14th European Signal Processing Conference (EUSIPCO 2006), Florence, Italy, September 4-8, 2006, copyright by EURASIP

SVD-BASED EQUALIZATION FOR ZERO PADDED MULTICARRIER SYSTEMS IN TIME-VARIANT FADING CHANNELS

Tiziano Bianchi and Fabrizio Argenti

Dipartimento di Elettronica e Telecomunicazioni, University of Firenze
Via S. Marta 3, I-50139, Firenze, Italy
phone: +39 055 4796485, fax: +39 055 472858, e-mail: {bianchi, argenti}@lenst.det.unifi.it

ABSTRACT

In this paper, we address the problem of equalization for filterbank transceivers in the presence of a dispersive time-variant channel. Filterbank transceivers can be adapted to the channel transfer function to yield intersymbol interference (ISI) cancellation. However, when the channel is time-variant, the transceiver should be changed whenever the channel evolves. In this paper, we will allow both the transmitter and the receiver to change and satisfy the interference-free condition, under the assumption of a zero-padded block transmission. Two transmitter-receiver pairs are proposed by using a singular value decomposition (SVD) of the channel matrix, and they are periodically adapted to the channel status relying on an SVD tracking algorithm. Simulation results show that minimum performance loss with respect to the ideal receiver can be achieved by the proposed approach, while it clearly outperforms systems based on a constant transmitter.

1. INTRODUCTION

Multicarrier techniques have been extensively employed in recent communication systems both in wired and in wireless scenarios. Filterbank transceivers are an attractive generalization of multicarrier systems [1]. With respect to classical systems, more degrees of freedom are available to design systems that are more robust to receiver impairments, such as synchronization errors and carrier frequency offset [2].

Filterbank transceivers are multichannel systems in which data transmission is usually block-based. Like multicarrier systems, they suffer from both inter-symbol interference (ISI) and intercarrier interference (ICI). The cancellation of both types of interference can be achieved imposing an interference-free condition, usually referred to as zero-forcing (ZF), on the overall system, comprehending the transmitter, the channel and the receiver [3]. In the case of a simple additive Gaussian noise channel, the theory of perfect reconstruction (PR) filterbanks yields ZF transceivers [4].

If we consider a dispersive channel, the ZF condition is also dependent on the channel distortion. Knowledge of the channel time spread and the use of a proper guard interval allows us to remove interblock interference (IBI) [5]. Moreover, the degrees of freedom involved in the filter design can be exploited in order to maximize some performance measure independently of the channel status [6].

In the most general case, both the transmitter and the receiver can be chosen to fulfill the ZF condition while satisfying some optimality criterion. If the channel is time-invariant, the ZF transceiver can be set-up at the beginning of the transmission and then it remains unchanged. When a time-variant channel is considered, the optimum transceiver should change whenever the channel evolves. This implies several problems. First, the transmitter and the receivers should be aware of common channel state information. Second, high computational cost to derive the new ZF receiver filterbank at each adaptation may be necessary.

The aim of this paper is investigating the implementation of an efficient transceiver in the case of transmission over a time-variant channel. We will assume that zero-padding is used at the transmitter and that both the transmitter and the receiver can change to satisfy the ZF condition. We will show that transmitter-receiver pairs outperforming the classical pseudo-inverse (PI) solution [5] can be computed under the ZF condition by using a singular value decomposition (SVD) of the channel matrix. Moreover, we will highlight that with a proper design of the transmit filters the proposed systems outperform any other system based on the PI approach. In order to provide a feasible implementation of the proposed transceiver in the case of a time-variant channel, a fast receiver adaptation based on SVD tracking is used. Simulation results show that minimum performance loss with respect to the optimum transceiver can be achieved by the proposed transceiver.

This work has been partially supported by PRIN 2005: "Situation and location aware design solutions over heterogeneous wireless networks"

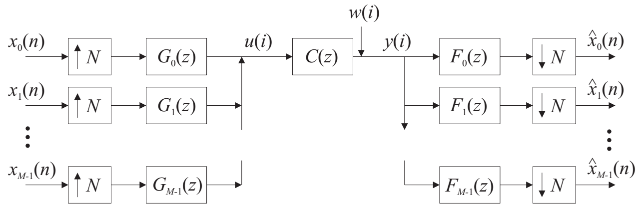


Figure 1: System model of M-subband filterbank transceiver

2. FILTERBANK TRANSCEIVER MODEL

The proposed system is shown in Fig. 1. The M input streams $x_k(n)$, $k = 0, 1, ..., M-1$ are upsampled by N and then fed into the M transmit filters $g_k(i)$ whose output are added together to produce the transmitted sequence $u(i)$. The discrete time channel is modeled as an FIR filter of length L_c followed by an additive white Gaussian noise $w(i)$. The output of the channel $y(i)$ is filtered by the M receive filters $f_k(i)$ and then each filter output is downsampled by N to produce the M output streams $\hat{x}_k(n)$. Usually, we have $N \geq M$, i.e., the system is nonmaximally decimated, with a redundancy $L = N - M$. In our system, we choose to have sufficient redundancy to avoid intersymbol interference, i.e., $L \geq L_c - 1$.

When a fading channel is considered, the coefficients of the FIR channel filter are modeled as complex random time-variant variables. The parameters of the fading channel are assumed to change slowly with time, and if an adequate time interval is observed, we can consider the channel as locally stationary. In the proposed system, the coefficients of the FIR channel are supposed to be constant over an interval of N_s samples, where $N_s \geq N$. In this way, for each set of M input symbols at the modulator, the N corresponding output samples are fed into a locally LTI system $C(z) = \sum_{n=0}^{L_c} c(n)z^{-n}$.

The system that has been previously described can be more conveniently represented as a Multiple-Input Multiple-Output (MIMO) system. The input of the MIMO system is the vector $\mathbf{x}(n) = [x_0(n), x_1(n), ..., x_{M-1}(n)]^T$. The output vector $\hat{\mathbf{x}}(n)$ is defined in a similar way. The noise vector process is given by $\mathbf{w}(n) = [w(nN), w(nN+1), ..., w(nN+N-1)]^T$ and we define both the transmitted vector $\mathbf{u}(n) = [u(nN), ..., u(nN+N-1)]^T$ and the received vector $\mathbf{y}(n) = [y(nN), ..., y(nN+N-1)]^T$.

The transmitter and receiver filters can be represented by using Nth order type-I and type-II polyphase components, respectively, to yield $G_k(z) = \sum_{l=0}^{N-1} G_{k,l}(z^N)z^{-l}$ and $F_k(z) = z^{-N+1}\sum_{l=0}^{N-1} F_{k,l}(z^N)z^l$, where $G_{k,l}(z) = \sum_n g_k(nN+l)z^{-n}$ is the lth polyphase component of the kth subband filter $g_k(n)$ ($F_{k,l}(z)$ is defined in a similar way).

The polyphase matrices yield a matrix representation of both the receiver and the transmitter. The $N \times M$ transmitter polyphase matrix is defined as

$$[\mathbf{T}(z)]_{m,n} = G_{n,m}(z). \tag{1}$$

whereas the $M \times N$ receiver polyphase matrix is given by

$$[\mathbf{R}(z)]_{m,n} = F_{m,n}(z).. \tag{2}$$

Finally, since we have $L_c < N$, the channel matrix is given by

$$\mathbf{C}(z) = \begin{bmatrix} c(0) & \cdots & 0 & z^{-1}c(L_c) & \cdots & z^{-1}c(1) \\ \vdots & \ddots & & & \ddots & \vdots \\ c(L_c-1) & & & & & z^{-1}c(L_c) \\ c(L_c) & & & & & 0 \\ \vdots & \ddots & & & \ddots & \vdots \\ 0 & \cdots & c(L_c) & c(L_c-1) & \cdots & c(0) \end{bmatrix}. \tag{3}$$

The resulting MIMO system is described by the polynomial matrix

$$\mathbf{\Psi}(z) = \mathbf{R}(z)\mathbf{C}(z)\mathbf{T}(z). \tag{4}$$

The condition that satisfies the zero-forcing (ZF) criterion, can be expressed as

$$\mathbf{\Psi}(z) = z^{-d_0}\mathbf{I}_M, \tag{5}$$

where d_0 indicates the transmission delay.

3. CHANNEL EQUALIZATION

Recently, it has been shown that zero-padding (ZP) precoders achieve better results than cyclic prefix (CP) ones in a fading channel environment [5]. For this reason, we will restrict our attention to the polynomial matrices $\mathbf{T}(z)$ and $\mathbf{R}(z)$ that arise when a zero padding precoding is used.

The transmitter polyphase matrix of a system using ZP precoding can be expressed as

$$\mathbf{T}(z) = \begin{bmatrix} \mathbf{G}(z) \\ \mathbf{0}_{L\times M} \end{bmatrix}, \tag{6}$$

where $\mathbf{G}(z)$ indicates the polyphase matrix of a maximally decimated synthesis filterbank. In the following, we will assume that $\mathbf{G}(z)$ satisfies the paraunitary property, i.e. $\tilde{\mathbf{G}}(z)\mathbf{G}(z) = \mathbf{I}_M$, where we define $\tilde{\mathbf{G}}(z) \triangleq \mathbf{G}^H(1/z^*)$. Moreover, $\mathbf{G}(z)$ is assumed independent of the channel.

When a ZP transmitter is used, we can decompose the channel matrix as $\mathbf{C}(z) = [\ \mathbf{C}_0 \quad \mathbf{C}_1(z)\ ]$, where $\mathbf{C}_0$ is a constant matrix (see eq. (3)). It is easy to show that

$$\mathbf{C}(z)\mathbf{T}(z) = \mathbf{C}_0\mathbf{G}(z). \tag{7}$$

The PR condition in (5) can be achieved by means of several receivers. In this paper we will focus on the solution based on the pseudo-inverse (PI) of $\mathbf{C}_0$, given by

$$\mathbf{R}(z) = \tilde{\mathbf{G}}(z)(\mathbf{C}_0^H\mathbf{C}_0)^{-1}\mathbf{C}_0^H. \tag{8}$$

In the following, this receiver will be denoted as PI receiver.

An alternative formulation of the transmitter-receiver pair described above can be found through the singular value decomposition (SVD) [7] of the channel matrix $\mathbf{C}_0$ given by

$$\mathbf{C}_0 = \mathbf{U}\begin{bmatrix} \mathbf{\Sigma} \\ \mathbf{0}_{(N-M)\times M} \end{bmatrix}\mathbf{V}^H \tag{9}$$

where $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices and $\mathbf{\Sigma}$ is an $M \times M$ diagonal matrix containing the singular values. In general, it is convenient to rewrite the left matrix as $\mathbf{U} = [\mathbf{U}_s\ \mathbf{U}_o]$, where $\mathbf{U}_s$ is $N \times M$ and $\mathbf{U}_o$ is $N \times (N-M)$, so that we obtain the more compact form $\mathbf{C}_0 = \mathbf{U}_s\mathbf{\Sigma}\mathbf{V}^H$.

If we do not restrict our interest to a fixed transmitter scheme, the transmitter polyphase matrix of our system can be expressed as

$$\mathbf{T}(z) = \begin{bmatrix} \mathbf{Q}\mathbf{G}(z) \\ \mathbf{0}_{L\times M} \end{bmatrix}, \tag{10}$$

where $\mathbf{Q}$ is an $M \times M$ channel-dependent precoding matrix. The design of $\mathbf{Q}$ offers some degrees of freedom that can be exploited for maximizing the performance of the system. Usually, it is imposed that the system has a constant average transmit power $\mathscr{P}_0 = M\sigma_x^2/N$, so that the precoding matrix $\mathbf{Q}$ has to satisfy $\mathrm{Tr}\left\{\mathbf{Q}^H\mathbf{Q}\right\} = M$.

In this paper, we have investigated two different solutions for the design of $\mathbf{Q}$, both satisfying a constant transmit power. The first one imposes $\mathbf{Q} = \mathbf{V}$ and, under the ZF criterion, leads to the transmitter-receiver pair

$$\mathbf{T}(z) = \begin{bmatrix} \mathbf{V}\mathbf{G}(z) \\ \mathbf{0}_{L\times M} \end{bmatrix} \tag{11}$$

$$\mathbf{R}(z) = \tilde{\mathbf{G}}(z)\mathbf{\Sigma}^{-1}\mathbf{U}_s^H. \tag{12}$$

The second one uses $\mathbf{Q} = \mathbf{V}\gamma\mathbf{\Sigma}^{-1/2}$, where $\gamma = \sqrt{M/\mathrm{Tr}\left\{\mathbf{\Sigma}^{-1}\right\}}$, and leads to the transmitter-receiver pair

$$\mathbf{T}(z) = \begin{bmatrix} \mathbf{V}\gamma\mathbf{\Sigma}^{-1/2}\mathbf{G}(z) \\ \mathbf{0}_{L\times M} \end{bmatrix} \tag{13}$$

$$\mathbf{R}(z) = \tilde{\mathbf{G}}(z)\gamma^{-1}\mathbf{\Sigma}^{-1/2}\mathbf{U}_s^H. \tag{14}$$

In the following, we will refer to the two systems as SVD-ZF1 and SVD-ZF2, respectively.

3.1 MSE Performance Analysis

By using the fact that $\mathbf{G}(z)$ is paraunitary, the MSE for the PI receiver can be derived as

$$\zeta_{\mathrm{PI}} = \sigma_w^2\mathrm{Tr}\left\{(\mathbf{C}_0^H\mathbf{C}_0)^{-1}\right\}, \tag{15}$$

whereas the MSE for the SVD-ZF1 and the SVD-ZF2 receivers are given by

$$\zeta_{\mathrm{SVD-ZF1}} = \sigma_w^2\mathrm{Tr}\left\{\mathbf{\Sigma}^{-2}\right\} \tag{16}$$

$$\zeta_{\mathrm{SVD-ZF2}} = \frac{\sigma_w^2}{M}\left(\mathrm{Tr}\left\{\mathbf{\Sigma}^{-1}\right\}\right)^2, \tag{17}$$

It is straightforward to demonstrate that the PI and the SVD-ZF1 receivers achieve the same MSE, since

$$\begin{aligned}\zeta_{\mathrm{PI}} =& \sigma_w^2\mathrm{Tr}\left\{(\mathbf{C}_0^H\mathbf{C}_0)^{-1}\right\} = \sigma_w^2\mathrm{Tr}\left\{\mathbf{V}\mathbf{\Sigma}^{-2}\mathbf{V}^H\right\} \\ =& \sigma_w^2\mathrm{Tr}\left\{\mathbf{\Sigma}^{-2}\right\} = \zeta_{\mathrm{SVD-ZF1}},\end{aligned} \tag{18}$$

where the second equality holds due to the fact that $\mathbf{V}$ in (9) is a unitary matrix. On the other hand, both the PI and the SVD-ZF1 systems achieve an higher value of MSE than the SVD-ZF2 system independently of the channel status. This can be explicitly verified by expressing the two MSEs as

$$\zeta_{\mathrm{SVD-ZF1}} = \sigma_w^2 \sum_i \mu_i^2 \quad (19)$$

$$\zeta_{\mathrm{SVD-ZF2}} = \frac{\sigma_w^2}{M}\left(\sum_i \mu_i\right)^2, \quad (20)$$

where $\mu_i = [\mathbf{\Sigma}]_{ii}^{-1}$, and reminding that, thanks to Jensen's inequality, $(\sum_i \mu_i/M)^2 \leq (\sum_i \mu_i^2)/M$, where the equality holds if and only if all the μ_i have the same value.

3.2 Error Probability Analysis

Although both PI and SVD-ZF1 receivers achieve the same MSE on the received vector of symbols, the variance of the noise components affecting individual symbols is in general different from subcarrier to subcarrier. As noticed in earlier papers on OFDM systems [8], this fact greatly influences the bit error probability. The bit error probability can be tightly approximated by

$$P_e \approx \alpha Q\left(\sqrt{\beta \gamma_b}\right), \quad (21)$$

where γ_b is the equivalent SNR per bit, whereas α and β are parameters that depend on the modulation. For example, for BPSK and QPSK we have $\alpha = 1$ and $\beta = 2$.

In the case of a simple AWGN channel, we have $\gamma_b = \mathscr{E}_b/N_0$. When considering a time-dispersive channel equalized by a ZF receiver, the equivalent SNR per bit measured at the kth subcarrier becomes

$$\gamma_b = \frac{\mathscr{E}_b}{N_0 \eta(k)}, \quad (22)$$

where $\eta(k)$ models the noise enhancement due to ZF equalization and can be expressed as $\eta(k) = \frac{1}{2\pi j}\oint \left[\mathbf{R}(z)\tilde{\mathbf{R}}(z)\right]_{k,k} z^{-1}dz$. Let us define the function

$$f(x) \triangleq \alpha Q\left(\sqrt{\frac{\beta \mathscr{E}_b}{N_0 x}}\right). \quad (23)$$

The above function is monotonically increasing in the variable x and it is convex for $0 < x < \beta\mathscr{E}_b/3N_0$ and concave for $x > \beta\mathscr{E}_b/3N_0$. Moreover, we can express the average BER of a multicarrier system having M subcarriers as

$$P_e = \frac{1}{M}\sum_{k=0}^{M-1} f\big(\eta(k)\big). \quad (24)$$

Now, if we assume to operate at an SNR value so that

$$\frac{\mathscr{E}_b}{N_0} > \frac{3}{\beta}\eta(k), \quad \forall k, 0 \leq k < M, \quad (25)$$

then all the functions $f(x)$ involved in (24) are in the convex region and, thanks to the Jensen's inequality, we have

$$\frac{1}{M}\sum_{k=0}^{M-1} f\big(\eta(k)\big) \geq f\left(\frac{1}{M}\sum_{k=0}^{M-1}\eta(k)\right) = f\left(\frac{\zeta}{M\sigma_w^2}\right), \quad (26)$$

where the leftmost equality holds if and only if all the $\eta(k)$ have the same value. Hence, given two receivers yielding the same MSE, equation (26) implies that for high SNR values the BER is minimized by designing the transmit filters so that the values of the $\eta(k)$ are constant over all the subcarrier index. In the following, we will say that these filters satisfy the equal gain (EG) property.

In the case of the PI receiver, the values of $\eta(k)$ are derived as

$$\begin{aligned}\eta_{\mathrm{PI}}(k) =& \frac{1}{2\pi j}\oint \left[\tilde{\mathbf{G}}(z)\mathbf{V}\mathbf{\Sigma}^{-2}\mathbf{V}^H\mathbf{G}(z)\right]_{k,k} z^{-1}dz \\ =& \sum_i \left[\boldsymbol{G}'(i)^H \mathbf{\Sigma}^{-2}\boldsymbol{G}'(i)\right]_{k,k} \\ =& \sum_i\sum_r \left|g'_{k,r}(i)\right|^2 \mu_r^2 = \sum_r \kappa'_{k,r}\mu_r^2,\end{aligned} \quad (27)$$

where we define $\mathbf{V}^H\mathbf{G}(z) = \sum_i \boldsymbol{G}'(i)z^{-i}$, $g'_{k,r}(i) = [\boldsymbol{G}'(i)]_{k,r}$, and $\kappa'_{k,r} = \sum_i |g'_{k,r}(i)|^2$. In order to have the values $\eta_{\mathrm{PI}}(k)$ independent of the index k, we should design the transmit filters so that $\kappa'_{k,r}$ does not depend on k. However, since $\mathbf{G}(z)$ is assumed to be channel-independent, there is no choice of $\mathbf{G}(z)$ satisfying the EG property. As a consequence, the PI receiver can not achieve the lower bound on the error probability given in (26).

On the other hand, when considering the SVD-based systems, the values of $\eta(k)$ can be expressed as

$$\begin{aligned}\eta_{\mathrm{SVD-ZF1}}(k) =& \frac{1}{2\pi j}\oint \left[\tilde{\mathbf{G}}(z)\mathbf{\Sigma}^{-2}\mathbf{G}(z)\right]_{k,k} z^{-1}dz \\ =& \sum_i \left[\boldsymbol{G}(i)^H\mathbf{\Sigma}^{-2}\boldsymbol{G}(i)\right]_{k,k} \\ =& \sum_i\sum_r |g_{k,r}(i)|^2 \mu_r^2 = \sum_r \kappa_{k,r}\mu_r^2\end{aligned} \quad (28)$$

$$\begin{aligned}\eta_{\mathrm{SVD-ZF2}}(k) =& \frac{\gamma^{-2}}{2\pi j}\oint \left[\tilde{\mathbf{G}}(z)\mathbf{\Sigma}^{-1}\mathbf{G}(z)\right]_{k,k} z^{-1}dz \\ =& \gamma^{-2}\sum_i \left[\boldsymbol{G}(i)^H\mathbf{\Sigma}^{-1}\boldsymbol{G}(i)\right]_{k,k} \\ =& \gamma^{-2}\sum_i\sum_r |g_{k,r}(i)|^2 \mu_r = \gamma^{-2}\sum_r \kappa_{k,r}\mu_r,\end{aligned} \quad (29)$$

where we consider $\mathbf{G}(z) = \sum_i \boldsymbol{G}(i)z^{-i}$ and $\kappa_{k,r} = \sum_i |g_{k,r}(i)|^2$. In this case, the values $\eta_{\mathrm{SVD-ZF1}}(k)$ and $\eta_{\mathrm{SVD-ZF2}}(k)$ can be made independent of the index k. In fact, the values $\kappa_{k,r}$ do not depend on the channel status, and several choices of $\mathbf{G}(z)$ satisfying the EG property can be found. In particular, two well known transmitters satisfying this condition are given by $\mathbf{G}(z) = \mathbf{W}^H$, where $\mathbf{W}$ is the DFT matrix and $\mathbf{G}(z) = \mathscr{H}$, where $\mathscr{H}$ is the Hadamard matrix [8]. Therefore, the following conclusions hold:

- given an SVD-ZF1 system equipped with filters satisfying the EG property, there always exists an SNR value $\gamma_{b,\mathrm{svd-zf1}}$ so that for $\gamma_b \geq \gamma_{b,\mathrm{svd-zf1}}$ the SVD-ZF1 system outperforms any other PI system;
- given an SVD-ZF2 system equipped with filters satisfying the EG property, there always exists an SNR value $\gamma_{b,\mathrm{svd-zf2}}$ so that for $\gamma_b \geq \gamma_{b,\mathrm{svd-zf2}}$ the SVD-ZF2 system outperforms both any other SVD-ZF1 system and any other PI system.

4. SVD TRACKING ALGORITHM

If we suppose to adapt both the transmitter and the receiver at each transmitted symbol, the complexity of the SVD computation would make the solution in (11) and (12) quite unfeasible for a practical implementation. To reduce the computational complexity, we could update the receiver at time intervals longer than a single symbol duration. However, if the time interval between two consecutive receiver updates becomes comparable to the channel coherence time, the performance of the system will decrease very quickly. For these reasons, in this paper we use an algorithm to track the SVD from the channel coefficient estimates, without computing it explicitly at each received symbol.

14th European Signal Processing Conference (EUSIPCO 2006), Florence, Italy, September 4-8, 2006, copyright by EURASIP

Consider the SVD of $\mathbf{C}_0$ rewritten as

$$\mathbf{C}_0 = [\ \mathbf{U}_s \quad \mathbf{U}_o\ ] \begin{bmatrix} \mathbf{\Sigma}_s \\ \mathbf{0} \end{bmatrix} \mathbf{V}_s^H. \tag{30}$$

The above equation is identical to (9) if $\mathbf{\Sigma}_s = \mathbf{\Sigma}$ and $\mathbf{V}_s^H = \mathbf{V}^H$.

Consider now a slowly time-varying channel. The channel coefficients are assumed constant within a symbol duration but they vary from one symbol to another. Relying on this model, if we denote with $\mathbf{C}_0^{(n)}$ the channel matrix as it appears at time n, we can express the channel matrix at time $n+1$ as

$$\mathbf{C}_0^{(n+1)} = \mathbf{C}_0^{(n)} + \Delta\mathbf{C}_0^{(n)}, \tag{31}$$

where $\Delta\mathbf{C}_0^{(n)}$ is a small matrix perturbation. Assuming that $\mathbf{U}_s^{(n)}$, $\mathbf{U}_o^{(n)}$, $\mathbf{\Sigma}_s^{(n)}$ and $\mathbf{V}_s^{(n)}$ are the matrices deriving from the SVD of $\mathbf{C}_0^{(n)}$, we can express the corresponding matrices at time $n+1$ in a similar way. It is possible to give a first-order approximation of the matrix perturbations of the above matrices as a function of $\Delta\mathbf{C}_0^{(n)}$. The equations take the following form

$$\Delta\mathbf{U}_s \approx \mathbf{U}_o\mathbf{U}_o^H \Delta\mathbf{C}_0 \mathbf{V}_s \mathbf{\Sigma}_s^{-1} \tag{32}$$

$$\Delta\mathbf{U}_o \approx -\mathbf{U}_s\mathbf{\Sigma}_s^{-H}\mathbf{V}_s^H \Delta\mathbf{C}_0^H \mathbf{U}_o \tag{33}$$

$$\Delta\mathbf{\Sigma}_s \approx \mathbf{U}_s^H \Delta\mathbf{C}_0 \mathbf{V}_s \tag{34}$$

$$\Delta\mathbf{V}_s \approx \mathbf{0}, \tag{35}$$

where the time index has been dropped since all quantities refer to the same index n. If we suppose to be able to estimate the channel perturbation $\Delta\mathbf{C}_0$ at each received symbol, then we can recursively apply (32)-(35) to track the SVD. The algorithm can be summarized in the following steps:

1. Compute the SVD of $\mathbf{C}_0$ to initialize the algorithm.
2. Estimate channel matrix perturbation $\Delta\mathbf{C}_0$.
3. Derive perturbations of SVD matrices according to (32)-(35).
4. Update matrices $\mathbf{U}_s$, $\mathbf{U}_o$ and $\mathbf{\Sigma}_s$.
5. Go back to step 2.

4.1 Algorithm Details

Since SVD perturbations are approximated, the algorithm gives at each iteration a coarser estimate of the channel SVD. Hence, the initialization step has to be performed periodically to reset the SVD. In the following, we will refer to the time interval between two initialization steps as T_{reset}.

The singular values update matrix $\Delta\mathbf{\Sigma}_s$ is only approximately diagonal and not necessarily real, even though $\mathbf{\Sigma}_s$ is real and diagonal. This requisite is not necessary in order to implement the SVD-based system. However, the proposed algorithm could benefit from a reduced computational complexity by requiring $\mathbf{\Sigma}_s$ to be at least diagonal.

In the following, we propose two variations of the SVD tracking algorithm. In the first one, the singular values update matrix is computed as $\Delta\mathbf{\Sigma}_s = \text{diag}\{\mathbf{U}_s^H \Delta\mathbf{C}_0 \mathbf{V}_s\}$, so that the matrix $\mathbf{\Sigma}_s$ is kept diagonal. This procedure will be referred to as *Tracker-1* algorithm. The second one uses at each step the expression for $\Delta\mathbf{\Sigma}_s$ as given in (34) and will be denoted as *Tracker-2* algorithm. For making comparisons, a system where the receiver is kept unchanged over T_{reset} as well as an ideal system, i.e., where the SVD is computed at each received symbol, have been considered. These systems will be denoted as *Fixed* and *Ideal*, respectively.

4.2 Complexity Issues

The computational complexity of Tracker-1 and Tracker-2 algorithms as a function of the number of subcarriers M and of the zero-padding length L are given by, respectively

$$\mathscr{C}_1 = 6L^2M + 8LM^2 \ \text{ flops} \tag{36}$$

$$\mathscr{C}_2 = 6L^2M + 10LM^2 + 8M^3/3 \ \text{ flops}. \tag{37}$$

As to the complexity of the Ideal system, we consider the cost of a direct SVD computation at each step with the Golub-Reisch algorithm [9], from which we obtain

$$\mathscr{C}_{\text{SVD}} = 14LM^2 + 22M^3 \ \text{ flops}. \tag{38}$$

For making a comparison, we derive also the complexity of a Moore-Penrose pseudo-inverse computation. In this case, the computational complexity can be expressed as

$$\mathscr{C}_{\text{PI}} = 3LM^2 + 13M^2/4 \ \text{ flops}. \tag{39}$$

$\mathscr{C}_1$	$\mathscr{C}_2$	$\mathscr{C}_{\text{PI}}$	$\mathscr{C}_{\text{SVD}}$
100%	233%	34%	1075%

Table 1: The complexity of Tracker-1 and Tracker-2 algorithms with respect to $\mathscr{C}_{\text{SVD}}$ and $\mathscr{C}_{\text{PI}}$, for $M = 64$ and $L = 16$.

In Tab. 1 we compare the cost of the proposed algorithms normalized to the value of $\mathscr{C}_1$, assuming $M = 64$ and $L = 16$. It is evident that the Tracker-1 algorithm yields a less burdensome implementation than the Tracker-2 one, since it relies on the inversion of a diagonal matrix. On the other hand, both Tracker-1 and Tracker-2 algorithms are characterized by a sensibly increased complexity with respect to the computation of a Moore-Penrose pseudo-inverse. Hence, this additional complexity is justified only if the SVD-based schemes can guarantee a sufficient performance gain with respect to the PI scheme.

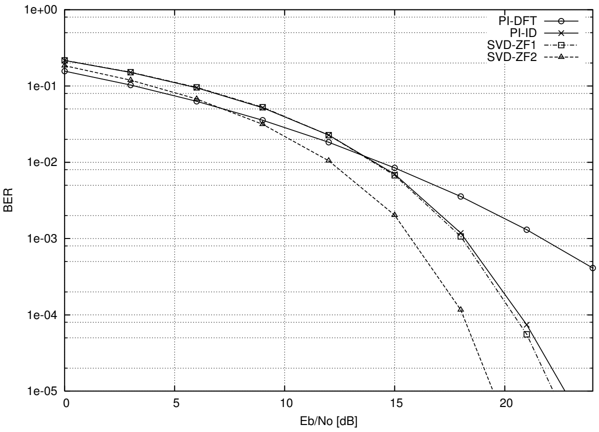
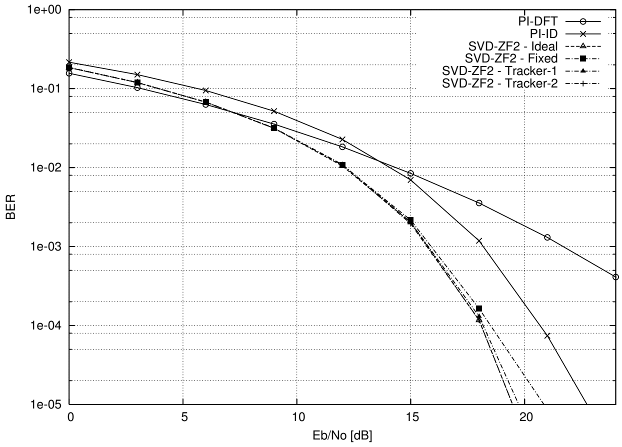


Figure 2: BER performance for different ideal systems.

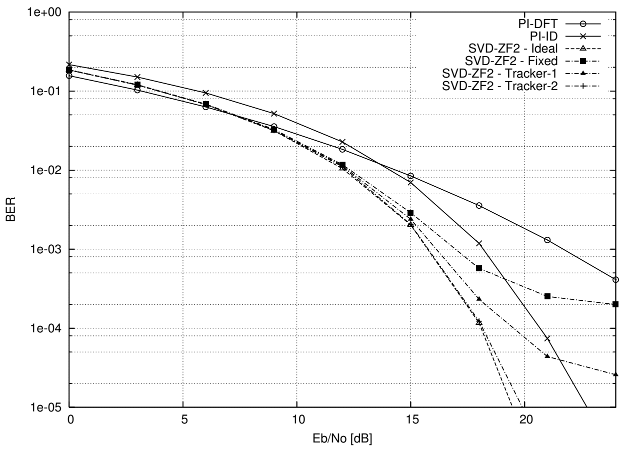
5. SIMULATION RESULTS

For the test of the proposed algorithm we have designed a system with 64 subcarriers equally spaced in a 20 MHz bandwidth, with a carrier frequency of 5 GHz. A guard time of 16 null samples is appended after each modulated symbol. The modulation used is QPSK. As to the PI system, we have considered a DFT-based system, i.e., with $\mathbf{G}(z) = \mathbf{W}_M^H$, denoted as PI-DFT, and a single carrier system, i.e., with $\mathbf{G}(z) = \mathbf{I}_M$, denoted as PI-ID. As to the SVD-based systems, for the sake of simplicity we have considered only DFT-based systems. The channel is modeled according to the specifications of model A in [10]. In particular, we suppose a maximum mobile speed of 3 m/s, corresponding to a channel coherence time Δt_c of 20 ms. The simulations have been conducted under the hypothesis of perfect channel knowledge.

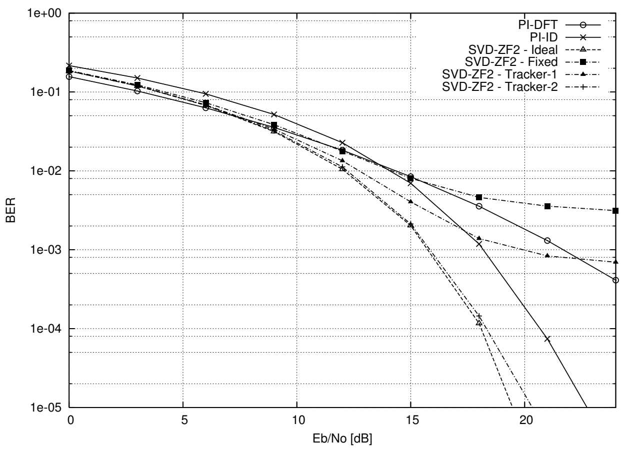
In Fig. 2, we compare the performance of the different systems assuming an ideal update of the channel status. As can be seen, when increasing the SNR value the proposed SVD-ZF2 system achieves the best BER performance. On the other hand, the



(a)



(b)



(c)

Figure 3: BER performance of tracking schemes with different T_{reset} values: (a) $T_{reset} = \Delta t_c/100$; (b) $T_{reset} = \Delta t_c/50$; (c) $T_{reset} = \Delta t_c/20$.

SVD-ZF1 system does not guarantee a sensible performance gain with respect to the single carrier PI system, even though it outperforms the PI system that is based on the same transmit filters.

According to the previous results, the performance of the two tracking algorithms has been evaluated using only the SVD-ZF2 scheme. The proposed systems have been tested with several choices of T_{reset}, ranging from $\Delta t_c/20$ to $\Delta t_c/100$. The bit error rate (BER) is reported in Fig. 3. As can be seen, the Tracker-2 algorithm yields results very close to the ideal case, confirming the tightness of the proposed approximation. In particular, the Tracker-2 algorithm allows us to outperform the PI system in all the proposed cases.

On the other hand, a sensible degradation can be observed when the Tracker-1 algorithm is used. We deem that the loss of performance of the Tracker-1 algorithm with respect to the Tracker-2 one is mainly due the fact that in our algorithm $\mathbf{V}_s$ is kept constant, since its perturbations are of order greater than one, and hence, after few steps, it produces only a coarse diagonalization of the channel matrix.

6. CONCLUSIONS

In this paper, different schemes of filterbank transceivers have been proposed to deal with a time-variant frequency-selective channel. The performance of the proposed schemes has been evaluated both analytically and by means of computer simulation. In particular, our analysis demonstrates that the scheme denoted as SVD-ZF2, with a proper design of the transmit filters, outperforms any other linear scheme based on zero-forcing equalization. A fast transceiver adaptation algorithm has been used based on SVD tracking. Simulation results show that minimum performance loss can be achieved with our tracking algorithm, while the complexity remains reasonable if compared with that of classical ZF approaches.

REFERENCES

[1] P. P. Vaidyanathan, “Filter banks in digital communications,” *IEEE Circuits Syst. Mag.*, vol. 1, no. 2, pp. 4–25, 2001.

[2] T. Bianchi, F. Argenti, and E. Del Re, “Performance of filterbank and wavelet transceivers in the presence of carrier frequency offset,” *IEEE Trans. Commun.*, vol. 8, no. 53, pp. 1323–1332, Aug. 2005.

[3] X.-G. Xia, “New precoding for intersymbol interference cancellation using nonmaximally decimated multirate filterbanks with ideal FIR equalizers,” *IEEE Trans. Signal Processing*, vol. 45, no. 10, pp. 2431–2441, Oct. 1997.

[4] M. Vetterli, “Perfect transmultiplexers,” in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Processing*, vol. 11, Tokyo, Japan, Apr. 1986.

[5] B. Muquet, Z. Wang, G. B. Giannakis, M. de Courville, and P. Duhamel, “Cyclic prefixing or zero padding for wireless multicarrier transmissions?” *IEEE Trans. Commun.*, vol. 50, no. 12, pp. 2136–2148, Dec. 2002.

[6] Y.-P. Lin and S.-M. Phoong, “BER optimized channel independent precoder for OFDM systems,” in *Conf. Rec. GLOBECOM ’02 - IEEE Global Telecommun. Conf.*, vol. 1, Taipei, Taiwan, Nov. 2002.

[7] O. Edfors, M. Sandell, J.-J. van de Beek, S. Wilson, and P. Borjesson, “OFDM channel estimation by singular value decomposition,” *IEEE Trans. Commun.*, vol. 46, no. 7, pp. 931–939, July 1998.

[8] Y.-P. Lin and S.-M. Phoong, “BER minimized OFDM systems with channel independent precoders,” *IEEE Trans. Signal Processing*, vol. 51, no. 9, pp. 2369–2380, Sept. 2003.

[9] G. H. Golub and C. F. V. Loan, *Matrix Computations*. Johns Hopkins University Press, 1989.

[10] J. Medbo and P. Schramm, “Channel models for HIPERLAN/2 in different indoor scenarios,” ETSI BRAN, Tech. Rep. 3ERI085B, Mar. 1998.