CORRELATED DISCRETE DISTURBERS CANCELATION IN DATA COMMUNICATION SYSTEMS

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ABSTRACT

In this paper we propose a method for suppressing discrete disturbers in data communication systems where the modulation scheme is implemented using the FFT (Fast Fourier Transform) algorithm. Similar to radio frequency interference (RFI) cancelation in the frequency domain, the compensation is performed after the FFT in the receiver. As opposed to the RFI methods it is not necessary to reserve some of the subchannels for the compensation purpose. However, the new method requires at least one reference tone and all discrete disturbers impairing the data transmission performance must be related to it. For example, this is the case for harmonics where the fundamental acts as reference tone. A detailed derivation of the compensation method is presented and illustrated by means of an example.

1. INTRODUCTION

Discrete disturbers are a serious problem for any kind of data transmission systems. Especially in FFT/IFFT based systems they impair not only the small frequency region at the center frequency but also the vicinity due to the leakage effect of the rectangular window [1]. This problem was addressed in detail in the context of RFI suppression [2]. RFI is considered to be a severe impairment for broadband transmission systems. In this paper we derive a new frequency domain compensation scheme for discrete disturbers which are harmonics of a strong signaling tone. This occurs for example in integrated voice and data applications where both data and voice are present at the same time on a single pair of copper wires.

In order to compensate for the harmonics in the frequency domain, the compensation for the leakage effect which arises whenever the center frequency of the disturber does not lie on the FFT grid will be analyzed in the next section. Then, the cancelation method for the harmonics can be derived and applied to a data communication system.

2. COMPENSATION FOR LEAKAGE

A sinusoidal disturber at the line can be expressed as

$$d[n] = |A|\cos(\omega_0 n + \varphi) = \mathrm{Re}(Ae^{j\omega_0 n}) = \\ = \frac{Ae^{j\omega_0 n} + A^* e^{-j\omega_0 n}}{2}, \quad (1)$$

where A is the complex amplitude and ω_0 the center frequency of the disturber. The corresponding discrete-time Fourier transform $D(e^{j\omega})$ is simply a pair of impulses at $+\omega_0$ and $-\omega_0$ which are repeated periodically with period 2π:

$$D(e^{j\omega}) = \sum_{k=-\infty}^{\infty} A\pi\delta(\omega - \omega_0 + 2\pi k) + \\ + \sum_{k=-\infty}^{\infty} A^*\pi\delta(\omega + \omega_0 + 2\pi k). \quad (2)$$

However, if $d[n]$ is multiplied by a rectangular window

$$d_w[n] = d[n]w[n] \quad (3)$$

with

$$w[n] = \begin{cases} 1, & 0 \le n \le N-1, \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

the corresponding Fourier transform $D_w(e^{j\omega})$ shows spectral smearing introduced by the window $W(e^{j\omega})$ (see [1]):

$$W(e^{j\omega}) = \frac{1 - e^{-j\omega N}}{1 - e^{-j\omega}}. \quad (5)$$

$D_w(e^{j\omega})$ is depicted in Fig. 1 and can be expressed as

$$D_w(e^{j\omega}) = \frac{A}{2}\frac{1 - e^{-j(\omega-\omega_0)N}}{1 - e^{-j(\omega-\omega_0)}} + \frac{A^*}{2}\frac{1 - e^{-j(\omega+\omega_0)N}}{1 - e^{-j(\omega+\omega_0)}}. \quad (6)$$

The shape becomes obvious if we consider that $D_w(e^{j\omega})$ results from a convolution of $D(e^{j\omega})$ and $W(e^{j\omega})$. Next, the N-FFT can be easily calculated:

$$D_w[k] = D_w(e^{j\frac{2\pi}{N}k}) = Aw_1 + A^* w_2, \quad (7)$$

where w_1 and w_2 are given by

$$w_1 = \frac{1}{2}\frac{1 - e^{-j(\frac{2\pi}{N}k - \omega_0)N}}{1 - e^{-j(\frac{2\pi}{N}k - \omega_0)}}, \quad (8)$$

$$w_2 = \frac{1}{2}\frac{1 - e^{-j(\frac{2\pi}{N}k + \omega_0)N}}{1 - e^{-j(\frac{2\pi}{N}k + \omega_0)}}. \quad (9)$$

Eq. (7) can be rewritten as a matrix multiplication if all complex numbers are split in their real and imaginary parts:

$$\begin{bmatrix} \mathrm{Re}(D_w[k]) \\ \mathrm{Im}(D_w[k]) \end{bmatrix} = \quad (10)$$

$$= \begin{bmatrix} \mathrm{Re}(w_1) + \mathrm{Re}(w_2) & \mathrm{Im}(w_2) - \mathrm{Im}(w_1) \\ \mathrm{Im}(w_1) + \mathrm{Im}(w_2) & \mathrm{Re}(w_1) - \mathrm{Re}(w_2) \end{bmatrix} \begin{bmatrix} \mathrm{Re}(A) \\ \mathrm{Im}(A) \end{bmatrix}$$

$$= \mathbf{W}(k, \omega_0) \begin{bmatrix} \mathrm{Re}(A) \\ \mathrm{Im}(A) \end{bmatrix}. \quad (11)$$

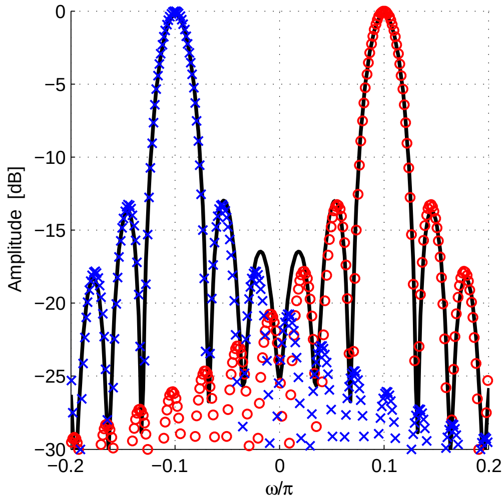

Figure 1: $D_w(e^{j\omega})$ (black, solid line), left hand part of $D_w(e^{j\omega})$ (red circles), right hand part of $D_w(e^{j\omega})$ (blue crosses).

The notation $\mathbf{W}(k,\omega_0)$ is used to express that the relationship between $D_w[k]$ and the complex amplitude A is a matrix multiplication which depends on k and ω_0. Note that a multiplication of two arbitrary complex numbers x and y can be expressed by a matrix multiplication with special symmetry properties:

$$\begin{bmatrix} \mathrm{Re}(x) & -\mathrm{Im}(x) \\ \mathrm{Im}(x) & \mathrm{Re}(x) \end{bmatrix} \begin{bmatrix} \mathrm{Re}(y) \\ \mathrm{Im}(y) \end{bmatrix}. \tag{12}$$

These symmetry properties are not given in Eq. (11) except if the term $A^* w_2$ in Eq. (7) is neglected. However, the computational complexity (2 multiplications, 2 additions) is the same in all cases.

From Eq. (7), Eq. (8) and Eq. (9) it is clear that the DFT $D_w[k]$, $k = 0,\ldots,N-1$, can be calculated if the complex amplitude A and frequency ω_0 of the sinusoidal signal is known. From the reversed relationship it is possible to express A if at least one $D_w[k]$ and ω_0 is known:

$$\begin{bmatrix} \mathrm{Re}(A) \\ \mathrm{Im}(A) \end{bmatrix} = \mathbf{W}^{-1}(k,\omega_0) \begin{bmatrix} \mathrm{Re}(D_w[k]) \\ \mathrm{Im}(D_w[k]) \end{bmatrix}. \tag{13}$$

Now, to compensate the leakage at the carrier l, the carrier k is multiplied by $\mathbf{W}^{-1}(k,\omega_0)$ to get the real and imaginary part of the amplitude A. A further multiplication by $\mathbf{W}(l,\omega_0)$ gives the desired signal which must be subtracted from the carrier l. This procedure is shown in Fig. 2 where $s[n]$ represents the desired data signal and $d[n]$ the disturber as given in Eq. (1). Of course, the two multiplications can be combined to reduce the computational complexity.

The presented technique can mainly be used if the disturber lies outside the frequency band of the data signal $s[n]$. If this is not the case one or more in-band tones must be reserved to detect the disturber and therefore, they cannot be used to transmit data [2].

In the next section we will consider the situation where not only the interference of the fundamental tone but also its harmonics should be canceled.

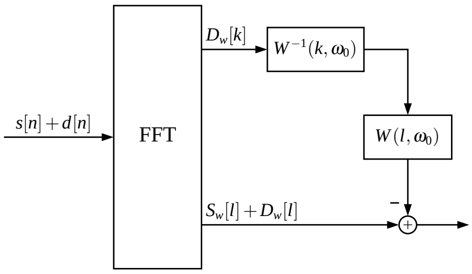

Figure 2: Leakage compensation.

3. COMPENSATION FOR HARMONICS

To compensate for the harmonics of a sinusoidal disturber after the FFT it is necessary that we establish a constant relationship between the carrier corresponding to the sinusoidal disturber and the carriers corresponding to the harmonics. For simplicity reasons it is first assumed that the sinusoidal disturber lies on the FFT grid which means that

$$\omega_0 = k\frac{2\pi}{N}, \quad k \in \mathbb{N}. \tag{14}$$

The disturbing signal is given by

$$d[n] = |A_1|\cos(\omega_0 n + \varphi_1) + \sum_{i=2}^{M} |A_i|\cos(i\omega_0 n + \varphi_i). \tag{15}$$

The phase difference of the carriers k and $k \cdot i$ is

$$\begin{aligned} \Delta\Phi = \Phi_1 - \Phi_i = (\omega_0 n + \varphi_1) - (i\omega_0 n + \varphi_i) = \\ = (1-i)\omega_0 n + \varphi_1 - \varphi_i. \end{aligned} \tag{16}$$

From Eq. (16) we see that $\Delta\Phi$ is dependent on the time index n. Therefore a direct compensation for the harmonic is hardly possible. However, in order to obtain a constant phase difference the argument $\Phi_1 = \omega_0 n + \varphi$ of the fundamental has to be multiplied by the factor i because then the time index n cancels in the difference $\Delta\Phi$:

$$\begin{aligned} \Delta\Phi = i\Phi_1 - \Phi_i = i(\omega_0 n + \varphi_1) - (i\omega_0 n + \varphi_i) = \\ = i\varphi_1 - \varphi_i = \text{const.} \end{aligned} \tag{17}$$

This means that the phase of the carrier k must be multiplied by i. This phase shifting operation can be efficiently implemented by using the CORDIC algorithm [3]. More easily, the i-th power of the carrier k could be computed instead of the phase shifting. Additionally, the power operation has the advantage that the amplitude is also corrected. This becomes obvious if the amplitudes of the harmonics of a sinusoidal signal are considered after it is processed by a polynomial (this polynomial should represent the nonlinearity generating the undesired harmonics):

$$x[n] = B_1 cos(\omega n) \quad (18)$$
$$y(x) = b_1 x + b_2 x^2 + b_3 x^3 + \cdots \quad (19)$$
$$d[n] = y(x[n]) = \frac{1}{2} b_2 B_1^2 + \frac{3}{8} b_4 B_1^4 + \ldots \quad (20)$$
$$(B_1 b_1 + \frac{3}{4} b_3 B_1^3 + \ldots) cos(\omega n) + \ldots$$
$$(\frac{1}{2} B_1^2 b_2 + \frac{1}{2} B_1^4 b_4 + \ldots) cos(2\omega n) + \ldots$$
$$(\frac{1}{4} B_1^3 b_3 + \frac{5}{16} B_1^5 b_5 + \ldots) cos(3\omega n) + \ldots$$

If for example only the third and fifth harmonic are dominant and the fifth harmonic is smaller than the third one Eq. (20) can be simplified

$$y([x[n]) \approx B_1 b_1 cos(\omega n) + \frac{1}{4} B_1^3 b_3 cos(3\omega n) + \ldots =$$
$$= A_1 cos(\omega n) + A_3 cos(3\omega n) + \ldots \quad (21)$$

From Eq. (21) it is clear that the amplitude $A_3 = \frac{1}{4} B_1^3 b_3$ is proportional to the third power of the amplitude B_1 of the fundamental. Therefore, the power operation compared to the phase shifting operation is advantageous, as was stated before.

Next, the case where the sinusoidal disturber does not lie on the FFT grid will be analyzed. To solve this problem the results from the previous sections will be used. The procedure can be split into 5 steps:

1. Calculate the complex amplitude A_1 from $D_w[k]$ by multiplication with $\mathbf{W}^{-1}(k, \omega_0)$ where k is the nearest integer to $\frac{\omega_0 N}{2\pi}$.
2. Perform the amplitude and phase compensation by calculation of the i-th power.
3. Multiply by a complex factor $c_i = A_i / A_1^i$. The output then corresponds to the complex amplitude A_i of the i-th harmonic. Generally, the factor c_i must be set via an adaptive algorithm, because it depends on many unknown quantities (nonlinear behavior of the power amplifier which generates the disturber, transfer function of the whole receive path, temperature, ...).
4. Map the signal with complex amplitude A_i onto the carrier l. This can be performed by an multiplication with $\mathbf{W}(l, i\omega_0)$ where l is the nearest integer to $\frac{i\omega_0 N}{2\pi}$.
5. Subtract the correction term from carrier l.

Fig. 3 shows the compensation scheme for $i = 3$ (3rd harmonic). The leakage effect caused by the fundamental tone is compensated for as well. As we will see in Sec. 4 this procedure leads to a high attenuation of the undesired harmonics. In practice, this is not always necessary. Therefore, the scheme in Fig. 3 could be simplified at the cost of reduced performance. For that purpose, the matrix multiplication ($\mathbf{W}^{-1}(k, \omega_0)$) used to estimate the complex amplitude A_1 is replaced by a complex multiplication with w_1^{-1}. This means that the term $A^* w_2$ in Eq. (7) is neglected. As we can see in Fig. 1 the influence of the left spectral peak is more and more negligible the higher the center frequency ω_0 is. Next, the ordering of the multiplication and the third power operation can be exchanged and w_1^{-1} must be replaced by w_1^{-i}.

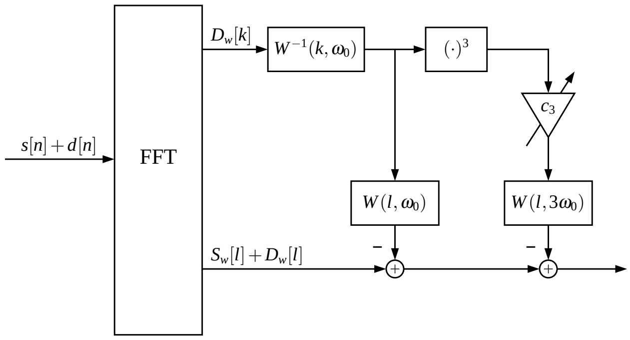

Figure 3: Compensation of 3rd harmonic.

It should be noted that generally, a matrix multiplication and a ith power operation do not commute. Only if the matrix multiplication corresponds to a complex multiplication (see Eq. (14)) can the ordering be exchanged. Finally, all remaining multiplications can be combined into one single matrix multiplication:

$$\mathbf{C}_i = \begin{bmatrix} c_{11_i} c_{12_i} \\ c_{21_i} c_{22_i} \end{bmatrix} = w_1^{-i} c_i \mathbf{W}(l, i\omega_0). \quad (22)$$

In addition, the two matrix multiplications for the leakage compensation can be combined into one single matrix multiplication:

$$\widetilde{\mathbf{W}}(k, l, \omega_0) = \mathbf{W}^{-1}(k, \omega_0) \mathbf{W}(l, \omega_0). \quad (23)$$

The simplified scheme is depicted in Fig. 4. In this case only one matrix multiplication is used for the compensation of the i-th harmonic. However, the 4 coefficients of the matrix C_i have to be tuned via an adaptive algorithm. Generally, is it recommendable to make the adaptive filter order as small as possible to get the best convergence behavior and smallest misadjustment [4]. Fig. 5 shows a configuration where only 2 coefficients (real and imaginary part of $\widetilde{c}_i$) are adapted as it is the case in Fig. 3. The coefficient $\widetilde{c}_i$ is given by

$$\widetilde{c}_i = w_1^{-i} c_i. \quad (24)$$

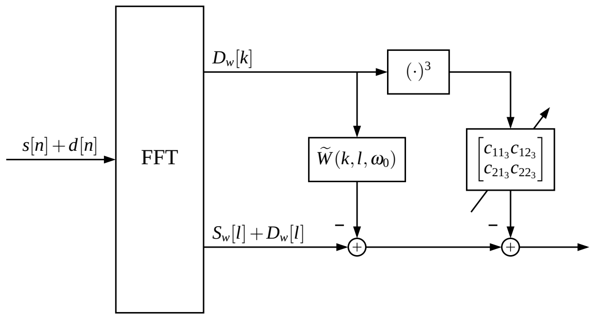

Figure 4: Simplified scheme for the compensation of the 3rd harmonic ($i = 3$), 4 coefficients.

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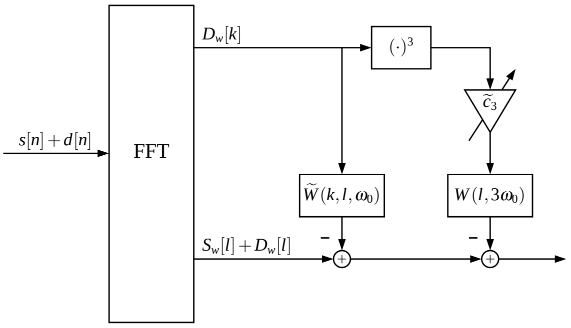

Figure 5: Simplified schemes for the compensation of the 3rd harmonic ($i = 3$), 2 coefficients.

4. EXAMPLE

In the following we will describe the impact of a signaling tone on the performance of a data transmission system. In Fig. 6 a simplified scheme of the considered system is shown. The used frequency bands are depicted in Fig. 7. From this it is clear that the signaling tone itself can be filtered out via a highpass (HP) but its harmonics will still impair the data transmission. Now, to compensate for the impact of the signaling tone, the methods introduced in the last section can be applied. For each harmonic the nearest affected carrier and if necessary also carriers in the neighborhood must be compensated. Some simulation results for the third harmonic are shown in Tab. 1. Different setups are considered:

- Setup 1: Configuration according to Fig. 3
- Setup 2: Configuration according to Fig. 4
- Setup 3: Configuration according to Fig. 5
- Setup 4: Configuration according to Fig. 4 but without the left hand leakage compensation branch.
- Setup 5: Configuration according to Fig. 5 but without the left hand leakage compensation branch.

In the case where a high attenuation of the harmonics is required setup 1 must be employed. Furthermore, we can observe that if the ratio A_3/A_1 is too high, the attenuation performance (for setup 1) degrades because not only the fundamental but also the harmonic causes leakage. Due to the fact that the harmonic overlaps with the data frequency spectrum a compensation for its leakage effect is rather difficult. If the ratio A_3/A_1 is small, only the leakage effect of the fundamental must be considered. If it is not compensated for, the attenuation is reduced significantly to very low values of 8 dB (see setup 4 and 5). As expected, the additional degree of freedom in setup 2 compared with setup 3 (4 adaptive coefficients instead of 2) does not improve the attenuation figures. From a practical point of view, the simplified configurations (setup 2 and 3) present a good compromise between performance and computational complexity. All results given in Tab. 1 correspond to the average attenuation over 50 FFT frames. The adaptation is performed by a signed LMS algorithm [4] where the difference between output and input of the decision device of the affected carrier is used as an error signal. Note that in this example the frequency of the disturber was selected in such a way that its third harmonic lies quite accurately on a carrier and, therefore, it is not necessary to compensate for carriers in the neighborhood.

5. CONCLUSION

We derived a new method for the compensation of harmonics of a signaling tone which impair the performance of a data communication system. The method operates in the frequency domain and was illustrated by means of an example. Generally, the proposed technique can be applied whenever discrete disturbers which are related to a reference tone interfere with the desired signal.

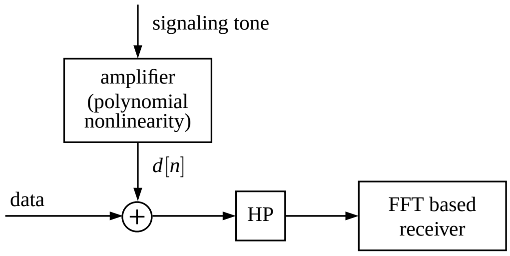

Figure 6: Simplified scheme of a data transmission system interfered with a discrete disturber.

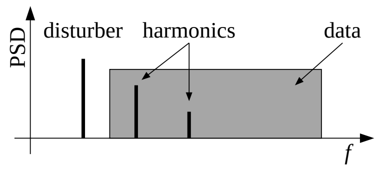

Figure 7: PSD (Power Spectral Density) of data signal and disturber.

Table 1: Attenuation figures depending on the ratio A_3/A_1 for different compensators

A_3/A_1	setup 1 att. [dB]	setup 2 att. [dB]	setup 3 att. [dB]	setup 4 att. [dB]	setup 5 att. [dB]
0.1	42.7	18.0	17.9	8.0	8.0
0.5	28.8	18.4	18.4	18.9	18.9
1.0	22.8	17.6	17.5	18.0	18.0

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