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A CLOSED FORM SOLUTION FOR THE BLIND SEPARATION OF TWO SOURCES FROM TWO SENSORS USING SECOND ORDER STATISTICS

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ABSTRACT

In this paper, we present a specific algorithm for the blind identification of a two input two output system. A closed form solution for the blind identification of the system is derived by exploiting the temporal coherence properties of the input sources. By exploiting the inherent indeterminacies of the blind processing, a simplified version is derived making the algorithm computationally cheaper and more suitable for hardware implementation. The weights of the zero forcing blind separator are then deduced. The performance of the proposed solutions with respect to the signal to noise ratio (SNR) and sample size are provided in the simulation section.

1. INTRODUCTION

Blind source separation (BSS) problem consists of identifying a linear system whose only output is observed. When an array of sensors samples the fields radiated by narrow band sources its output is classically modelled as an instantaneous spatial mixture of a random vector whose components are the source signals, possibly corrupted by additive noise. Source separation may be obtained by first identifying the directional vectors associated to each source and then by projecting the array signal onto the estimated vectors. This is a standard program in array processing except that in *blind source separation* problem we perform system identification without resorting to the knowledge of the array manifold. Hence, blind source separation is essentially unaffected by errors in the propagation model or in array calibration.

When the specifications of a blind identification problem are known in advance, e.g. number of sensors and sources involved. One can design specific BSS algorithms for the problem at hand. In this case, closed form solutions become possible. In particular, the two-input two-output case has attracted a lot of attention in the literature, e.g. [3, 4, 5, 6], due to its simplicity and its numerous potential applications.

Note that such solutions are more suitable for hardware implementation where iterations are often avoided. It is well known that in VLSI implementation, divisions and square roots are more complex to implement than multiplications and require more space and time resources [1, 2]. Hence, a challenge is to provide a solution to the BSS problem that does involve a minimum of division and square root computations. We show in this paper how we can take advantage of the inherent indeterminacies of the BSS problem to meet this challenge. Herein, we propose closed form solutions for the blind identification of a two input two output system together with its zero forcing separator by exploiting the temporal coherence of the source signals.

2. PROBLEM FORMULATION

2.1. Signal model

Consider an array of 2 sensors receiving signals from 2 narrow band sources. The array output denoted $\mathbf{x}(t)$ is a 2×1 random vector. Corrupted by additive white noise denoted $\mathbf{n}(t)$, it is classically modelled as:

$$\mathbf{x}(t) = \mathbf{y}(t) + \mathbf{n}(t) = \mathbf{H}\mathbf{s}(t) + \mathbf{n}(t) \qquad (1)$$

where $\mathbf{s}(t)$ is a 2×1 vector whose p-th component denoted $s_p(t)$ is the signal emitted by the p-th source. The 2×2 matrix:

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$$

is assumed to have full rank but otherwise unknown. The source signals are temporally colored second order stationary, mutually uncorrelated processes.

The purpose of source separation is to recover the source signals from the array output $\mathbf{x}(t)$ without knowledge of the mixture matrix $\mathbf{H}$. The benefit of such a 'blind' approach is that source separation is essentially unaffected by errors in the propagation model or in array calibration. Source separation techniques based on second order statistics require only the main assumption of uncorrelated source signals. The additive noise $\mathbf{n}(t)$ is assumed to be spatially and temporally white and uncorrelated with the source signals.

2.2. Concept of blind identifiability

In blind context, complete identification of the mixture matrix $\mathbf{H}$ is impossible as shown by the following relation:

$$\mathbf{x}(t) = \mathbf{H}\mathbf{s}(t) + \mathbf{n}(t) = \sum_{p=1}^{2} \frac{\mathbf{h}_p}{\alpha_p}\alpha_p s_p(t) + \mathbf{n}(t) \quad (2)$$

where $\alpha_p \in \mathbb{R}$ and $\mathbf{h}_p$ denotes the p-th column of $\mathbf{H}$. Hence, the exchange of a fixed scalar factor between a source signal and the corresponding column of $\mathbf{H}$ leaves the observations unaffected.

Advantage can be taken of this indeterminacy, without any loss of generality, by assuming unit variance source signals, so that the dynamic range of the sources is accounted for by the magnitude of the corresponding column of $\mathbf{H}$. Consequently, blind identification of $\mathbf{H}$ is understood as the determination of a matrix equal to $\mathbf{H}$ up to permutation (which comes from the fact that the source numbering is arbitrary) and diagonal matrices. The crucial point is that these indeterminacies do not impede source separation. If the mixture matrix $\mathbf{H}$ is estimated up to permutation and diagonal matrices, it still allows to determine the source signals up to the corresponding fixed permutation and scalar factor. In the sequel, we exploit these indeterminacies to derive a BSS algorithm free from divisions.

3. THE PROPOSED SECOND ORDER BLIND IDENTIFICATION SOLUTION

Consider the following sampled version of the data model (1),

$$\mathbf{x}(n) = \mathbf{H}\mathbf{s}(n) + \mathbf{n}(n), \quad \mathbf{s}(n) = [s_1(n)\ s_2(n)]^T. \quad (3)$$

In the above expression, $\mathbf{x}(n)$, $\mathbf{n}(n)$ and $\mathbf{s}(n)$ are the sampled version of $\mathbf{x}(t)$, $\mathbf{n}(t)$ and $\mathbf{s}(t)$, respectively, where we have omitted to specify the sampling rate for ease of notation. ^{T} denotes the transpose operator.

The correlation matrices of $\mathbf{x}(n)$ are given by,

$$\begin{aligned}
\mathbf{R}_{x_1x_1} &= h_{11}^2\mathbf{R}_{s_1s_1} + h_{12}^2\mathbf{R}_{s_2s_2} + \sigma^2\mathbf{I} &(4)\\
\mathbf{R}_{x_2x_2} &= h_{21}^2\mathbf{R}_{s_1s_1} + h_{22}^2\mathbf{R}_{s_2s_2} + \sigma^2\mathbf{I} &(5)\\
\mathbf{R}_{x_1x_2} &= h_{11}h_{21}\mathbf{R}_{s_1s_1} + h_{12}h_{22}\mathbf{R}_{s_2s_2} &(6)
\end{aligned}$$

where $\mathbf{x}(n) = [x_1(n)\ x_2(n)]^T$, $\mathbf{I}$ is the $N\times N$ identity matrix, and $\mathbf{R}_{x_ix_j}$, $i,j = 1,2$ is defined by

$$\mathbf{R}_{x_ix_j} = E([x_i(1),\cdots,x_i(N)]^T[x_j(1),\cdots,x_j(N)]) \quad (7)$$

$E(.)$ being the expectation operator and N is some chosen window length[1]. The above expressions are derived under the assumptions of Section 2.1.

[1] We choose N as a power of 2 so that a division by N becomes a simple bit shifting.

Let us define the operators $off(.)$ and $tr(.)$ by

$$\begin{aligned}
off(\mathbf{M}) &= \sum_{i\neq j} M_{ij} &(8)\\
tr(\mathbf{M}) &= \frac{1}{N}\sum_{i} M_{ii} &(9)
\end{aligned}$$

where $\mathbf{M}$ is any square matrix of dimension $N\times N$ and M_{ij} are the entries of $\mathbf{M}$. By applying these operators to equations (4), (5) and (6), we get the following set of relations,

$$\begin{aligned}
F_1 &= off(\mathbf{R}_{x_1x_1}) = h_{11}^2R_1 + h_{12}^2R_2 &(10)\\
F_2 &= off(\mathbf{R}_{x_2x_2}) = h_{21}^2R_1 + h_{22}^2R_2 &(11)\\
F_{12} &= off(\mathbf{R}_{x_1x_2}) = h_{11}h_{21}R_1 + h_{12}h_{22}R_2 &(12)\\
T_1 &= tr(\mathbf{R}_{x_1x_1}) = h_{11}^2 + h_{12}^2 + \sigma^2 &(13)\\
T_2 &= tr(\mathbf{R}_{x_2x_2}) = h_{21}^2 + h_{22}^2 + \sigma^2 &(14)\\
T_{12} &= tr(\mathbf{R}_{x_1x_2}) = h_{11}h_{21} + h_{12}h_{22} &(15)
\end{aligned}$$

where $R_i = off(\mathbf{R}_{s_is_i})$, $i = 1,2$. In (13), (14) and (15), we have used the fact that, under unit-variance assumption, $tr(\mathbf{R}_{s_is_i}) = 1$, $i = 1,2$.

By solving equations (10)-(15), we obtain the following expressions of the mixing matrix entries,

$$\begin{aligned}
h_{11} &= \sqrt{\frac{F_1 - (T_1-\sigma^2)\beta}{\gamma}} &(16)\\
h_{22} &= \sqrt{\frac{(T_2-\sigma^2)\alpha - F_2}{\gamma}} &(17)\\
h_{12} &= \frac{T_{12}\alpha - F_{12}}{\gamma h_{22}} &(18)\\
h_{21} &= \frac{F_{12} - T_{12}\beta}{\gamma h_{11}} &(19)
\end{aligned}$$

where

$$\alpha = \frac{a+c}{b}, \quad \beta = \frac{a-c}{b}, \quad \gamma = \frac{2c}{b}$$

with

$$\begin{aligned}
a &= 2F_{12}T_{12} - (F_{11}(T_{22}-\sigma^2) + (T_{11}-\sigma^2)F_{22}) &(20)\\
b &= 2(T_{12}^2 - (T_{11}-\sigma^2)(T_{22}-\sigma^2)) &(21)\\
c^2 &= (F_{11}(T_{22}-\sigma^2) - (T_{11}-\sigma^2)F_{22}))^2 + 4(F_{12}(T_{22} \\
&\quad -\sigma^2) - T_{12}F_{22})(F_{12}(T_{11}-\sigma^2) - T_{12}F_{11}). &(22)
\end{aligned}$$

An estimate of the noise variance σ^2 is needed for a robust estimation of the channel coefficients. It can be obtained by the eigen-decomposition of the data covariance matrix [7] if a third sensor is available. Otherwise, σ^2 can be estimated using only two sensors before data recording begins.

Another alternative is to estimate σ^2 by choosing the value that minimizes in the least squares sense w.r.t. σ^2 the intercorrelation at different time lags between the two outputs of $\mathbf{H}(\sigma^2)^{-1}\mathbf{x}(t)$. This solution is not considered here as it increases the cost of the proposed algorithm. Note that in practice, the temporal correlation matrices of the data are replaced by their time-averages. To track possible non-stationarity, these temporal correlation matrices can be estimated adaptively using e.g. an exponential memory.

Remark: It is clear from equations (10) to (12), that for sources with identical spectral shapes (i.e. $R_1 = R_2$), these equations are reduced to equations (13) to (15). Subsequently, the latter become insufficient to solve the identification problem. In this case, one has to use higher order blind identification techniques [8, 9, 10].

4. A SIMPLIFIED SECOND ORDER BLIND IDENTIFICATION SOLUTION

In this section we take advantage of the inherent indeterminacies of the blind source separation problem stated in Section 2.2 to further reduce the computational load of the proposed solution by eliminating all the division operations. These simplifications should allow an adequate architecture of the proposed algorithm when implemented on hardware devices (e.g. FPGA, ASIC).

Let us rewrite expressions (16) to (19) of the mixing matrix entries in the following form,

$$\mathbf{H} = \begin{pmatrix} \sqrt{\frac{F_1-(T_1-\sigma^2)\beta}{\gamma}} & \frac{(T_{12}\alpha-F_{12})\sqrt{\gamma}}{\gamma\sqrt{(T_2-\sigma^2)\alpha-F_2}} \\ \frac{(F_{12}-T_{12}\beta)\sqrt{\gamma}}{\gamma\sqrt{F_1-(T_1-\sigma^2)\beta}} & \sqrt{\frac{(T_2-\sigma^2)\alpha-F_2}{\gamma}} \end{pmatrix} \quad (23)$$

By taking advantage of the inherent indeterminacies of the blind processing, a new solution to the blind identification of the mixing matrix is obtained by multiplying matrix $\mathbf{H}$ with the following diagonal matrix

$$\begin{pmatrix} \gamma b\sqrt{\frac{F_1-(T_1-\sigma^2)\beta}{\gamma}} & 0 \\ 0 & -\gamma b\sqrt{\frac{(T_2-\sigma^2)\alpha-F_2}{\gamma}} \end{pmatrix}. \quad (24)$$

This leads to the following solution

$$\mathbf{H}_s = \begin{pmatrix} bF_1 - (T_1-\sigma^2)d_1 & bF_{12} - T_{12}d_2 \\ bF_{12} - T_{12}d_1 & bF_2 - (T_2-\sigma^2)d_2 \end{pmatrix} \quad (25)$$

where $d_1 = a - c$ and $d_2 = a + c$. Note that the obtained solution does not involve any division operation and reduces in the same time the number of square root operations needed for the channel identification.

5. SOURCE SIGNAL RECOVERY

In this Section, our objective is to determine the weights of the spatial filter

$$\mathbf{W} = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} \quad (26)$$

to achieve our task of source signal recovery. Several minimization and maximization criteria can be used to optimize the weights of the spatial filter [11]. This includes the maximization of signal to interference and noise ratio, and linearly constrained minimum variance of the filter output. Herein, we compute the zero forcing spatial filter which maximizes the signal to interference (SIR) at the output of the filter. Taking into account the inherent indeterminacies of blind source separation, the zero forcing solution is given by

$$\mathbf{WH} = \mathbf{PD}$$

where $\mathbf{P}$ and $\mathbf{D}$ are a permutation matrix and a diagonal matrix, respectively. A solution to (26) is given by

$$\mathbf{W} = \begin{bmatrix} h_{22} & -h_{12} \\ -h_{21} & h_{11} \end{bmatrix} \quad (27)$$

where the h_{ij} are computed either from expression (23) or from expression (25), according to the chosen solution.

6. SIMULATION RESULT

We first present a sample run of the proposed solutions: Two speech signals sampled at 8000 Hz are mixed by the following matrix,

$$\mathbf{H} = \begin{bmatrix} 1.0 & 1.0 \\ 1.0 & 0.8 \end{bmatrix}. \quad (28)$$

The plots of the two individual speech signals and their observed mixtures are shown in Figure 1. The separated sources are plotted in Figure 2, where the SOBI algorithm [7] is used for the purpose of comparison. It is clear that the proposed BSS solutions work well in this case and give for this sample run similar result as the SOBI algorithm.

Next, we access the performance of the proposed solutions through Monte Carlo runs. The performance is characterized in terms of signal rejection. After blind identification, the estimated source signals are $\hat{\mathbf{s}}(t) = \hat{\mathbf{W}}\mathbf{x}(t) = \hat{\mathbf{W}}\mathbf{H}\mathbf{s}(t) + \hat{\mathbf{W}}\mathbf{n}(t)$ where $\hat{\mathbf{W}}$ is an estimate of the spatial filter matrix. The matrix $\hat{\mathbf{P}}$ defined by $\hat{\mathbf{P}} = \hat{\mathbf{W}}\mathbf{H}$ should be close to some permutation matrix times a diagonal matrix (permutation and scale indeterminacies). The p-th estimated source signal is :

$$\hat{s}_p(t) = \sum_{q=1,n} \hat{\mathbf{P}}_{pq} s_q(t) \quad (29)$$

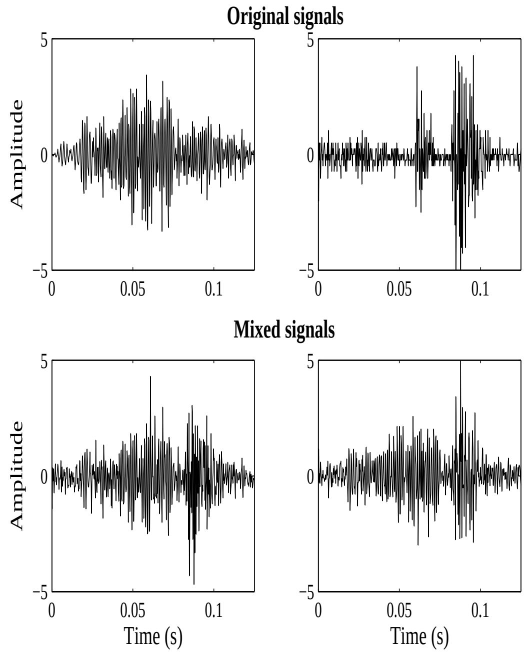

Fig. 1. Example of mixed speech signals.

and contains the q-th source signal at level $\frac{|\hat{\mathbf{P}}_{pq}|^2}{|\hat{\mathbf{P}}_{pp}|^2}$. A measure of the global quality of a separation is the overall rejection level:

$$I_{perf} \stackrel{\text{def}}{=} \sum_{q \neq p} \frac{|\hat{\mathbf{P}}_{pq}|^2}{|\hat{\mathbf{P}}_{pp}|^2} \tag{30}$$

where we have assumed for convenience that $\hat{\mathbf{P}}$ is close to diagonal rather than to some other permutation matrix.

In our performance study, we consider two sources mixed by the matrix of equation (28). The additive gaussian noise has covariance $\mathbf{R}_n = \sigma^2\mathbf{I}$. The source signals have unit variance and each one is generated by filtering a white Gaussian process by two different auto-regressive models. The overall rejection level is evaluated over 500 realizations. In these simulations, the proposed solutions assume no noise (i.e. in equations (16) to (19) and (25), σ^2 is set to zero).

In Figure 3, the rejection level I_{perf} is plotted in dB against Signal to Noise Ratio (SNR) in dB for a sample size of 512. In Figure 4, the Signal to Noise Ratio (SNR) is kept constant at 30 dB. The curves show the rejection level I_{perf} in dB plotted against the sample size.

Plots of Figures 3 and 4 show significant increase in performance for a sufficient number of samples and high SNR. Also, one can observe that the proposed solution and its simplified version have similar performance. When compared with the SOBI algorithm, one observes a loss of 2 dB in performance but a significant saving in the computation cost since in contrast to the proposed solutions, the SOBI algorithm involves eigen decomposition for the whitening process

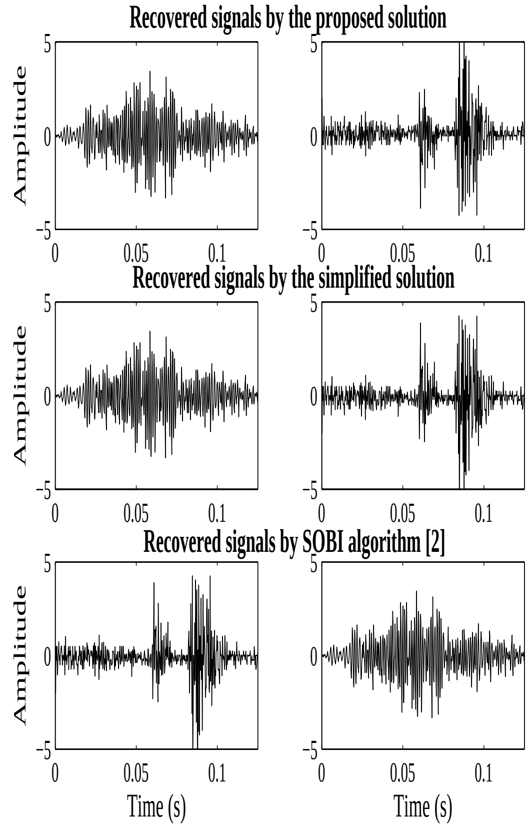

Fig. 2. A sample run on speech signals.

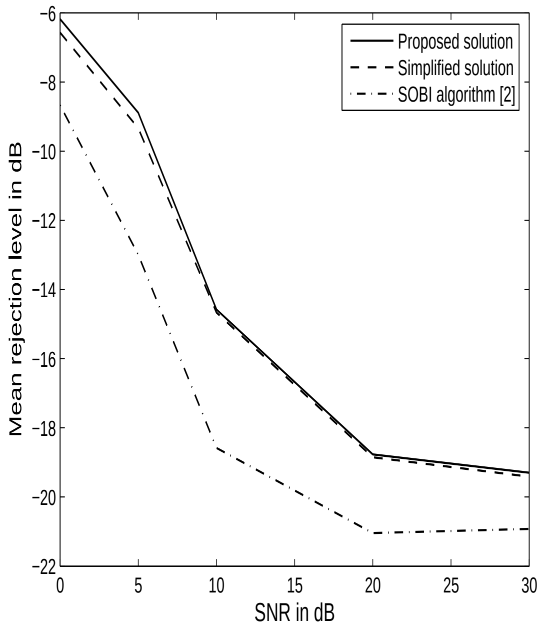

Fig. 3. Mean rejection level vs SNR.

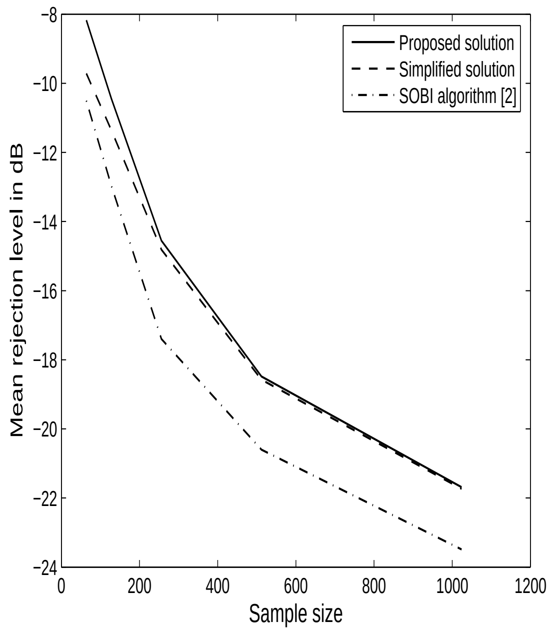

Fig. 4. Mean rejection level vs Sample size.

and computation of the Givens rotations for the joint diagonalization process [7]. Hence, the traditional tradeoff that we often have between the achievable performance and the simplicity in the implementation.

7. CONCLUSION

In this paper, we have proposed a direct solution to the specific problem of the blind separation of two sources from two sensors using second order statistics. We have shown how, by exploiting the indeterminacies of the BSS problem, one can simplify the proposed closed form solution to provide a simple solution with no division operations. Such solution allows adequate architectures when implemented on hardware devices.

8. REFERENCES

[1] J.-P. Deschamps and G. Sutter, “Finite field division implementation,” in *Proc. International Conference on Field Programmable Logic and Applications*, pp. 670–674, Aug. 2005.

[2] A.E. Bashagha, “Novel radix-2k square root module”, *IEE Proceedings on Circuits, Devices and Systems*, Vol. 148, no. 4, pp. 190–196, Aug. 2001.

[3] H. Pan, D. Xia, S.C. Douglas and K.F. Smith, “A scalable VLSI architecture for multichannel blind deconvolution and source separation Signal Processing Systems” , *IEEE Workshop on SIPS*, pp. 297–306, Oct. 1998.

[4] A. Belouchrani and M.G. Amin, “A two-sensor array blind beamformer for direct sequence spread spectrum communications”, *IEEE Transactions on Signal Processing*, Vol. 47, no. 8, pp. 2191–2199, Aug. 1999.

[5] Yuchang Cao, S. Sridharan and A. Moody, “Multichannel speech separation by eigendecomposition and its application to co-talker interference removal”, *IEEE Transactions on Speech and Audio Processing*, Vol. 5, no. 3, pp. 209–219, May 1997.

[6] V. Zarzoso, F. Herrmann and A.K. Nandi, “Weighted closed-form estimators for blind source separation”, *Proceedings of the 11th IEEE Signal Processing Workshop on Statistical Signal Processing*, pp. 456–459, Aug. 2001.

[7] A. Belouchrani and K. Abed Meraim and J.-F Cardoso and E. Moulines, “A blind source separation technique using second order statistics,” *IEEE Trans. on SP*, vol. 45, no.2, pp. 434–444, Feb. 1997.

[8] P. Comon, “Independent component analysis,” in *Proc. Int. Workshop on Higher-Order Stat., Chamrousse, France*, pp. 111–120, 1991.

[9] A. Belouchrani and J.-F. Cardoso, “Maximum likelihood source separation for discrete sources,” in *Proc. EUSIPCO*, pp. 768–771, 1994.

[10] L. Albera, A. Ferreol, P .Chevalier and P .Comon, “ICAR: a tool for blind source separation using fourth-order statistics only,” *IEEE Trans. on SP*, vol. 53, no.10, pp. 3633–3643, Oct. 2005.

[11] R. Monzingo and T. Miller, *Introduction to Adaptive Arrays*. Wiley and Sons, New York, 1980.