ABRupt NOISE REJECTION USING WAVELET TRANSFORM FOR ELECTROMAGNETIC WAVE OBSERVATION SIGNALS

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ABSTRACT
This paper presents a method of realizing abrupt noise rejection in an observed electromagnetic (EM) wave signal by using the wavelet transform (WT) technique. Our goal is to better process the EM waves that radiate from the earth’s crust in order to predict earthquakes.

The proposed method involves the multi-scale wavelet transform domain with a second-order derivation property. Typical noise rejection methods that use WT set the threshold according to the noise power. Unfortunately, this approach can suppress the precursor radiation since its radiated energy and period are similar to the abrupt noise. The method proposed herein uses local maximum points on the WT coefficient (wavelet maxima), which means that no threshold is needed to suppress abrupt noise. It is shown that the proposed method can reject abrupt noise without dropping the precursor signal.

1. INTRODUCTION
Attention is being placed on the electromagnetic waves that radiate from the earth’s crust in advance of earthquakes and volcanic activity. Such EM waves are observed in the Extremely Low Frequency (ELF) band of 223Hz. Our research is directed towards identifying the precursor signals of earthquakes[5]-[7]. Captured signals include noise created by spurious events in the magnetosphere or the ionized layer and lightning radiation in the tropics, and so on. While various precursor signal detection methods have been proposed, their detection sensitivity is not really sufficient[8]-[13]. To improve detection sensitivity, reducing these various types of noise is critical.

In EM wave analysis, however, it is difficult to reject the abrupt noise in the power adjustment signal, near field lighting, and sensor errors due to the strong similarity between the features of these noises and those of the typical precursor signal[11]. In particular, the method of statistical analysis is extremely degraded by abrupt noises[12]. In actual observation, the precursor signal shows two kinds of feature; First is the signal increase during several days before an earthquake, another is the spike radiation for several tens of minutes so that signals defined as abrupt noise are much shorter than the duration time of the precursor radiation.

Current abrupt noises elimination in the EM wave is based on WT since abrupt noises are observed nonstationary. In [11], wavelet coefficients in the low level (high frequency component) are removed and denoised signal is synthesized through the inverse wavelet transform. This method, however, reduces spike radiation induced by precursor of the earthquake as well as undesired abrupt noises in the EM wave. Other denoising techniques using wavelet decomposition has been applied for image and ultrasonography speckle denoising, i.e. Achiem et al. proposed the symmetric alpha-stable distribution and a Bayesian processor in [4]. This method is adequate for removing a speckle noise. It is considered that the characteristic parameter estimation becomes complex since the empirical distribution of EM wave is not symmetric and it fluctuates due to daily, seasonal trend and location of the antenna. This paper proposes a simple method that uses a wavelet maxima feature to realize abrupt noise rejection. The proposed method shows good performance in rejecting only the abrupt noise from EM waves.

2. ELECTROMAGNETIC WAVE CAPTURE
2.1 General conditions
We gathered data on electromagnetic wave radiation in the ELF band (223Hz) collected at over forty observation stations in Japan. Each observation station captures the east-west, north-south, and vertical components and averages them over 6 second intervals. Thus 14400 data points are collected per day for each component. We use the data averaged over 150 seconds for convenience (i.e. 576 points per day per component). The typical electromagnetic wave radiated from the earth’s crust in an earthquake event has a field strength of about 1 pico tesla normalized by the square root of the frequency ($pT \sqrt{Hz}$) in the ELF band.

2.2 Observation data
The observed signal contains several noise components. Given that the sampled time is $k$ and the observation signal is $y(k)$, $y(k)$ is described as consisting of background noise $T(k)$, precursor signal $P(k)$, and other noises $w(k)$ which include abrupt noises. A simple model of the observed signal can be expressed as:

$$y(k) = T(k) + P(k) + w(k).$$  (1)

The features of these components are given below.

- **Background noise:** The dominant background noise in the ELF band is lightning radiation from the tropics. It passes between the ionized layer and the surface of the earth. It is weak in the daytime and strong at night because of the properties of propagation path. Furthermore,
The abrupt noise consists of the power adjustment signal, which behave as Low-Pass-Filter (LPF) or High-Pass-Filter (HPF), respectively, \( j \) is the band level. Input signal \( c^j(k) \) is divided into two wavelet coefficients, \( c^{j+1}(k) \) and \( d^{j+1}(k) \). These coefficients are given by

\[
c^{j+1}(k) = \sum_k g(n)c(k-2^n) \\
d^{j+1}(k) = \sum_k h(n)c(k-2^n)
\]

where \( c^{j+1}(k) \) is an approximate component of \( c^j(k) \), \( d^{j+1}(k) \) is a detail component of the \( c^j(k) \). The observed signal is subjected to \( c^0(k) \), with \( N \) sample points, and the calculation algorithm is repeated 4 times to extract a characteristic feature of the abrupt noise. An implementation is shown in Fig.2.

The impulse response of the second-order derivation used in this paper is shown in Table 1. It is used in Eq.(2) and Eq.(3) for calculation convenience.

**Table 1: Filter coefficient**

<table>
<thead>
<tr>
<th>( k )</th>
<th>( g(n) )</th>
<th>( h(n) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.25</td>
<td>-0.25</td>
</tr>
<tr>
<td>0</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>-1</td>
<td>0.25</td>
<td>-0.25</td>
</tr>
</tbody>
</table>

The abrupt noise is defined as a signal increase that does not continue for more than 2 samples. However, a signal longer than 3 samples is a useful abrupt signal. We define the amplitude of abrupt noise and signal as \( A \) and their length as \( L \). We also assume that \( A \) is normally distributed random number with mean= 4, variance= 1 in this paper. The abrupt noise of \( A, L = 1, 2 \) added to observed EM wave and its wavelet coefficients \( d^1(k), ..., d^4(k) \) are shown in Fig.3(a) and (b), respectively. In Fig.3(a), a local maximum point (wavelet maxima) on \( d^1(k), ..., d^4(k) \) corresponds to the point of abrupt noise in the input signal. Moreover, the value of wavelet coefficients \( d^1(k), ..., d^4(k) \) and \( d^2(k) \), ..., \( d^4(k) \) in Fig.3(a) and (b), respectively, are attenuated to 1/2 as \( j \) increases. As an example of a useful abrupt signal, the wavelet coefficient with \( A, L = 3 \) is shown in Fig.3(c). In Fig.3(c), the wavelet maxima from \( d^2(k) \) to \( d^4(k) \) correspond to the center of an abrupt signal, wavelet coefficient \( d^3(k), d^4(k) \) is attenuated to 1/2. From these results, the point of abrupt noise, \( (L \leq 2) \), corresponds to a wavelet maxima, moreover, wavelet coefficients from \( d^2(k) \) to \( d^4(k) \) are attenuated to 1/2.

These features show that signals from sources 3. and 4. can be identified by their temporal characteristics since the precursor radiation is much longer. In this paper, shorter signals (sources 3. and 4.) are defined as abrupt noise.

### 3. WAVELET TRANSFORM

#### 3.1 Algorithm

We introduce a method that uses WT for signal decom- position. Assume that \( g(n) \) and \( h(n) \) are wavelet filter coefficients, the impulse response of the second-order derivation used in this paper is shown in Table 1. It is used in Eq.(2) and Eq.(3) for calculation convenience.
4. METHOD OF ABRUPT NOISE REJECTION

4.1 Proposed method

The characteristic features of the abrupt noise are obtained from the wavelet maxima and the attenuation characteristics. The proposed algorithm for rejecting an abrupt noise is detailed below.

- **STEP1:** Wavelet coefficients $d^j(k)$ are calculated. ($j = 1, \ldots, 4$)

- **STEP2:** Points $k_L$ that correspond to wavelet maxima in $d^j(k)$ ($j = 1, \ldots, 4$) are obtained.

- **STEP3:** Moreover, points $k_M$ that satisfy the following expression in $j = 2, \ldots, 4$ are identified as abrupt noise.

$$1.5d^{j+1}(k_L) < d^j(k_L) < 2.5d^{j+1}(k_L) \quad (4)$$

- **STEP4:** The wavelet coefficient $d^j(k_M)$, which is detected as an abrupt noise, is removed using the following expression.

$$c^0_{\text{new}}(k_M) = c^0(k_M) - \sum_{j=1}^{4} d^j(k_M) \quad (5)$$

where $x = 6$ since abrupt noise impacts the lower wavelet coefficients.

These steps are applied to the entire observed signal. The procedure of the proposed method is shown in Fig.4. In the case of $L = 2$, the larger point in the abrupt noise is identified by these steps. It is assumed that the abrupt noise is rejected by calculating a procedure twice since the larger (lower) value is rejected by calculating a procedure once (twice). We assume that the number of calculating a procedure is $I$.

4.2 Typical conventional methods

The proposed method is evaluated against two conventional alternatives: wavelet coefficients with threshold and median filter processing.

- **Wavelet coefficient with threshold (WT method)**

  High frequency components $D^j(k) = \sum_{j=1}^{4} d^j(k)$ including abrupt noise are extracted by WT. When $D^j(k)$ exceeds 1.0, $D^j = \sum_{j=1}^{N} d^j(k)$ is treated as abrupt noise and reduced.

  $$c_{\text{new}}^0(k) = c^0(k) - D^j(k) \quad (\text{if } D^j(k) > 1.0). \quad (6)$$

- **Median filter**

  The median filter considers each value in the input signal in turn and looks at its nearby neighbors to decide whether or not it is representative of its surroundings. This method replaces the value with the median of those values. In this paper, window size $W$ is 5, the median filter is processed when the high frequency component $D^j(k) > 1.0$ so as to reject only abrupt noise.

5. RESULTS

The three noise rejection methods are applied to an observed signal that included short-time precursor radiation and abrupt noise. The signal captured on July 5th by Gifu Kawai station and the results of the noise rejection methods are shown in Fig.5. The horizontal axis represents hours, the vertical axis is the level of the observed signal ($pT/\sqrt{Hz}$).

The parameters of the abrupt noise used in this experiment were $A, L = 1, 2$. The abrupt noise was added at hour 3 and 6 on the horizontal line, short-time radiation occurs at hour 20. The evaluation factor of noise rejection ability, $MSE$, is given by

$$MSE = \frac{\sum_{k=1}^{N} (S_{\text{in}}(k) - S_{\text{out}}(k))^2}{N} \quad (7)$$

where $k$ is sampled time, $N$ is the number of samples per day ($N = 576$), $S_{\text{in}}(k)$ and $S_{\text{out}}(k)$ are the observed EM wave and noise rejected signal, respectively.
Figure 5: Noise rejection results of the observed EM wave including artificial abrupt noise. (a) The observed EM wave. (b) Abrupt noise was artificially added to (a). (c) Result of WT method. (d) Result of median filter. (e) Result of proposed method (I = 1).

Figure 6: The result of proposed method (I = 2).

Figure 7: Noise rejection results of the observed EM wave including artificial abrupt noise. (a) The observed EM wave with abrupt signals. (b) Abrupt noise was artificially added to (a). (c) Result of WT method. (d) Result of median filter. (e) Result of proposed method (I = 1).
An analysis of the plots indicates that all the methods examined provided abrupt noise rejection at hour 3, see Fig.5(c)-(e). The conventional methods, Fig.5(c),(d), suppressed the short-time radiation and reduced the abrupt noise. Fig.5(e) shows that the short-time radiation was not reduced, but the abrupt noise at hour 6 was not rejected. This is due to the larger value in the abrupt noise \( L = 2 \) was rejected by calculating a procedure once, however, it is not enough to apply the procedure once for denoising the abrupt noise \( L = 2 \). The result of proposed method \( I = 2 \) is shown in Fig.6. This figure shows that the abrupt noise can be rejected by applying the procedure twice. It also improved \( MSE \).

We evaluated another signal that included abrupt noise and abrupt signals. The observed signal with artificial abrupt signals \( (L > 2) \) and the result of the three noise rejection methods are shown in Fig.7. The horizontal axis represents the hours, the vertical axis is the level of the observed signal \( pT/\sqrt{Hz} \). The abrupt noise has parameters of \( A, L = 1.2 \), those of the abrupt signals are \( A, L = 3.4 \). Abrupt noise was added in the first 12 hours, abrupt signals were added in the last 12 hours. In this experiment, abrupt signals should not be reduced. The conventional methods reduced the abrupt noise in the first 12 hours as shown in Fig.7(c),(d). The abrupt signal in the last 12 hours was almost completely eliminated by the WT method and was smoothed by the median filter. Fig.7(e) shows that the abrupt signal is not reduced, but abrupt noises were not rejected since the abrupt noise of \( L = 2 \) is not rejected to apply the procedure of proposed method once.

The signal showing a result of proposed method \( I = 2 \) is shown in Fig.8(a). The abrupt noise of \( L = 2 \) was rejected in this figure, but the abrupt noise at hour 8 is not. This is due to the burstiness of the abrupt noise since the wavelet maxima are influenced by other abrupt noises. Fig.8(b), which is result of proposed method \( I = 3 \), shows that the abrupt noise was rejected completely. Therefore, the abrupt noise can be rejected by applying the procedure three times. The proposed method offers lower \( MSE \) than other methods.

It has been proven that the proposed method is effective in rejecting abrupt noise without reducing the precursor radiation. The burst noise can be rejected by applying the proposed method \( I \geq 2 \).

6. CONCLUDING REMARKS

In this paper, we proposed an abrupt noise reduction method uses wavelet decomposition. This method does not require the use of a threshold, unlike traditional methods. Experiments have shown the effectiveness of the proposed method in abrupt noise rejection. The abrupt noise of \( L = 2 \) and burst noise are not rejected by applying the procedure once. However, these noises can be rejected by applying the proposed method \( I \geq 2 \). A remaining problem is to improve the removal performance of burst noise and to obtain the optimal processing times.

REFERENCES