produces the same BER performance for all users as expected. On the other side, we note that the proposed algorithm produces different BER performance for all users. This is due to the fact that when the weights for user i is obtained by maximizing the signal to leakage ratio, it tends to reduce the leakage to all the other users. However, since we use the weight vectors of users 1 to $i-1$ in the design of weight vector of user i, users 1 to $i-1$ are more likely to benefit in terms of interference suppression rather than users $i+1$ to K. In order to gain from this effect we are encouraged to carry out further iterations. Also carrying out further iterations insures that the average BER of all users is the same and therefore guarantees the same QoS for all users.

Similarly, in Figure 3, the results presented consider the case with 6 transmitting antennas and 5 users each with 3 receiving antennas each. But in this figure the average BER of all users is depicted, but for various number of iterations. We note that the performance is greatly improved when the number of iterations is increased. But the performance converges roughly around 20 iterations and further iterations have marginal improvement on the BER performance.

To understand the proposed algorithm better, we look at the SINR outage (or cumulative distribution function), which is plotted to show and compare the distribution of the SINR achieved at the output of the receiver, which is given by (7). Figure 4 and 5 show the SINR outage for the proposed algorithm as compared to the SINR outage of [1] and the conventional single user beamforming solution [9]:

$$\mathbf{w}_i^o \propto \mathscr{P}\left(\mathbf{H}_i^H\mathbf{H}_i\right) \qquad (21)$$

In Figure 4 the proposed algorithm achieves SINR of larger than 20dB for 80% of the channel realizations at an SNR of 10 dB. Similarly in figure 5 the proposed algorithm achieves SINR of larger than 15dB for 90% of the channel realizations at an SNR of only 5 dB. Where as both SLR and conventional beamforming have relatively poor outage performances.

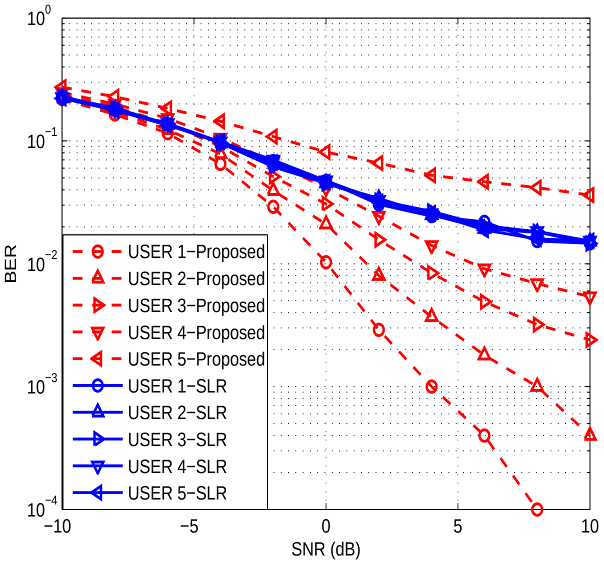

Figure 2: The BER performance for all the USERS is plotted as a function of the signal to noise ratio (SNR) for a Multiuser MIMO system with N = 6 transmit antennas and K = 5 users, each equipped with M_i = 3 receive antennas.

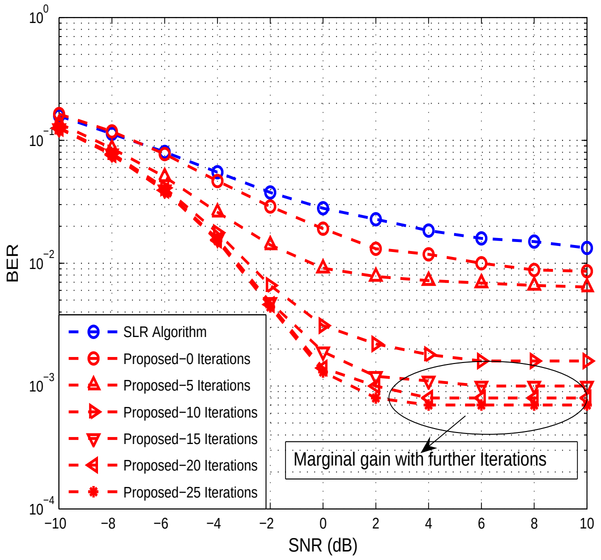

Figure 3: The average BER performance is plotted for SLR and the proposed algorithm for various iterations as a function of signal to noise ratio (SNR) for a Multiuser MIMO system with N = 6 transmit antennas and K = 5 users, each equipped with M_i = 3 receive antennas.

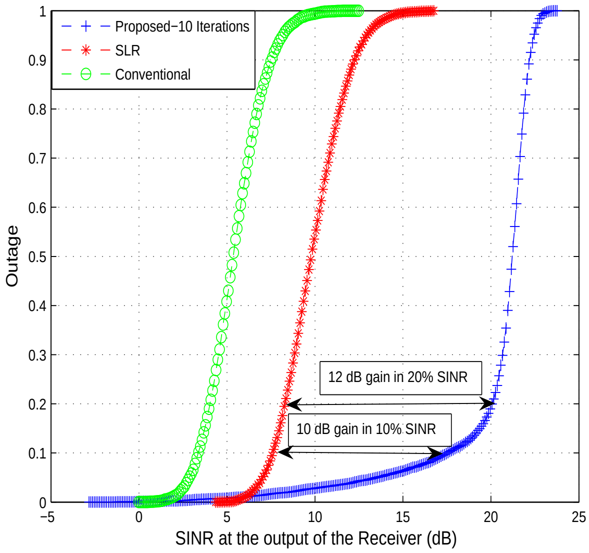

Figure 4: SINR outage for conventional beamforming, SLR and the proposed algorithm at the 10^{th} iteration at a signal to noise ratio (SNR) of 10 dB for a Multiuser MIMO system with N = 6 transmit antennas and K = 5 users, each equipped with $M_i = 3$ receive antennas.

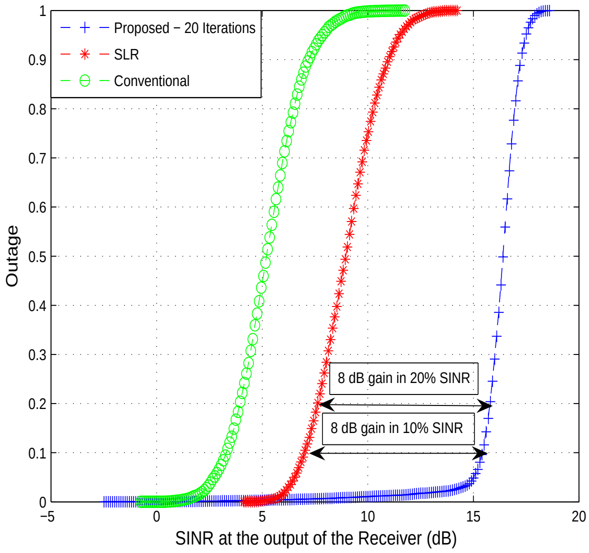

Figure 5: SINR outage for conventional beamforming, SLR and the proposed algorithm at the 20^{th} iteration at a signal to noise ratio (SNR) of 5 dB for a Multiuser MIMO system with N = 6 transmit antennas and K = 5 users, each equipped with $M_i = 3$ receive antennas.

6. CONCLUSIONS

We proposed modifications to a recently proposed signal to leakage based design for multi user beamformers. Our method explicitly considered the interference present at the beamformer output instead of the interference present at the output of an array of antennas of any user. We demonstrated a significant improvement in the BER performance using the proposed modifications. To further increase the performance, we proposed an iterative optimization approach which also guarantees a lower error floor and equal BER performance for all users.

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