ML SYMBOL SYNCHRONIZATION FOR GENERAL MIMO-OFDM SYSTEMS IN UNKNOWN FREQUENCY-SELECTIVE FADING CHANNELS


UMOP/GESTE CNRS FRE 2701, University of Limoges/ENSIL
16, rue Atlantis, Parc ESTER, 87068 Limoges cedex, France
{saemi, meghdadi, cances, zahabi, dumas}@ensil.unilim.fr

ABSTRACT
This paper proposes a new synchronization algorithm for a general MIMO-OFDM system over frequency selective fading channels, based on maximum-likelihood principle. By knowing the exact place of the training sequence packet, we are able to align the start of FFT window within the OFDM symbol. Symbol timing joint with channel estimation is performed in this paper in order to find the exact place of the beginning of the training sequence. In addition, loss in system performance due to synchronization error is derived for MIMO systems and used as a performance criterion. We provide simulation results to illustrate that the performance of the proposed algorithm is quite satisfying.

1. INTRODUCTION
The technique of Multiple Input Multiple Output (MIMO) combined with Orthogonal Frequency Division Multiplexing (OFDM) in communication systems for multimedia applications have gained considerable interest in recent years. Although OFDM is well known for its ability to combat inter-symbol interference (ISI) introduced by multipath channels, incorrect positioning of the FFT window within an OFDM symbol reintroduces ISI during data demodulation, causing serious performance degradation [1]. Symbol synchronization is therefore one of the important tasks that should be performed at the receiver. A number of methods for OFDM symbol synchronization have been proposed in the literature [1]-[16]. A lot of them are based on preamble repetition scheme introduced by Moore [2]. He described a technique to estimate frequency offset by using a repeated training sequence and derived maximum likelihood estimation procedure. Then, this idea was employed for time synchronization by Schmidl [3], Speth [1] and Keller and Hanzo [4]. The principle of exploiting a double training sequence preceded by a cyclic prefix (CP) for frame synchronization was originally suggested for single-carrier transmission in [5]. Weinfurther in [6] introduced and compared several metrics to detect repeated preamble in a received signal for frame synchronization. A simple MIMO extension of Schmidl’s algorithm was proposed in [7], and another MIMO extension of repeated preamble algorithm was utilized by Zelst [8]. But all of these algorithms have one disadvantage in common. As a matter of fact, these algorithms use auto-correlation criterion that results in a plateau during cyclic prefix [16],[9]. Therefore, the precise packet arrival time is hard to estimate and a symbol timing procedure is necessary. Some authors propose the use of periodic structure of cyclic prefix in OFDM symbols [10],[11], but these kinds of algorithms can not be extended over MIMO systems easily.

On the other hand, there are some symbol synchronization techniques that are specifically designed for IEEE 802.11a WLANs standard [12]-[15]. Although some of these algorithms like [15], work well but they have benefited from the structure of this standard so they can not be applied to a generic OFDM system and they have been designed for SISO systems.

Simple correlator can be easily implemented at the receiver, but its performance is poor in dispersive channels [17], indicating that more sophisticated synchronization algorithms are required. This paper is our effort to establish a more complex time synchronization algorithm which elaborates on ideas from [15]. In fact we extend the method of [15], to develop a maximum-likelihood (ML) symbol synchronizer for a general MIMO-OFDM system on frequency selective channels. To the best of our knowledge, there is no published paper that proposes ML symbol synchronization/channel estimation for MIMO-OFDM systems.

The rest of this paper is as follows. In section 2, we describe our signal model for which the ML algorithm is tailored. Section 3 introduces the ML symbol timing algorithm. The loss in system performance due to synchronization error is defined as the performance criterion, this criterion was used in [15] based on the ideas of [11],[20]. We extend this criterion over MIMO systems in section 4. Section 5 presents the simulation results and finally conclusions are drawn in section 6.

2. SIGNAL MODEL
Consider a MIMO-OFDM communication system with $N_t$ transmit and $N_r$ receive antennas. At each receiving antenna, a superposition of faded signals from all the transmit antennas plus noise is received.

Let the baseband equivalent signal of preamble at $n^{th}$
transmitter be \( s_n(t) \), and \( h_{nm}(t) \) be the equivalent channel between \( n \)th transmit antenna and \( m \)th receive antenna. The received signal \( r_m(t) \) at \( m \)th receive antenna is sampled at \( t = kT_s + lT_r \), where \( T_s \) is the sampling time, and \( e_0 \in \{0,1\} \) is the unknown time offset induced by the combination of the channel first path delay and the sampling phase offset. According to [18], if the equivalent channel bandwidth \( (B_s) \) satisfies: \( B_s < 1/T_r - B_t \) (where \( B_t \) is the bandwidth of \( s(t) \)), then by the equivalence of digital and analog filtering for band-limited signals, the sampled received signal can be expressed as:

\[
r_{m,k} = \sum_{n=1}^{N} \sum_{l=0}^{L-1} s_n(kT_s - lT_r) h_{nm}(lT_r) + w_{m,k}
\]

where \( r_{m,k} \triangleq r_m(kT_s + e_0 T_r) \), \( w_{m,k} \triangleq w_m(kT_r + e_0 T_r) \) and \( w_m(t) \) is the additive stationary Gaussian noise at \( m \)th receive antenna which is independent of the other antennas.

The fading multi-path channel is considered to be quasi static. Supposing a channel with length \( L \), we have \( N_s \times N_r \) paths, each of which can be modeled by an equivalent FIR complex filter of order \( L - 1 \) with \( h_{nm}(l) = h_{nm}(lT_r) \) as the taps with \( l = 0, 1, ..., L - 1 \). These taps are assumed to be independent zero mean complex Gaussian random variables with variance \( 1/2P(l) \) per dimension. The ensemble \( P(l) \) with \( l = 0, 1, ..., L - 1 \) is called the power delay profile (PDP) of the channel and its total power is assumed to be normalized to \( \sigma_n^2 = 1 \), which is the average channel attenuation. Therefore (1) can be written as:

\[
r_{m,k} = \sum_{n=1}^{N} \sum_{l=0}^{L-1} s_n(kT_s - lT_r) h_{nm}(lT_r) + w_{m,k}
\]

Let \( r_{m,k} \) be a received-signal vector of size \( N \) sampled from time \( k \) to \( N + k - 1 \) at \( m \)th antenna:

\[
r_{m,k} = \begin{bmatrix} r_{m,k} & r_{m,k+1} & \cdots & r_{m,k+N-1} \end{bmatrix}_{N \times 1}
\]

Suppose the training sequence of \( n \)th transmit antenna is \( \begin{bmatrix} c_{n,0} & c_{n,1} & \cdots & c_{n,N_p-1} \end{bmatrix} \) where \( N_p \) is the length of training sequence. If the length of cyclic prefix for OFDM symbol is \( N_{CP} \), we have \( \begin{bmatrix} c_{n,N_p-N_{CP}} & \cdots & c_{n,N_p-1} \end{bmatrix} \) as the cyclic prefix of training sequence. For \(-N_{CP} \leq k \leq N_p - N \) we have:

\[
r_{m,k} = \sum_{n=1}^{N} \begin{bmatrix} c_{n,0} & \cdots & c_{n,k+N_p-1} \end{bmatrix} h_{nm} + w_{m,k}
\]

where:

\[
C_{n,k} = \begin{bmatrix} c_{n,0} & c_{n,1} & \cdots & c_{n,k+N_p-1} \\
c_{n,k+N_p-N_{CP}} & c_{n,N_p-N_{CP}+1} & \cdots & c_{n,k+N_p-N_{CP}+N_p-1} \\
c_{n,k+N_p-N_{CP}+N_p} & c_{n,N_p-N_{CP}+N_p+1} & \cdots & c_{n,k+N_p-N_{CP}+N_p+N_p-1} \\
\vdots & \vdots & \ddots & \vdots \\
c_{n,k+N_p-N_{CP}-1} & c_{n,N_p-N_{CP}-N_{CP}} & \cdots & c_{n,k+N_p-N_{CP}-N_{CP}+N_p-1} \end{bmatrix}_{N \times N_{CP}}
\]

\[
h_{nm} = [h_{nm}(0), h_{nm}(1), \ldots, h_{nm}(L-1)]^T_{L \times 1}
\]

and \( w_{m,k} \) is a column vector containing the noise samples at the \( m \)th receive antenna with covariance matrix \( \sigma_n^2 I_N \) (\( I_N \) is the \( N \times N \) identity matrix).

Stacking the received vectors from all the \( N_r \) receive antennas and using Kronecker product, we have:

\[
r_{obs} = (I_N \otimes C) h_0 + w_k
\]

where:

\[
r_k = \begin{bmatrix} r_{1,k}^T & r_{2,k}^T & \cdots & r_{N_r,k}^T \end{bmatrix}_{N \times N_r}
\]

\[
C_k = \begin{bmatrix} C_{1,k} & C_{2,k} & \cdots & C_{N_r,k} \end{bmatrix}_{N \times N_r}
\]

\[
h_k = \begin{bmatrix} h_{1,k}^T & h_{2,k}^T & \cdots & h_{N_r,k}^T \end{bmatrix}_{N \times N_r}
\]

\[
w_k = \begin{bmatrix} w_{1,k}^T & w_{2,k}^T & \cdots & w_{N_r,k}^T \end{bmatrix}_{N \times N_r}
\]

**3. ML SYNCHRONIZER**

We assume that the arrival of the preamble can be identified by detecting the received signal energy in the frame synchronization procedure (e.g. using the method in [2]-[8] or a coarse cross correlation). It means that, the coarse frame synchronization has succeeded to detect a packet arrival but the precise beginning is not known. This could be due to dispersive effect of the propagation channel and the use of a simple correlator algorithm.

Suppose the beginning of the preamble, as shown in Figure 1, is chosen as the time reference, i.e. \( k = 0 \). The receiver takes a signal vector of size \( N \) from estimated instant in frame synchronization procedure as the observation vector \( (r_{obs}) \). Assume \( k_0 \) as time offset of observation vector with respect to beginning of the training sequence (Figure 1). We have to perform symbol timing in order to find the exact place of the beginning of the training sequence packet. By knowing the exact place of the training sequence packet, we are able to align the start of FFT window within the OFDM symbol. The received observation vector, \( r_{obs} \), begins from \( k_0 \) sample from the beginning of the training sequence packet. The objective of our algorithm is to estimate \( k_0 \) from the observation vector \( r_{obs} \).

Suppose \( (k_0 \in [-N_{CP}, N_p - N]) \), from equation (7), the joint ML estimate of \( k_0 \) and \( h_0 \) can be obtained by maximizing:
\[ p(r_{\text{obs}} | k, h) = \frac{1}{(2\pi\sigma^2)^{N}} \exp \left\{ \frac{(r_{\text{obs}} - (I_N \otimes C_k)h)^T}{\sigma^2} \right\} \] (12)

or equivalently minimizing the following metric:

\[ J(r_{\text{obs}} | k, h) = (r_{\text{obs}} - (I_N \otimes C_k)h)'^HY^H \begin{pmatrix} \Sigma_I \end{pmatrix} - (I_N \otimes C_k)h \] (13)

where \( k \) and \( h \) are the trial values for \( k_0 \) and \( h_0 \) respectively. Setting the partial derivatives of \( J(r_{\text{obs}} | k, h) \) with respect to \( h \) to zero, we obtain the ML estimate for \( h_0 \) (when \( k \) is fixed) as [19]:

\[ \hat{h} = [(I_N \otimes C_k)HY(H \otimes C_k)]^{-1}(I_N \otimes C_k)Yr_{\text{obs}} \] (14)

In order that this equation leads to unique solution, the number of observation points must be more than the number of unknown channel coefficients: \( N \times N > L \times N \times N \). Substituting equation (14) into equation (13), after some straightforward manipulations and dropping the irrelevant terms, the delay \( k_0 \) can be estimated by maximizing the following likelihood function:

\[ \Psi(k) = r_{\text{obs}}'^HY(I_N \otimes C_k)HY(I_N \otimes C_k)^{-1}(I_N \otimes C_k)^{-1}(I_N \otimes C_k)r_{\text{obs}} \] (15)

Using the well-known properties of the Kronecker product \((A \otimes B)^{-1} = A'^{-1} \otimes B'^{-1}\), \((A \otimes B)^{-1} = A'^{-1} \otimes B'^{-1}\) and \((A \otimes B)(C \otimes D) = (AC) \otimes (BD)\), we have:

\[ \Psi(k) = r_{\text{obs}}'^HY(I_N \otimes C_k)(C_k'^{-1}C_k^{-1})HYr_{\text{obs}} = \sum_{m=1}^{N} r_{m,\text{obs}}'^HY(C_k'^{-1}C_k^{-1})HYr_{m,\text{obs}} \] (16)

where \( r_{m,\text{obs}} \) is the observation vector at \( m \)th receive antenna. The delay \( k_0 \) from the received signal vector \( r_{\text{obs}} \) can be estimated as follows:

\[ \hat{k}_0 = \arg \max_k \Psi(k) \] (17)

It is worth to mention that:

- In our algorithm there is no constraint over structure of training sequence. It means that we can use only one preamble or two consecutive preambles or any alternative structure.
- The cyclic prefix for preamble can be utilised or not. The only condition is \((k_0 \in [\{-N_{CP}, N_{P} - N\}]\). This interval is justified by the fact that the size of multipath channel is less than the cyclic prefix size. Though, by choosing \( N \) less than \( N_{P} + N_{CP} \) we have a margin with length \( N_{P} + N_{CP} - N \) for detecting of the arrival of the preamble.
- The orthogonality between the preambles of different antennas is not required.
- In the implementation of the algorithm the result of \( C_k'^{-1}C_k^{-1} \) for different values of \( k \) can be stored in a lookup table. So for symbol timing of each packet using (16) we need to \( N_f(N_f^2+N) \) multiplications and \( N_f(N_f^2-1) \) sum.

4. SYMBOL SYNCHRONIZATION PERFORMANCE CRITERION

In a Rayleigh multipath fading channel, the channel may contain some small taps at the beginning, so the starting position of the channel is not clear. It can be defined in so many ways: as the first non-zero tap of the channel, as the first tap with energy larger than a certain threshold, as the position of the strongest path or any other definition. In MIMO case the problem is even more complicated because we have several different paths. Therefore, the symbol boundary of a received OFDM symbol is not well defined. Even if we choose one of the above definitions as the reference position, there is no guarantee that a certain synchronization algorithm giving estimates close to the reference position would provide good performance in OFDM systems. Moreover in OFDM systems, due to the existence of cyclic prefix, some timing offset can be tolerated as long as the samples within the FFT window are influenced by only one transmitted OFDM symbol. Therefore the criterion that the synchronization error has to be within certain limits of a fixed reference point is not an appropriate performance measure for OFDM systems in frequency selective fading channels. Hence, according to [15] we use the loss in system performance due the synchronization error as a criterion.

With reference to Figure 2, supposing that the fast Fourier transform (FFT) window starts at position \( k_x \), the signal at the sub-carrier \( p \) after FFT operation has been written in [20] for SISO case. Using the linearity properties we extend it over MIMO system, so the signal at the sub-carrier \( p \), at the \( m \)th receiver, after FFT operation, \( z_{m,p} \), can be written as:

\[ z_{m,p} = e^{j2\pi(p/N_f)p}\sum_{n=1}^{N_f} a_m(k_p)H_{m,n,p} + l_{m,p} + \eta_{m,p} \] (18)

where \( a_m(p) \) is the transmitted data from \( n \)th transmit antenna at sub-carrier \( p \), \( H_{m,n,p} \) is the channel transfer function between \( n \)th transmit and \( m \)th receive antenna at sub-carrier \( p \), \( \eta_{m,p} \) is the noise sample at sub-carrier \( p \) and \( m \)th receive antenna. \( N_f \) is the number of FFT points in the OFDM system. \( a_m(k_p) \) is the attenuation caused by the synchronization error, which is approximated for SISO in [20] and we use it for MIMO system as well:

\[ a_m(n_{x}) = \sum_{l=0}^{k_x-1} h_{m,l} \left( I_{FFT} - \Delta \epsilon \right) \] (19)

where:

\[ \Delta \epsilon = \begin{cases} k_x - l & k_x > l \\ (N - k_x) & k_x < -(N - l) \\ 0 & \text{otherwise} \end{cases} \] (20)

and \( l_{m,p} \) is the ISI plus inter-carrier interference (ICI) term at sub-carrier \( p \) and \( m \)th receive antenna because of timing
offset that can be well approximated by Gaussian noise with power [20]:

\[
\sigma^2_{\text{off}}(k_e) = \sum_{n=1}^{N} \sum_{l=0}^{L-1} |h_{n,l}(l)|^2 \left( 2 \frac{\Delta E_f}{N_{\text{FFT}}} \right)^2 \tag{21}
\]

The signal to interference-plus-noise ratio (SINR) per receiver can be written as:

\[
\text{SINR}(k_e) = \frac{1}{N_r} \sum_{m=1}^{N_r} \sum_{l=1}^{L} |\hat{a}_{n,p} H_{m,p}|^2 \left( \frac{\Delta E_f}{N_{\text{FFT}}} \right)^2 \tag{22}
\]

We can rewrite also SINR by using (18):

\[
\text{SINR}(k_e) = \frac{1}{N_r} \sum_{m=1}^{N_r} \sum_{l=1}^{L} \left( |\hat{a}_{n,p} H_{m,p}|^2 - \frac{\sigma^2_{\text{off}}(k_e)}{\sigma^2_{\text{off}}(k_e) + \sigma^2_u} \right) \tag{23}
\]

When we know the channel coefficients, which is the ideal case, we can calculate \( k_e \) of the ideal MIMO synchronizer by maximizing SINR. Note that for each of receive antennas \((m, \tau)\), \( |\hat{a}_{n,p} H_{m,p}|^2 \) is independent of \( m \) and \( p \) and thus we can maximize the following term to maximize SINR.

\[
\Gamma(k_e) = \frac{1}{N_r} \sum_{m=1}^{N_r} \frac{1}{\sigma^2_{\text{off}}(k_e) + \sigma^2_u}
\]

Using the equations (23), (19)-(21) we can calculate \( k_e \) of the ideal MIMO synchronizer numerically by maximizing \( \Gamma(k_e) \). The ideal symbol synchronizer can be served as a reference to our practical synchronization algorithms. For a particular realization of channel, let \( k_e \) be the start of FFT window estimated by our symbol synchronization algorithm and \( k_d \) be that of the ideal symbol synchronizer. Then the loss of SINR, defined as the ratio of SINR obtained from the ideal symbol synchronizer to that from non-ideal synchronizer is given by:

\[
\text{SINR}_{\text{loss}}(k_e) = \frac{\text{SINR}_{\text{ideal}}(k_e)}{\text{SINR}(k_e)}
\]

Noting that \( |\hat{a}_{n,p} H_{m,p}|^2 \) is independent of \( m \) and \( n \) and

\[
\frac{1}{N_r} \sum_{m=1}^{N_r} \left| H_{m,p} \right|^2 = \frac{1}{L} \sum_{l=1}^{L} h_l^2 \tag{22}
\]

we have:

\[
\text{SINR}_{\text{loss}}(k_e) = \frac{1}{N_r} \sum_{m=1}^{N_r} \frac{\sum_{l=1}^{L} a_{n,p}^2 \sigma^2_{\text{off}}(k_e) + \sigma^2_u}{\sum_{l=1}^{L} \sigma^2_{\text{off}}(k_e) + \sigma^2_u}
\]

As in [15] and [11] we assume that a synchronization failure happens when the probability of loss in SINR is greater than a certain thresholds, that is:

\[
P_f(\Delta \gamma) = P(10 \log_{10}(\text{SINR}_{\text{loss}}) > \Delta \gamma) \tag{27}
\]

where \( P_f(\Delta \gamma) \) is the probability of synchronization failure when the tolerable system degradation (in dB) is \( \Delta \gamma \).

5. SIMULATION RESULTS

We provide a few Monte Carlo simulation results to illustrate the effectiveness of our new algorithm. In all of the simulations, the channel between a certain transmit and receive antenna is modelled using an exponentially decaying power delay profile (PDP) with independent Rayleigh fading on every tap according to section II. We assume that each channel has \( L = 15 \) Gaussian distributed coefficients with a mean power of \( \sigma^2 = \sigma^2_0 \exp(-lT_s/\tau_{\text{max}}) \) \( l=1,\ldots,L \) where \( T_s \) similar to IEEE 802.11a standard is 50 nS, and and \( \sigma^2_0 \) is such that \( \sum_{l=1}^{L} \sigma^2_l = 1 \). The channel is fixed during transmission of one packet and independent of that of another packet. Furthermore it is assumed that the length of FFT window or the OFDM symbol similar to IEEE 802.1a is 64 and there is a cyclic prefix for each OFDM symbol with length 16. We use a \( 4 \times 4 \) MIMO system and it is assumed that the average total TX power \( P \) is distributed among the TX antennas such that \( \sigma^2_i = P/N_i \). The SNR per receive antenna is \( P/\sigma^2_i = N_i \sigma^2_0/\sigma^2_i \). As preamble, we use a single sequence with the length of an OFDM symbol and according to IEEE802.11, we use a cyclic prefix of length 32 for our preamble. The preamble is generated randomly for each transmit antenna. Our sampling vector length, \( N_s \), is 64. So, according to section 3, the index \( k \) in (17) belongs to \([32,0]\) and also \( k_0 \) in equation (12) is treated as a uniform random variable over this interval and value of \( k_0 \) was randomly generated in each iteration. For each simulation run, the loss of SINR is calculated using (26), where the ideal symbol synchronizer uses \( k_d \) such that (24) is maximized. Each point of result is obtained by averaging over 10° Monte-Carlo runs. Figure 3 represents the result of [15] and [14] for IEEE 802.11a specification SISO system. The channel models are considered with 100nS and 30nS delay spread respectively. Figure 4 presents our simulation results for a \( 4 \times 4 \) MIMO system, but in comparison with Figure 3 we have considered the channel models with 100 and 150 nS delay spread and the curve shows the \( P_f(0.05dB) \) in contrast with \( P_f(0.5dB) \) for the case of SISO. As it can be seen in the figure, the probabilities of failure are very small. Note that as it is described in [15] the curves of \( P_f \) in general have a U shape form. This is because at low SNRs, the simulation is not accurate due to high level of noise, while at high SNRs, although the estimated positions can be quite accurate, a small of shift with respect to ideal position leads to a large amount of loss in SINR (see (26)).

Finally we want to mention that although we can not compare directly our results with that of [15] and [14] because of different assumptions, but it is evident from the figures that the performance of the proposed algorithm is much better than the SISO case. This improvement in
performance is quite reasonable since we have used a 4×4 MIMO system instead of SISO one.

6. CONCLUSION

The problem of symbol timing for MIMO-OFDM systems is addressed in this paper. For this purpose a new synchronization algorithm over unknown frequency selective channel is proposed based on the maximum-likelihood principle. In our algorithm there is no constraint for the preamble structure. Furthermore in this paper we calculated the loss in system performance due to synchronization for MIMO–OFDM systems and we used it as a performance criterion. The suitability of the proposed synchronization approach was shown through performance simulations of a MIMO-OFDM system, which resulted in an acceptable probability of synchronization failure.

REFERENCES