OPTIMAL DESIGN OF FIR FILTER WITH SP2 COEFFICIENTS BASED ON SEMI-INFINITE LINEAR PROGRAMMING METHOD

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ABSTRACT

In this paper, we propose a new design method of FIR filters with Signed Power of Two (SP2) coefficients. In the method proposed here, the design problem of FIR filters is formulated as a discrete semi-infinite linear programming problem (DSILP), and the DSILP is solved using a branch and bound technique. We will guarantee the optimality of the solution obtained. Hence, it is possible to obtain the optimal discrete coefficients. It is confirmed that the optimal coefficients of linear phase FIR filter with the SP2 coefficients could be designed fast with enough precisions by the computational experiments.

1. INTRODUCTION

Digital signal processing deals with the representation of signals as ordered sequencers of numbers and the processing of those sequences. Typical reasons for signal processing include: estimation of characteristic signal parameters, elimination or reduction of unwanted interference, and transformation of a signal into a form that is in some sense more informative.

There are two methods for the realization of FIR filter, one is a software realization method and another is a hardware realization by using digital circuits. In hardware implementation of FIR filters, the filter coefficients corresponding to multiplier coefficients are presented as the finite word length numbers. When the coefficients are simply rounded to the nearest discrete number, precision of filters are degraded from the one with the optimal real coefficients. Therefore, design methods of FIR filters with discrete coefficients have been widely researched [12], [6]. There are no design methods of designing filters that could be easily adapted to special design specifications. So each filter has to be designed, in principle, by a complete mathematical design procedure. It is the aim of all design methods to approximate a desired frequency response as close as possible by a finite number of FIR filter coefficients. Recently, many studies on a design method for linear phase FIR filters with discrete coefficients have been published [11], [8] in which, a numerical representation by a sum of signed power of two (SP2) has been used in several methods. [7], [13], [9], [8]. It is a reason that a small number of non-zero digits is often required for a representation of the coefficients in a VLSI implementation of the filters. There exist a lot of studies to obtain an approximated solution for this design problem. See, for example, Ito et. al [5], W. -S. Lu [11]. They proposed to use a semidefinite programming (SDP) relaxation method for the design problem. However, if we do not have an optimal solution for the design problem, we cannot mention the performance of the approximation method precisely.

Since the design problem is formulated as a discrete semi-infinite linear programming problem, the most practical methods to solve the problem is to use the branch and bound (B & B) method. And, there are some methods using B & B method for the design problem, for example, based on LP, Remez algorithm, and so on. Cho et. al [1] proposed a B & B method based on LP focusing only on the active constraints to decrease the computational time. However, they did not assure the optimality of the solution obtained by the algorithm.

In this paper, we propose a new design method of linear phase FIR filters with SP2 coefficients which guarantees the optimality of the solution obtained. In the method proposed, the design problem is formulated as a discrete semi-infinite linear programming problem (DSILP) and solved by B & B method. In the B & B method, a branching tree is generated and, on each node, it is necessary to solve semi-infinite linear programming problem (SILP) [3].

It is shown by the results of some computational experiments for the filter designing problem, the developed algorithm is rather practical.

2. PROBLEM FORMULATION

In this section, we introduce the design method of digital FIR filters with SP2 coefficients.

2.1 Design problem of FIR digital filters with continuous coefficients

In this paper, we deal with a design problem of FIR digital filters with SP2 coefficients that minimize the maximal error, i.e., minimize the following function:

$$e = \max_{\omega \in \Omega} |H(e^{j\omega}) - H_d(\omega)| \qquad (1)$$

where $H_d(\omega)$ is the desired frequency response function and $\Omega = [0,\omega_p] \cup [\omega_s,\pi]$. Here, $[0,\omega_p]$ denotes a passband and $[\omega_s,\pi]$ denotes a stopband.

In the first, we consider the continuous coefficient case. Then the design function of the FIR filter is:

$$H(e^{j\omega}) = \sum_{k=0}^{N-1} h_k e^{-jk\omega}. \qquad (2)$$

Now, we assume N is odd filter number. Given a budget of total number of power-of-two terms M, a certain number of SP2 terms, m_k , is allocated to the k-th target discrete-coefficient d_k. Then we denote the frequency response $H(e^{j\omega})$ as follows.

$$H(e^{j\omega}) = \sum_{k=0}^{N-1} d_k e^{-jk\omega} \qquad (3)$$

The allocation of SP2 terms is determined, for example, by Lu [10], Ito et. al [4].

We assume that the absolute value of each SP2 coefficient $\{d_k\}$ is in $\{0\} \cup [2^{-U}, 2^0]$ where U is a natural number. Then, with a given term allocation m_k, the discrete coefficients d_k in the equation(3) can be expressed as,

$$d_k = \sum_{i=1}^{m_k} b_i^{(k)} 2^{-q_i^{(k)}}. \qquad (4)$$

Since each SP2 coefficient d_k is consisted of m_k nonzero digits, the relation of $m_0,\ldots,m_k$ and M is represented as the following equation.

$$\sum_{k=0}^{N-1} m_k = M. \qquad (5)$$

Here, we have $b_i^{(k)} \in \{-1,1\}$ and $1 \le q_i^{(k)} \le U,\ (1 \le i \le m_k,\ 0 \le k \le N-1)$.

Omitting the linear phase factor $e^{-(N-1)/2j\omega}$, the frequency response of a symmetrical impulse response filter with N odd is given by

$$H(\omega) = \sum_{k=0}^{K} d_k \cos k\omega. \qquad (6)$$

Here $K = (N-1)/2$ and this equation is called a magnitude response. Then the number of filter coefficients we consider is $K+1$. Suppose a desired response $H_d(\omega)$ is given as follows

$$H_d(\omega) = \begin{cases} S, & \omega \in [0,\omega_p], \\ 0, & \omega \in [\omega_s,\pi]. \end{cases} \qquad (7)$$

Where S is a scaling factor, ω_p is the passband cutoff frequency, and ω_s is the stopband cutoff frequency, respectively. Then, the optimal problem to approximate $H(\omega)$ to $H_d(\omega)$ in a min-max sense can be written as

$$\min_{d_0,\ldots,d_K} \max_{\omega\in\Omega} |H(\omega) - H_d(\omega)| \qquad (8)$$

where $\Omega = [0,\omega_p] \cup [\omega_s,\pi]$ is the approximation band.

If we introduce a new variable δ that corresponds to the L_∞-approximation error , it is easy to convert the above min-max problem to the following minimization problem, that is a semi-infinite programming problem with SP2 coefficients (DSILP).

$$\begin{array}{lrcll} \min & \delta & & & \\ \text{sub.to} & H(\omega)+\delta & \ge & H_d(\omega), & \omega \in \Omega, \\ & -H(\omega)+\delta & \ge & -H_d(\omega), & \omega \in \Omega \end{array} \qquad (9)$$

3. AN ALGORITHM FOR SOLVING DSILP

Our aim is to solve DSILP (9), but it is impossible to solve (9) directly, since it is an NP-hard problem. Hence, we solve SILP ignoring the constraints that each coefficient is an SP2. Here we denote again the variables h_k's instead of d_k's. Then DSILP is relaxed to a standard SILP and we can use several standard methods to solve the SILP, see for example [3]. Since SILP is a continuous optimization problem, an obtained optimal solution does not always satisfy the condition that each coefficient is an SP2. Hence it might be a infeasible solution for DSILP. Hence, we have to combine SILP and a B & B method to obtain an optimal solution for DSILP. We explain the main idea how to get an optimal solution for DSILP by combining SILP and B & B method. If there are some $\bar{h}_i$'s that are not SP2 in an optimal solution for SILP, then select one $\bar{h}_j$ that is not SP2 and generate two subproblems, which one has an additional constraint $h_j \le \lfloor \bar{h}_j \rfloor$ and the other has an additional constraints $h_j \ge \lceil \bar{h}_j \rceil$. Here $\lfloor \bar{h}_j \rfloor$ is the maximum SP2 coefficients that is less than or equal to $\bar{h}_j$ and $\lceil \bar{h}_j \rceil$ is the minimum SP2 coefficients that is greater than or equal to $\bar{h}_j$.

To solve SILP problem, we exploited the 3-phase method, and we introduce the algorithm shortly in the following.

[An algorithm for solving SILP by 3-phase method]

INPUT: $N, \omega_p, \omega_s, S, M, m_0, \ldots, m_K$
OUTPUT : $\bar{\mathbf{h}} = (\bar{h}_0,\ldots,\bar{h}_K), \bar{\delta}$,
(Phase 1):
Generate a discretized linear programming problem with discretizing parameter q.
Solve the discretized linear programming problem and obtain, $\bar{\mathbf{h}}, \bar{\delta}, \bar{\mathbf{y}}, \omega_0, \ldots, \omega_K$ where $\bar{\mathbf{h}}$ is an optimal primal variable vector, $\bar{\mathbf{y}}$ is an optimal dual variable vector for the discretized linear programming problem and $\omega_0,\ldots,\omega_K$ are the frequencies that correspond to the active constraints in the descretized linear programming problem.
(Phase 2):
Delete the variables $\bar{y}(\omega_l)$ that are zero. For each pair (ω_i,ω_j) whose ω_i and ω_j are very close and $\bar{y}(\omega_l)$,

$\overline{y}(\omega_j) \neq 0$:
do
$\overline{y}(\omega_i) \leftarrow \overline{y}(\omega_i) + \overline{y}(\omega_j)$,
$y_\alpha(\omega_j) \leftarrow 0$ and delete $\overline{y}(\omega_j)$,
$\omega_i \leftarrow (\omega_i + \omega_j)/2$.
end
(Phase 3):
Solve the SILP using Newton method or quasi Newton method with using $(\underline{\mathbf{h}}, \delta, (\overline{y}(\omega_{i_1}), \ldots, \overline{y}(\omega_{i_k}, \omega_{i_1}, \ldots, \omega_{i_k})$ as the initial solution.
Here, $\overline{y}(\omega_{i_1}), \ldots, \overline{y}(\omega_{i_k})$ and $\omega_{i_1}, \ldots, \omega_{i_k}$ are the variables left in the operation of phase 2.
Output the solution of the Newton/quasi Newton method.
Now, we describe the B & B method for solving DSILP in the following:

[B & B procedure for DSILP]

INPUT: $N, \omega_p, \omega_s, S, M, m_0, \ldots, m_K$
OUTPUT :$h_0, \ldots, h_K, \delta$,
$k \leftarrow 0$,
$\overline{z} \leftarrow$ high value.
Generate DSILP (9), and set SILP $P(0)$ by relaxing the condition to be SP2 numbers.
$\mathscr{P} \leftarrow \{P(0)\}$.
while $\mathscr{P} \neq \emptyset$ **do**
Select $P \in \mathscr{P}$.
$\mathscr{P} \leftarrow \mathscr{P} \backslash \{P\}$.
Solve SILP P by 3 Phase method.
if $\delta < \overline{z}$
then
if the optimal solution $(\overline{\mathbf{h}}, \overline{\delta})$ of P is a solution with SP2 coefficients
then
$\overline{z} \leftarrow \overline{\delta}$,
$\mathbf{h}^* \leftarrow \overline{\mathbf{h}}$,
else
select j that $\overline{h}_j$ is not an SP2, and generate $P(k+1)$ by adding a constraint
$h_j \geq \lceil \overline{h}_j \rceil$ to P,
generate $P(k+2)$ by adding a constraint
$h_j \leq \lfloor \overline{h}_j \rfloor$ to P,
$\mathscr{P} \leftarrow \mathscr{P} \cup \{P(k+1), P(k+2)\}$,
$k \leftarrow k+2$.
end if
end if
end while
Output $h_0^*, \ldots, h_K^*, \overline{z}$.

4. NUMERICAL EXPERIMENTS

We executed some computational experiments to certify the performance of the proposed filter design method. We consider a low pass filter with the odd length and the symmetric characteristic with $S = 1$, that is:

$$\Omega = [0, \omega_p] \cup [\omega_s, \pi], \tag{10}$$

$$H_d(\omega) = \begin{cases} 1, & 0 \leq \omega \leq \omega_p, \\ 0, & \omega_s \leq \omega \leq \pi. \end{cases} \tag{11}$$

The approximation errors from the proposed scheme are calculated for the following three sets of parameters, (A), (B), (C) for $N = 9, \ldots, 41$. Discretizing parameter q to generate the discretized linear programming problem is $4(K+1)$.

	M	ω_p	ω_s	U
(A)	$2(K+1)$	0.3π	0.35π	16,
(B)	$2(K+1)$	0.4π	0.41π	16,
(C)	$2(K+1)$	0.4π	0.43π	16,

We set each $m_k = 2$. The CPU used is mobile Pentium III 650 MHz and memory is 192 M bytes. We used glpk (Ver.4.4) [2] to obtain continuous solutions and to solve subproblems in Branch and Bound. In Figure 1 and 2, we show the objective value of our method and of continuous solutions for $K = 4, 6, \ldots, 20$. The expression "Continuous" in these figures means the optimal continuous solution and "CSD" means the SP2 solution of our method.

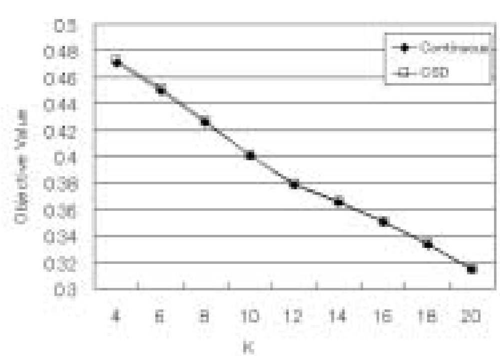

Figure 1: $\omega_p = 0.4\pi$, $\omega_s = 0.41\pi$

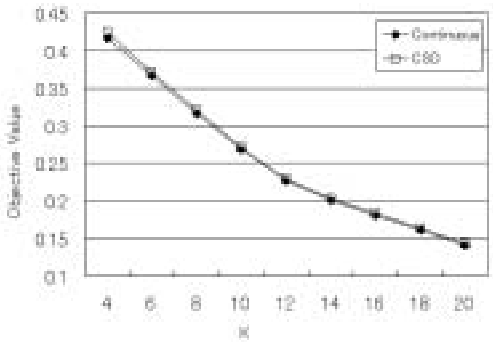

Figure 2: $\omega_p = 0.4\pi$, $\omega_s = 0.43\pi$

In these figures, it was confirmed that the objective values of our method are close to the one of the continuous SILP. In general, it is known that the transferband gets narrow, it is difficult to design FIR filter, but in Figure 1, the objective value by our method is still very close the one of the continuos SILP in spite that transferband is narrow.

In Figure 3, ..., Figure 6, the magnitude responses are shown for $\omega_p = 0.4\pi$, $\omega_s = 0.41\pi$ and Figure 7, ..., Figure 10 show the magnitude responses for $\omega_p = 0.3\pi$, $\omega_s = 0.35\pi$ and $\omega_p = 0.4\pi$, $\omega_s = 0.43\pi$.

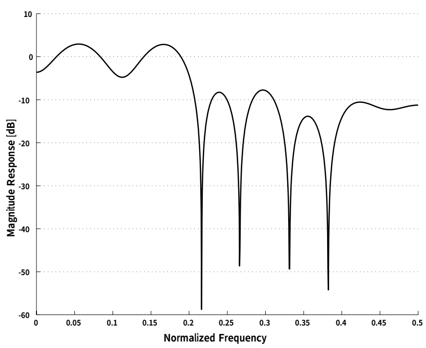

Figure 3: $K = 14, \omega_p = 0.4\pi$, $\omega_s = 0.41\pi$

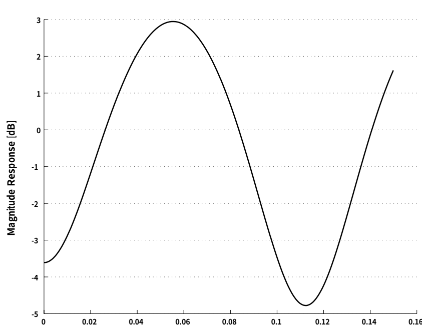

Figure 4: $K = 14, \omega_p = 0.4\pi$, $\omega_s = 0.41\pi$

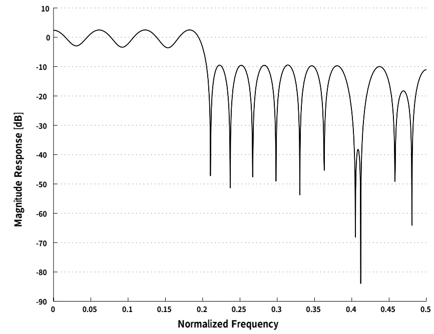

Figure 5: $K = 18 \omega_p = 0.4\pi$, $\omega_s = 0.41\pi$

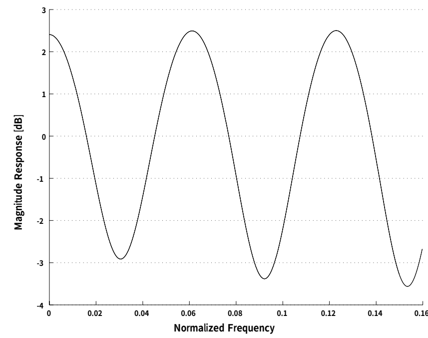

Figure 6: $K = 18 \omega_p = 0.4\pi$, $\omega_s = 0.41\pi$

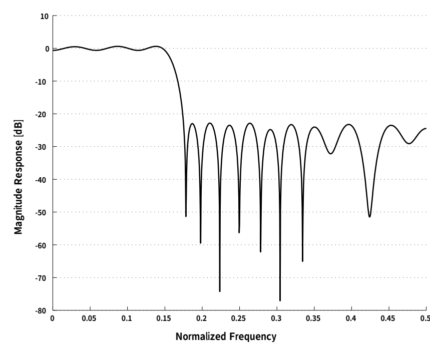

Figure 7: $K = 20, \omega_p = 0.3\pi$, $\omega_s = 0.35\pi$

In Figure 3, ..., Figure 6, it is observed that almost equi-ripple characteristic are obtained in both of two cases $K = 14$, and $K = 18$. Especially, in case of $K = 18$, it is shown that the magnitude response is almost equi-ripple. In Figure 7, ..., Figure 10, these magnitude responses show that our method is efficient in not only stopband but also passband. In case of $K = 14$, it is shown that the magnitude response in passband is small and in case of $K = 16$, the magnitude response in stopband is almost equi-ripple.

In these results, it is shown that our method to design FIR filter is effective on obtaining of equi-ripple magnitude responses.

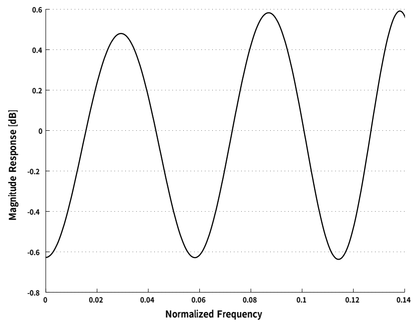

Figure 8: $K = 20, \omega_p = 0.3\pi,\ \omega_s = 0.35\pi$

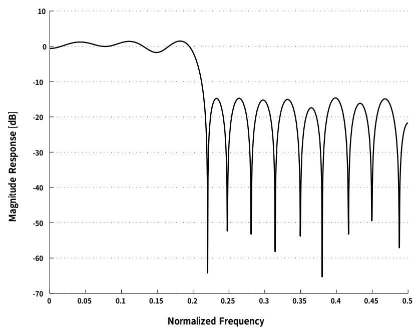

Figure 9: $K = 16, \omega_p = 0.4\pi,\ \omega_s = 0.43\pi$

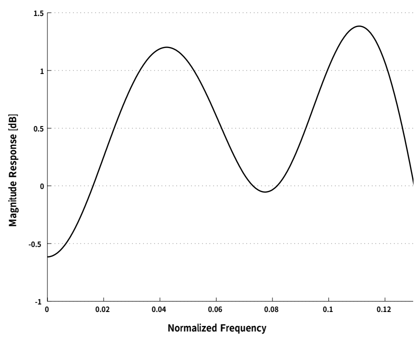

Figure 10: $K = 16, \omega_p = 0.4\pi,\ \omega_s = 0.43\pi$

5. CONCLUSION

In this paper, we proposed a new design method of FIR filters with SP2 coefficients. In this method, it is possible to obtain an optimal discrete coefficients. It is confirmed that the optimal coefficients of linear phase FIR filter with the SP2 coefficients could be designed with enough precisions through the computational experiments.

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