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USE OF STATISTICAL HYPOTHESIS TEST FOR MINES DETECTION IN SAS IMAGERY

Lionel Bombrun[1], *Frédéric Maussang*[1], *Eric Moisan*[1], *and Alain Hétet*[2]

[1]Images and Signals Laboratory – CNRS
LIS/ENSIEG – Domaine Universitaire – BP 46
38402 Saint-Martin-d'Hères Cedex, France
phone: +33 476 826 420, fax: +33 476 826 384
email: {lionel.bombrun ; frederic.maussang ; eric.moisan}@lis.inpg.fr

[2]Groupe d'Etudes Sous-Marines de l'Atlantique
DGA/DET/GESMA – BP 42
29240 Brest Armées, France
phone: +33 298 228 600, email: alain.hetet@dga.defense.gouv.fr

ABSTRACT

Synthetic Aperture Sonar (SAS) imagery is currently used in order to detect underwater mines laying on or buried in the sea bed. But the low signal to noise ratio characterizing these images leads to a high number of false alarms. In this paper, a new method of detection based on a statistical hypothesis test is presented. The proposed method can be divided into two main steps. Firstly, a statistical model of the speckle noise is described. A statistical hypothesis test is then performed and an evaluation of the performances is proposed.

1. INTRODUCTION

The detection of underwater mines in the sea bed is a significant problem. Indeed, mines used during battles can derive and appear near coasts where the sea traffic is considerable. Damages caused by an explosion can be a real disaster. Low frequency Synthetic Aperture Sonar (SAS) is an effective way to solve this problem. SAS systems give high resolution images of the sea bed with low frequency emission [1].

But these images are built with speckle which gives a granular aspect and disturbs its interpretation. Classical methods of detection and classification propose to use shadows of the sought objects [2]. These methods cannot be employed in buried objects detection because the shadows are not visible. Therefore, the method presented in this paper is based on a segmentation of the echoes reflected by the objects. Some papers deal with the use of smoothing filters [3], other on statistical model and a representation in the mean-standard deviation plan [4]. Authors proposed to use higher order statistics [5]. Parameters can then be combined to detect efficiently the sought objects [6].

A detection method using statistical hypothesis test is presented in this paper. The first step consists in a statistical model description of the speckle (section 2). Thanks to this model, a statistical hypothesis test based on the Kolmogorov distance is then proposed (section 3). Finally, an evaluation of the performances is proposed (section 4).

The detection method is tested on data recorded by GESMA during the BMC'99 at Lanvéoc (Finistère, France). This image represents a sea bed region of about 11m by 13m, with a resolution of 6cm in both dimensions (Fig. 1). This image contains three buried mines (b, c, and e), one buried rock (d) and one mine laying on the sea bed (f) and an undefined object (a).

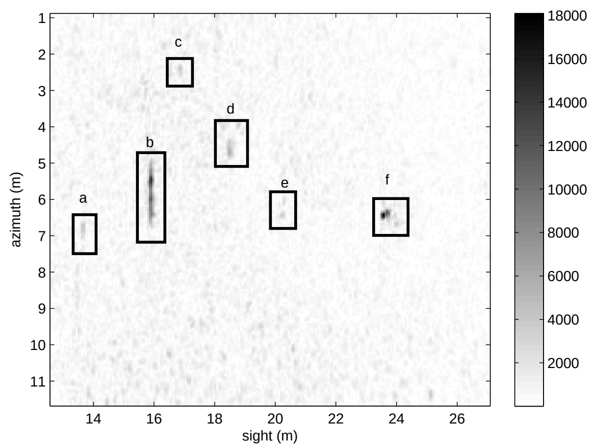


Figure 1: SAS image containing buried objects.

2. STATISTICAL MODELS

2.1 Speckle noise and the Rayleigh law

Images provided by a coherent system like SAS are built with speckle. The study of the response ρ of a resolution cell is necessary to obtain a statistical model of the amplitude. The sensor receives the constructive and destructive interferences of all the waves reflected by the N_d diffusers contained in a resolution cell [7]:

$$\rho = \sum_{i=1}^{N_d} a_i \exp\{j\varphi_i\} = A\, e^{j\phi} = X + jY \qquad (1)$$

We assume X and Y independent Gaussian random variables. The amplitude $A = \sqrt{X^2+Y^2}$ is then described by a Rayleigh probability density function defined as follows:

$$f_Y(y) = \frac{y - min}{\alpha^2} \exp\left\{-\frac{(y-min)^2}{2\alpha^2}\right\} u(y - min) \qquad (2)$$

with u the Heaviside function ($u(x) = 1$ if $x > 0$, $u(x) = 0$ else), α the scale parameter and min the offset. This last parameter takes into account the minimum of the amplitude if it is not null.

This model is suitable when the number of elementary scatters N_d on a resolution cell is large. But SAS providing high resolution images, the number of diffusers present in a resolution cell decreases significantly. The amplitude A is therefore better described by a Weibull probability density function.

2.2 The Weibull law

2.2.1 *Definition*

The probability density function of a Weibull law is defined with three parameters as follows:

- β the scale parameter
- c the shape parameter
- min the offset

$$f_Y(y) = \frac{c}{\beta}\left(\frac{y-min}{\beta}\right)^{c-1} \exp\left\{-\left(\frac{y-min}{\beta}\right)^{c}\right\} u(y-min) \quad (3)$$

2.2.2 *Estimators*

The parameters β, c, and min of the Weibull law are estimated using the maximum likelihood method. $\widehat{\beta}_{ML}$, $\widehat{c}_{ML}$, and $\widehat{min}_{ML}$ are respectively given by the following equations [2]:

$$\widehat{c}_{ML} = \lim_{k\to+\infty} c_k \quad (4)$$

with $c_k = F(c_{k-1})$, $c_0 = 1$, and:

$$F(x) = \frac{N \sum_{i=1}^{N} \widetilde{y}_i^{\,x}}{N \sum_{i=1}^{N} (\widetilde{y}_i^{\,x} ln(\widetilde{y}_i)) - \sum_{i=1}^{N} ln(\widetilde{y}_i) \sum_{k=1}^{N} \widetilde{y}_k^{\,x}} \quad (5)$$

$$\widehat{\beta}_{ML} = \left(\frac{1}{N}\sum_{i=1}^{N} \widetilde{y}_i^{\,\widehat{c}_{ML}}\right)^{\frac{1}{\widehat{c}_{ML}}} \quad (6)$$

$$\widehat{min}_{ML} = y_{min} - 1 \quad (7)$$

N denotes the number of pixels, y_i the value of pixel i, y_{min} the minimum of all the y_i and $\widetilde{y}_i = y_i - \widehat{min}_{ML}$. The term -1 in the $\widehat{min}_{ML}$ estimation comes from the non-nullity of the probability to have the minimum value.

2.3 Choice of the model

Fig. 2 represents the Rayleigh and Weibull distributions estimated by a maximum likelihood estimator on gray levels distribution of SAS data. Graphically, the Weibull probability density function seems to be a better model of the amplitude than the Rayleigh probability density function. A distance criterion enhances this idea. The Kolmogorov distance (d_K) is displayed for Rayleigh and Weibull models. This criterion is defined as follows:

$$d_K = \sup_x |F_N^*(x) - F(x)| \quad (8)$$

where $F(x)$ represents the theoretical cumulative distribution function and $F_N^*(x)$ the empirical cumulative distribution function.

The Kolmogorov distance is bigger with the Rayleigh model than with the Weibull one. This leads to the conclusion the description of the speckle by a Weibull law is better. More complex models have been proposed in the literature (K distribution for example [8]), but the Weibull model is a good trade-off between performance of description and complexity of estimation. Later on, the Weibull distribution is the only law taken into account to obtain a statistical model.

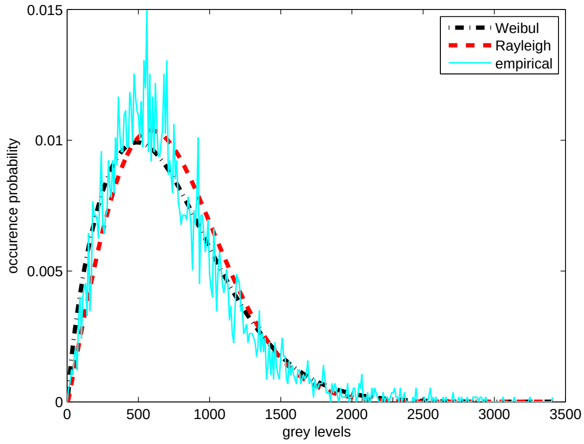


Figure 2: Description of the amplitude by Rayleigh ($d_K = 0.064$) and Weibull ($d_K = 0.027$) distributions (N=5600).

Table 1 displays the Weibull law parameters estimated on two distinct regions of the SAS image assumed to contain no object. Note that the scale parameter β is twice as high at the bottom of the image as at the top. Therefore, a global model cannot be performed. But, note that the efficiency of the model is approximatively the same for the two regions (d_K).

	min	β	c	d_K
top	1.95	413.7	1.95	0.032
bottom	6.31	784.7	1.80	0.039

Table 1: Description of the amplitude with a Weibull probability density function in distinct parts of the SAS data.

3. THE STATISTICAL HYPOTHESIS TEST

3.1 Hypothesis test

A statistical hypothesis test is based on statistical data allowing to decide between two hypothesis: the null hypothesis denoted H_0 and the alternate hypothesis denoted H_1 [9]. The test is defined as follows:

$$\begin{cases} H_0: & \forall k \in \{1,\dots,n\}\ F(x_k) = F_n^*(x_k) \\ H_1: & \exists k \in \{1,\dots,n\}\ F(x_k) \neq F_n^*(x_k) \end{cases}$$

where $\{x_1, \ldots, x_n\}$ represents n independent samples of a random variable X, $F(x)$ is the theoretical cumulative distribution function, and $F_n^*(x)$ denotes the empirical cumulative distribution function.

Our problem is to determine whether there is a mine in the SAS data or not. Then it seems to be quite logical to work with the cumulative repartition function of the signal "mines". However, it is impossible to have an accurate model of the signal "mines" because of the few number of pixels corresponding to mines echoes. Consequently, $F(x)$ is the theoretical cumulative distribution function of the signal "sea bed" we can model with a Weibull probability density function. H_1 is then the default hypothesis and a criterion will be used to estimate the distance with the hypothesis H_0.

3.2 Principle

The detection method presented in this article is based on a segmentation of the echoes. The first step of this method consists in estimating the Weibull parameters in a sliding square window using the equations 4, 6, and 7. The theoretical cumulative repartition function is then assumed to be locally known. A distance criterion is then used to determine if the model with a Weibull probability density function is acceptable or not.

The criterion we have decided to choose is the Kolmogorov distance (equation 8) thanks to its simplicity and its properties. Indeed, the Kolmogorov distance is independent from the estimated statistical model, and then from the characteristics of the sea bed, if the hypothesis "sea bed" is true. The non-stationarity of the sonar image highlighted in section 2.3 is then not a problem in this test: the Kolmogorov distance has the same behavior when a Weibull law is an accurate statistical model of the current region.

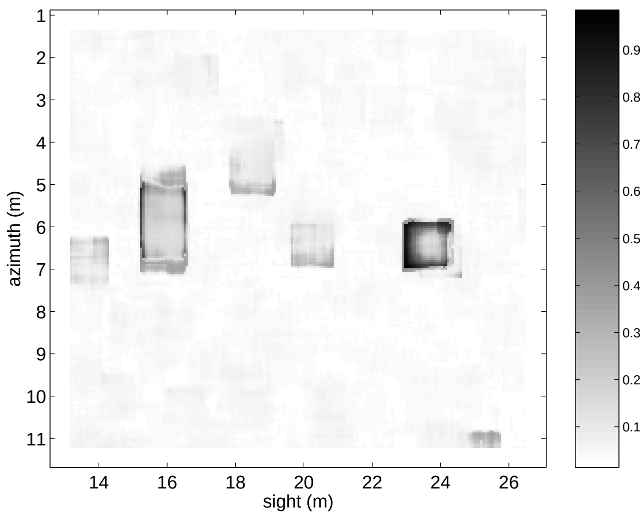


Figure 3: Result after the statistical hypothesis test.

3.3 Post-processing

As we can see on Fig. 3, the detection of an object is quite large. Indeed, objects are detected as long as there is at least one pixel relevant of a mine in the square window. If l represents the width of the window ($n = l^2$) and m the width of an echo, this object will be detected with a rectangular of $l+m-1$ pixels width. Therefore, a post-processing step is necessary to rebuild the mine dimension. We propose to perform a morphological operator: an erosion. The operator is a square $l \times m$, equal to the previous computation window.

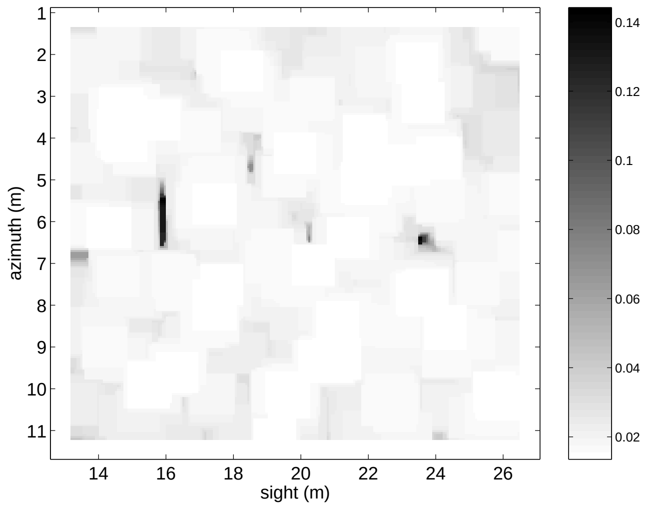


Figure 4: Result after erosion.

Some experiments have been performed to explain the choice of the size of the computation window:

- If the size of the window is too small (11×11 for example), there is not samples enough to have an accurate model of the speckle noise. The Weibull law parameters are then not estimated successfully. Therefore, the statistical hypothesis test is meaningless.
- If the size of the window is too large (31×31 for example), the number of pixels corresponding to mine echoes are not large enough to be significant. The statistical hypothesis test does not allow an accurate detection of the echoes.

Finally, a square window of 21×21 size seems to be a good trade-off between these two cases.

3.4 Validation of the method: comparison with a threshold in gray levels

Finally, a threshold on the Fig. 4 allows the detection of mines in SAS data. On Fig. 5, the results obtained with our segmentation method and with a simple threshold for the same false alarm probability (defined in section 4) are plotted ($p_{fa} = 2.10^{-3}$). The method presented is suitable: all the objects are detected and false alarms are not isolated, contrary to a simple threshold. Indeed, the number of 8-connected objects (n_{co}) is almost five times as high for the simple threshold as for the hypothesis test.

Results obtained with our segmentation method (Fig. 5(a)) are extremely promising. Indeed, some objects were badly visible on Fig. 1 (SAS image) and appear clearly on the resulting image. For example, the buried mine 'e' on Fig. 1 is better identified with our algorithm than with a simple threshold.

4. EVALUATION OF THE PERFORMANCES

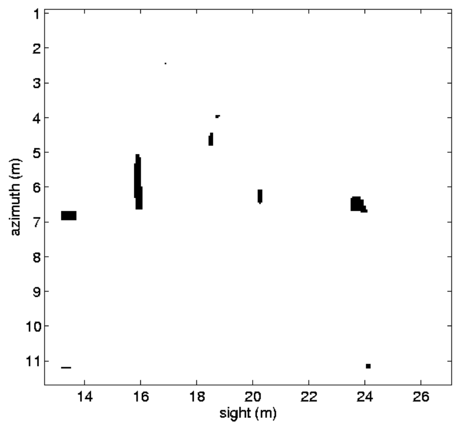
4.1 Definition

To highlight the relevance of this segmentation method, it is tested on several SAS data and compared with a simple threshold performed on the gray levels of the original image. Receiver Operating Characteristic (ROC) curves are computed to make the comparison, plotting the evolution of the detection probability (p_d) versus the false alarm probability (p_{fa}) when the threshold value increases. If A is the image containing the regions selected as echoes by the threshold process (Fig. 6(b)) and B the segmented region with our algorithm (Fig. 6(a)), the detection probability and the false alarm probability can be defined by:

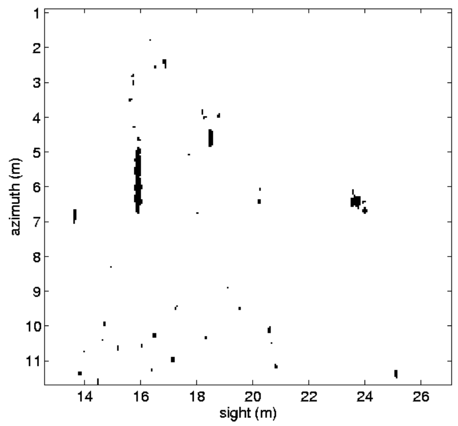
$$p_d = \frac{N_{A\cap B}}{N_A} \tag{9}$$

$$p_{fa} = \frac{N_{\bar{A}\cap B}}{N_{\bar{A}}} = \frac{N_B - N_{A\cap B}}{N_{\bar{A}}} \tag{10}$$

where N_X is the number of pixels considered in the region X.

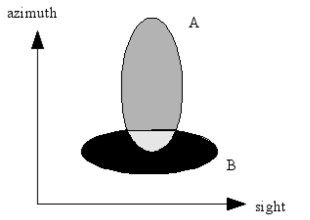


(a) Hypothesis test
($p_d = 0.523$, $n_{co} = 9$)

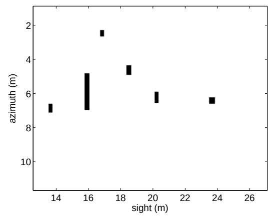


(b) Threshold in gray levels
($p_d = 0.504$, $n_{co} = 41$)

Figure 5: Comparison with a threshold in gray levels ($p_{fa} = 2.10^{-3}$)

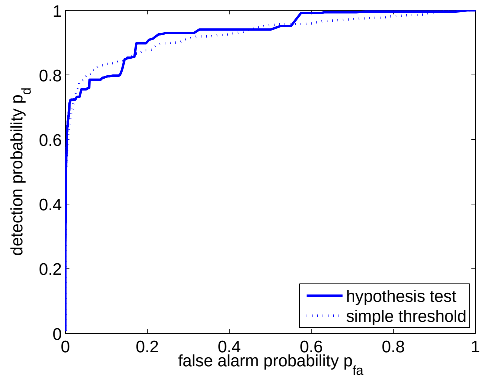


(a) Evaluation of p_d and p_{fa}

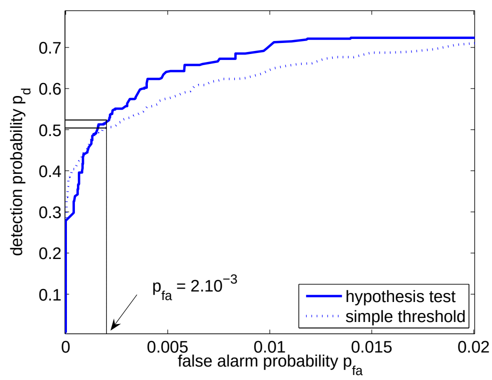


(b) Reference image (A)

Figure 6: Evaluation of the performances
(A: regions selected on the reference image,
B: regions selected by the proposed algorithm / the threshold)



(a) ROC curves



(b) zoom in low p_{fa}

Figure 7: ROC curves: hypothesis test / threshold

4.2 Results

On Fig. 7, we can see ROC curves, one for our segmentation method based on a statistical hypothesis test and one for a simple threshold on Fig. 1. For very low false alarm probability (lower than 0.02), our method presented in this paper is more efficient than a simple threshold on SAS data. It confirms the results obtained in section 3 for $p_{fa} = 2.10^{-3}$.

5. CONCLUSION AND PERSPECTIVES

A detection method in SAS imagery, using statistical hypothesis test, has been proposed in this paper. This method uses the echoes reflected by the objects and a statistical model of the sea bed by a Weibull probability density function. An analysis of the size of the square window has been studied. The performance of the method can be underlined. Indeed, contrary to a simple threshold on the SAS data, false alarms are not isolated and our segmentation method leads to a better detection of buried objects which echoes have a low signal to noise ratio.

An idea to improve this method would be the use of a more complex and appropriate probability density function. For example, we may obtain a better model of the speckle noise with a K distribution than with a Weibull probability density function. A comparison with the use of a Rayleigh assumption can also be performed. The perspectives of this work include the recognition and classification of the detected objects.

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