CHANNEL ESTIMATION IN THE PRESENCE OF MULTIPATH DOPPLER BY MEANS OF PSEUDO-NOISE SEQUENCES

Olivier Rabaste and Thierry Chonavel

ENST Bretagne, Signal & Communications Department
BP832, 29285 Brest Cedex, FRANCE
email: {olivier.rabaste,thierry.chonavel}@enst-bretagne.fr

ABSTRACT

This paper addresses the problem of multipath channel estimation when channel paths are subject to Doppler shifts. The proposed method builds a rough approximation of the signal ambiguity function by means of a filter bank. Each output of the filter bank is deconvolved by means of an MCMC approach, that provides estimates of the path delays. The estimation of amplitudes and Doppler frequencies is then carried out at each detected path delay. The method is able to cope with simultaneous paths subject to distinct Doppler offsets and with interferences occuring among paths. Cramer-Rao Lower Bounds are derived and presented with simulation results.

1. INTRODUCTION

For most applications involving multipath channel propagation, the channel impulse response $h(t)$ is of the form $h(t) = \sum_{p=1}^{P} \alpha_p \delta(t-\tau_p)$, where P is the number of paths and α_p and τ_p respectively denote the complex attenuation and the time delay of path p. However, this model is suitable only when the ratios $f_{d_p} = \frac{v_p f_c}{c}$ $(p = 1,\ldots,P)$ are negligible, where v_p is the relative celerity between the transmitter and the receiver along path p, f_c the carrier frequency and c the wave speed in the propagation medium. When these conditions are not fulfilled, Doppler effects occur, that is, the carrier frequency is shifted by f_{d_p}. Doppler compression effect [1] on the transmitted signal $e(t)$ can be neglected in many situations where the ratio v_p/c is negligible. Then the received signal $r(t)$ can be written as

$$r(t) = \sum_{p=1}^{P} \alpha_p e^{j2\pi f_{d_p}(t-\tau_p)} e(t-\tau_p) + n_0(t), \qquad (1)$$

where f_{d_p} denotes the Doppler shift of path p, and $n_0(t)$ represents some additive Gaussian noise with autocorrelation function $\Gamma_{n_0}(t)$. This situation may arise in applications such as underwater acoustics [2] or satellite communications [3]. Finally, channel estimation amounts to detect the number P of paths and estimate of the parameters $(\alpha_p, \tau_p, f_{d_p})_{p=1,P}$.

In this paper, we will assume that the transmitted signal is known by the receiver, so that a matched filtering step can be performed for helping the estimation process. The output signal resulting from the application of a matched filter to a Doppler shifted signal can be expressed by means of a well-known time-frequency transformation: the ambiguity function [4, 5], defined as

$$A_s(\tau, f) = \int_{-\infty}^{+\infty} s(t)\, s^*(t-\tau)\, e^{-j2\pi f t}\, dt.$$

This ambiguity function can roughly be built by demodulating the received signal at different frequencies spanning a certain frequency interval $[-f_{max}, f_{max}]$, where f_{max} accounts for the frequency support of $A_s(\tau, f)$ and for the Doppler range, and by applying a matched filter to each output. Processing the received signal (1) in such a way results in an output signal expressed in the time-frequency domain as

$$x(t, f) = \sum_{p=1}^{P} \alpha_p e^{-j\pi(f-f_{d_p})\tau_p} A(t-\tau_p, f-f_{d_p}) + n(t, f), \quad (2)$$

where f is the demodulating frequency and $n(t, f)$ is a complex circular gaussian noise.

In applications where severe Doppler effects arise so that the carrier frequency shift cannot be neglected, such as radar applications, chirp signals are often chosen because of their robustness against this kind of distorsions [4]. However they bring some ambiguity between the time delay and the Doppler shift: a Doppler-shifted signal will appear at a shifted time delay at the output of the matched filter. Thus, for such signals, time delay and Doppler shift parameters cannot be estimated separately. Other widely used signals in the context of multipath channel estimation are the pseudo-noise (PN) sequences because of their quasi-dirac-shape periodic autocorrelation function. For such signals no ambiguity occurs: the maximum of the ambiguity function at a specific frequency shift will appear at the true time delay. Thus time delay estimation can be carried out independently from the Doppler shift estimation.

In this paper, we propose to estimate the multipath channel in two steps by exploiting the properties of the PN sequence ambiguity function. First, delayed paths are detected at the matched-filtered output corresponding to each demodulating frequency. This is achieved by means of a method we proposed in [6] for recovering Doppler free multipath channels parameters. In this precedent paper, the channel is modeled as a Bernoulli-Gaussian process and the deconvolution is performed via an MPM algorithm [7]. In a second step, we propose to estimate the amplitude and Doppler shift parameters corresponding to each estimated time delay by means of a descent algorithm.

The paper is organised as follows: Section 2 highlights the interesting properties of the PN sequence ambiguity function. The Bernoulli-Gaussian model and the MPM algorithm

This work was supported by the SHOM (Brest, France).

are recalled in Section 3. Then the method proposed for estimating amplitudes and Doppler shifts, as well as the number of possible simultaneous paths, is explained in Section 4, together with an iterative procedure for improving parameter estimation via paths interference contributions removal. Cramer-Rao Lower Bounds and simulation results are provided in Section 5. A conclusion is given in Section 6.

2. PN SEQUENCE AMBIGUITY FUNCTION

The PN sequence $s(t)$ can be written as

$$s(t) = \frac{1}{\sqrt{NT_b}} \sum_{k=0}^{N-1} c_k \mathbb{1}_{\left[-\frac{T_b}{2}, \frac{T_b}{2}\right]}(t - kTb),$$

where $\mathbb{1}_{\Omega}(\tau) = 1$ if $\tau \in \Omega$ and 0 otherwise.. Then, the ambiguity function of $s(t)$ is

$$A_s(\tau, f) = e^{-j\pi f\tau} \sum_{k=0}^{N-1} \sum_{n=0}^{N-1} c_k c_n \frac{e^{-j\pi f(k+n)T_b}}{\pi f T_b N} \times$$
$$\sin\left(\pi f\left(T_b - |\tau + (n-k)T_b|\right)\right) \mathbb{1}_{[(k-n-1)T_b, (k-n+1)T_b]}(\tau).$$

From this expression, we can remark that:

- the zero Doppler cut $A_s(\tau, 0)$ of the ambiguity function provides the exact autocorrelation function $\lambda(t)$ of $s(t)$, that has a triangular shape:

$$\lambda(t) = \begin{cases} 1 - (N+1)\frac{|t|}{NT_b} & \text{if } t \in [-T_b, T_b], \\ -\frac{1}{N} & \text{otherwise.} \end{cases}$$

- for $f \neq 0$, $\tau \rightarrow A_s(\tau, f)$ differs from $\tau \rightarrow A_s(\tau, 0)$. However the frequency support of $f \rightarrow A_s(\tau, f)$ is about $[-\frac{1}{NT_b}, \frac{1}{NT_b}]$ and on this interval

$$A_s(\tau, f) \approx \left(1 - \left|\frac{\tau}{NT_b}\right|\right) \sum_{n=0}^{N-1} c_n^2 e^{-j2\pi f n T_b} \mathbb{1}_{[-T_b, T_b]}(\tau),$$

  that is, the triangular shape of $\tau \rightarrow A_s(\tau, f)$ is approximately preserved when $f \neq 0$. This property is interesting because it enables to look for the presence of paths via the MPM algorithm by considering only one particular waveform that doesn't depend on the frequency.

For the path p, the ambiguity function is translated around (τ_p, f_{d_p}). In order to observe these time-frequency translated versions of A_s, the received signal is demodulated at different frequencies $(f_1, \ldots, f_K)$ spanning $[-f_{max}, f_{max}]$, and a matched filter is applied to each output, as shown in Figure 1. From the properties of the ambiguity function of PN sequences, each output of this filter bank can be considered as the result of the convolution between $\lambda(t)$ and an equivalent channel impulse response $h_{f_m}(t)$ $(m = 1, \ldots, K)$ that may present multiple paths. It is therefore possible to apply a deconvolution technique for each output in order to recover $h_{f_m}(t)$.

3. FILTER BANK OUTPUT DECONVOLUTION VIA THE MPM ALGORITHM

After sampling, the output signal corresponding to a fixed frequency f_m can be written

$$\mathbf{x}_{f_m} = \mathbf{S}_\lambda \mathbf{h}_{f_m} + \mathbf{n}_{f_m},$$

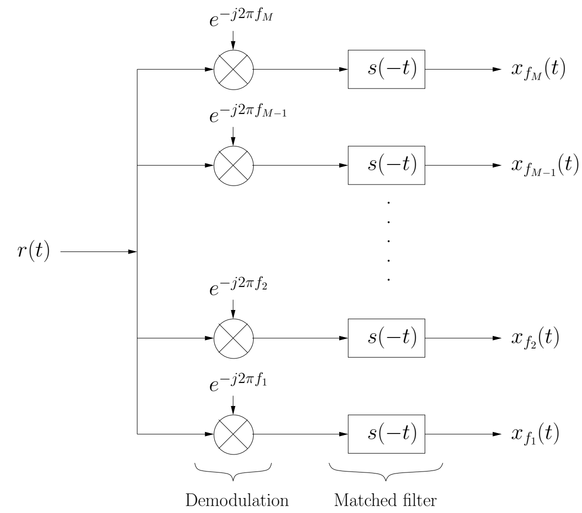

Figure 1: Processing of the received signal by a filter bank.

where $\mathbf{S}_\lambda$ is the convolution matrix associated with $\lambda(t)$. For the sake of simplicity, the subscript f_m will be replaced by m in the following.

3.1 Bernoulli-Gaussian channel modelling

The channel vector sparseness is accounted for via a Bayesian prior [6]. More precisely, we consider a Bernoulli-Gaussian model $\mathbf{z}_m = (\mathbf{h}_m, \mathbf{q}_m)$, where $\mathbf{q}_m$ is an underlying state vector of independent Bernoulli random variables $q_m(k)$ $(k = 1, \ldots, L)$ that defines the presence or the absence of a path at time index k with probability of presence $\mu = P(q_m(k) = 1)$. Conditionaly to $q_m(k)$, the entries $h_m(k)$ of $\mathbf{h}_m$ are then modeled via a gaussian mixture such that

$$p(h_m(k)|q_m(k) = a) \sim \mathcal{N}(0, \sigma_a^2) + j\mathcal{N}(0, \sigma_a^2), \qquad a = 0, 1,$$

where $\sigma_1 >> \sigma_0$. Note that this model presents a good robustness with respect to the lack of knowledge of its parameters $(\mu, \sigma_1^2, \sigma_0^2)$, and for many applications, rough values are available. Alternatively a hierachical Bayesian approach [8] could be considered that amounts to introduce prior distributions for these parameters. For the BG model, typical priors can be found for instance in [9].

With the BG model, the posterior log-likelihood of $\mathbf{z}_m$ is then

$$L(\mathbf{z}_m|\mathbf{x}_m) = -(\mathbf{x}_m - S_\lambda \mathbf{h}_m)^H \mathbf{A}_{n_m} (\mathbf{x}_m - S_\lambda \mathbf{h}_m) - \frac{\mathbf{h}_m^H \mathbf{D}_{\mathbf{q}_m} \mathbf{h}_m}{2\sigma_1^2}$$
$$- \frac{\mathbf{h}_m^H (1 - \mathbf{D}_{\mathbf{q}_m}) \mathbf{h}_m}{2\sigma_0^2} + \mathbf{q}_m^H \mathbf{q}_m \ln\left(\frac{\mu}{1-\mu} \frac{\sigma_0^2}{\sigma_1^2}\right) + C,$$

where C is a constant term, $\mathbf{D}_{\mathbf{q}_m}$ represents the diagonal matrix with k-th entry equal to $q_m(k)$ and $\mathbf{A}_{n_m} = \mathbf{\Gamma}_{n_m}^{-1}$ is the inverse of the covariance matrix of the noise $\mathbf{n}_m$. If the noise $n_0(t)$ in Eq.(1) is white, then the correlation function of $\mathbf{n}_m$ has a triangular shape, due to matched filtering.

3.2 The MPM approach

The direct maximization of this log-likelihood to obtain a MAP estimator of $\mathbf{z}_i$ is untractable. It is therefore necessary to implement simulation methods. In [6], we have proposed a solution based on the MPM algorithm. It aims at simulating the pdf of $\mathbf{z}_m$ conditional to $\mathbf{x}_m$. Since this pdf is not computable directly, the algorithm uses a Gibbs sampler to simulate realisations $z_m^i(k)$ of samples $z_m(k)$ following the *a posteriori* marginals $p(z_m(k)|\mathbf{x}_m, \mathbf{z}_m(-k))$, where $\mathbf{z}_m(-k) = (z_m(0), \ldots, z_m(k-1), z_m(k+1), \ldots, z_m(L))$. After a burn-in period corresponding to $i < I_0$, the Gibbs sampler reaches "thermal equilibrium" [7], that is, the generated samples are distributed according to the *a posteriori* pdf $p(\mathbf{z}_m|\mathbf{x}_m)$. These samples are used to compute the A Posteriori Mean estimators $\hat{\mathbf{q}}_m$ and $\hat{\mathbf{h}}_m$ of $\mathbf{q}_m$ and $\mathbf{h}_m$:

$$\hat{q}_m(k) = 1 \quad \text{if} \quad \frac{1}{I-I_0}\sum_{i=I_0+1}^{I} q_m^{(i)}(k) > s, \quad 0 \text{ otherwise},$$
$$\hat{h}_m(k) = \frac{\sum_{i=I_0+1}^{I} q_m^{(i)}(k) h_m^{(i)}(k)}{\sum_{i=I_0+1}^{I} q_m^{(i)}(k)} \quad \text{if} \quad \hat{q}_m(k) = 1, \quad 0 \text{ otherwise}.$$

The threshold s is usually set to 0.5 since it then minimizes the Bayes risk when uniform equal costs are chosen. Using the Neyman-Pearson approach, it is however possible to set a desired false-alarm probability P_{FA}; then the threshold that minimizes the detection probability is given by

$$s = \left[1 + \frac{1-\mu}{\mu} \exp\left(\frac{1}{2}\left(\frac{\sigma_0^2}{\sigma_1^2} - 1\right)\left[Q^{-1}\left(\frac{\sigma_0 P_{FA}}{\sqrt{2\pi}}\right)\right]^2\right)\right]^{-1},$$

where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{+\infty} e^{-\frac{t^2}{2}}\, dt$ is the usual Q-function.

The MPM approach was compared to other methods for estimating multipaths channel impulse responses such as the Matching Pursuit, EM and ICM methods, and proved to yield better estimation performance than these methods in terms of the mean-square error of the reconstructed noiseless received signal.

4. AMPLITUDE AND DOPPLER SHIFT ESTIMATION

From the properties of the PN sequence ambiguity function discussed above, the MPM algorithm provides accurate time delay. However the amplitudes of a given path contribution decrease away from the true Doppler frequency on the filter bank outputs. In addition, the frequency step between consecutive filters of the matched filter bank is not chosen very small for computational rapidity purpose. Moreover, paths with distinct Doppler shifts but similar time delay may be encountered. For all these reasons further processing is required for amplitude and Doppler offset estimation.

4.1 Estimation via the Levenberg-Marquardt algorithm

At a given time delay τ_p, the K filter bank outputs provide a vector denoted by $\mathbf{x} = [x_1, \ldots, x_K]^T$, the entries of which correspond to respective demodulation frequencies $(f_1, \ldots, f_K)$. Assuming that r paths are simultaneously present at this time delay and denoting by $\theta_r = [\alpha_{p,i}, f_{d_{p,i}}]_{i=1,r}$ the corresponding parameters, the contribution of all these paths to the received signal $\mathbf{x}$ is given, as a function of f, by

$$g_{p,r}(f, \theta_r) = \sum_{i=1}^{r} \alpha_{p,i} e^{-2j\pi f \tau_p} A_s(0, f - f_{d_{p,i}}).$$

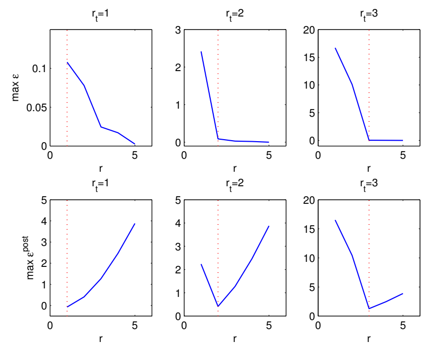

Figure 2: Reconstruction error and penalized reconstruction error for true number of paths $r_t = 1, 2, 3$.

Here, estimating θ_r according to the Maximum Likelihood (ML) criterion does not provide a good estimate, because it requires the inversion of the noise covariance matrix which is ill-conditioned due to the strong correlation of the noise samples among Doppler filters outputs. We propose therefore to consider the Least Square Error (LSE) criterion. The LSE estimator is given by

$$\hat{\theta}_r = \arg\min_{\theta_r} \sum_{k=1}^{K} \left|g_{p,r}(f_k, \theta_r) - x_k\right|^2.$$

This criterion can be minimized by means of a Levenberg-Marquardt algorithm [10, 11], whose principle is based on the Newton's algorithm but takes into account the possible non invertibility of the Hessian matrices.

However this method requires the knowledge of the number of simultaneous paths r. When r is unknown, running the algorithm for different values of r is not very helpful since the application

$$r \to \mathscr{E}(\mathbf{x}, r, \hat{\theta}_r) = \sum_{k=1}^{K} \left|g_{p,r}(f_k, \hat{\theta}_r) - x_k\right|^2,$$

decreases as r increases. Thus, some penalization of $\mathscr{E}(\mathbf{x}, r, \hat{\theta}_r)$ has to be introduced. If we interpretate $\mathscr{E}(\mathbf{x}, r, \hat{\theta}_r)$ as the opposite log-likelihood of some gaussian density, such a penalty factor can be seen as some bayesian prior upon r. A convenient and common choice [8] consists then in modelling r as a Poisson random variable with parameter ξ. This choice can be briefly justified as follows: from the Poisson theorem [12], it can be shown that many sequences of distribution for describing the paths frequency locations lead asymptotically to a Poisson distribution for the number of paths falling into a given interval. The corresponding posterior log-likelihood of r is then given by

$$L(r|\mathbf{x}) \propto L(\mathbf{x}|r) + L(r) \propto -\frac{\mathscr{E}(\mathbf{x}, r, \theta_r)}{2\sigma_f^2} + \log\left(\frac{\xi^r}{r!}\right) + C,$$

where C is a constant term. Then the corresponding penalized reconstruction error criterion is

$$\mathscr{E}^{post}(\mathbf{x}, r, \theta_r) = \frac{\mathscr{E}(\mathbf{x}, r, \theta_r)}{2\sigma_f^2} - \log\left(\frac{\xi^r}{r!}\right),$$

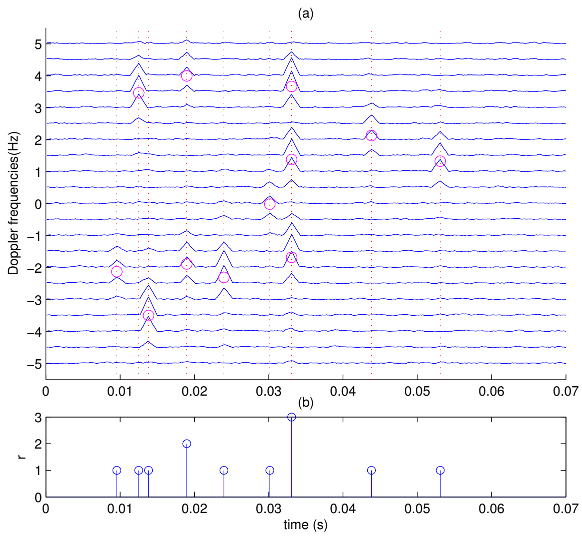

Figure 3: (a) Filter bank output (b) Number of simultaneous paths r. $\circ$: paths positions.

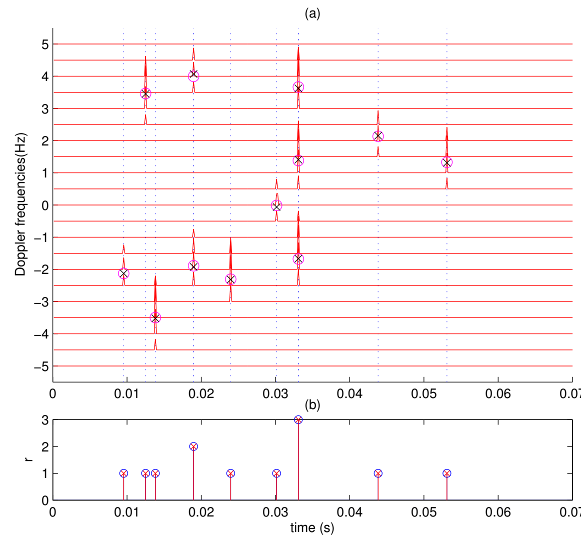

Figure 4: (a) MPM output (b) Number of simultaneous paths r. $\circ$: true values, $\times$: estimated values.

and the minimization is performed over (r, θ_r). For the simulations, we have chosen the parameter ξ close to 1 ($\xi = 1.2$) because simultaneous paths are quite uncommon in practice. Figure 2 represents the reconstruction error and the penalized reconstruction error as functions of r when its true value is $r_t = 1, 2$ and 3. Clearly the minimum of $\mathscr{E}(\mathbf{x}, r, \hat{\theta}_r)$ decreases with r while that of $\mathscr{E}^{post}(\mathbf{x}, r, \hat{\theta}_r)$ shows a global minimum at r_t.

A practical example is presented in Figures 3 and 4 for an SNR of -10 dB and a PN sequence with a period of 63 symbols: this sequence is transmitted 10 times and then averaged after matched filtering. Figure 3 shows the output of the matched filter bank together with the true paths positions. The shape of the PN sequence ambiguity function spreading along the frequency axis can be clearly seen around each path, with the maxima aligned on the same time delay. Figure 4 presents the output of the MPM algorithm for each demodulation frequency, and the results obtained with the estimation procedure that we propose. Clearly the algorithm recovers the number of simultaneous paths and good parameter estimation is achieved.

4.2 Interference cancellation

In the procedure presented above, we look for simultaneous paths present at a given time delay τ_p, but we neglect possible interference among paths affected by distinct time delays. In practice, it appears that these interferences may lead to some degraded results at high SNR. It is therefore necessary to take into account this effect in the processing. We propose thus the following processing scheme:

1. For each estimated time delay τ_p, apply the above estimation procedure to obtain a first estimate of the number of simultaneous paths $\hat{r}_p$ and of the parameters $\hat{\theta}_{\hat{r}_p} = [\hat{\alpha}_{p,1}, \hat{f}_{d_{p,1}}, \ldots, \hat{\alpha}_{p,\hat{r}_p}, \hat{f}_{d_{p,\hat{r}_p}}]$,
2. While convergence of the parameter estimates has not been reached, execute for each delay τ_p:
   (a) Computation of other paths contributions:

$$x_{-p}(t,f) = \left[\sum_{\substack{k=1\\k\neq p}}^{P} \sum_{i=1}^{\hat{r}_k} \hat{\alpha}_{k,i} e^{j2\pi \hat{f}_{d_{k,i}}(t-\hat{\tau}_k)} e^{-j2\pi ft} s(t-\hat{\tau}_k)\right] \star s(-t)$$

   (b) Interference cancellation:

$$x_{new}(t,f) = x(t,f) - x_{-p}(t,f)$$

   (c) Estimation update:

$$(\hat{r}, \hat{\theta}_{\hat{r}}) = \arg\min_{(r,\theta_r)} \mathscr{E}^{post}(\mathbf{x}_{new}, r, \theta_r).$$

At high SNR, where performance is little affected by noise, the Interference Cancellation (IC) achieves better performance than the estimation procedure without distinct delay interference removal, as shown on Figure 5.

Note also that at high SNR, the initial estimation of r provides the true value and its re-estimation during the convergence loop is unnecessary.

Let us finally remark that this IC method is close to the EM (Expectation Maximization) procedure introduced in [13]. Indeed, the received data $x(t,f)$ can be rewritten as $x(t,f) = \sum_p x_p(t,f)$ with

$$x_p(t,f) = \sum_{k=1}^{r_p} \alpha_{p,k} e^{-j\pi(f-f_{d_{p,k}})\tau_p} A(t-\tau_p, f-f_{d_{p,k}}) + n_p(t,f),$$

where r_p is the number of simultaneous paths at time τ_p. The complete data model is then given by $(x(t,f), \{x_p(t,f)\}_p)$, the expectation step corresponds to steps (a) and (b) and the maximization step to (c).

5. CRAMER-RAO LOWER BOUNDS AND PERFORMANCE

In this section, we compare parameter estimation procedures proposed in section 4 for frequency and amplitude estimation on multipath channels with the corresponding CRLBs. Performance of delay parameters estimation has already been

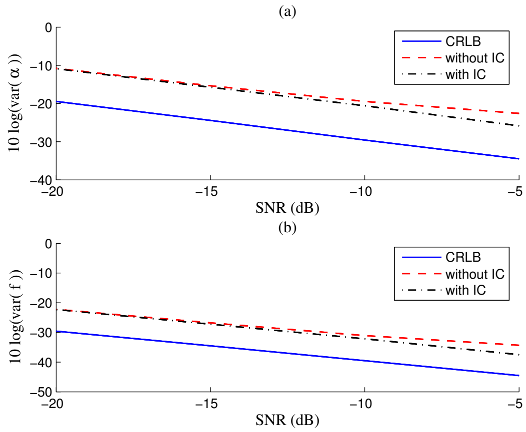

Figure 5: CRLB and estimated variances for (a) amplitude estimation and (b) Doppler shift estimation, with and without interference cancellation.

presented in [6]. We provide here analytical formulas for amplitude and frequency estimators. Since in that case the noise covariance is ill-conditioned as mentioned above, the exact CRLB cannot be computed and we resort to assuming that the noise samples are uncorrelated. Letting $\theta = [\alpha_r, \alpha_i, f_d]$ where α_r, α_i and f_d represent the amplitude real part and imaginary part and the Doppler shift respectively, the Fisher information matrix terms can then be proved to be

$$\mathbf{J}_{11} = \mathbf{J}_{22} = \frac{1}{\sigma_f^2}\sum_f \mathrm{sc}^2(f),$$

$$\mathbf{J}_{12} = \mathbf{J}_{21} = 0,$$

$$\mathbf{J}_{33} = \frac{(\alpha_r^2+\alpha_i^2)}{\sigma_f^2}\sum_f\left(\pi^2 T_b^2(N-1)^2\mathrm{sc}^2(f) + \left(\frac{d\mathrm{sc}(f)}{df}\right)^2\right),$$

$$\mathbf{J}_{13} = \mathbf{J}_{31} = \frac{1}{\sigma_f^2}\sum_f\left(-\pi T_b(N-1)\alpha_i \mathrm{sc}^2(f) - \frac{\alpha_r}{2}\frac{d\mathrm{sc}^2(f)}{df}\right),$$

$$\mathbf{J}_{23} = \mathbf{J}_{32} = \frac{1}{\sigma_f^2}\sum_f\left(\pi T_b(N-1)\alpha_r \mathrm{sc}^2(f) - \frac{\alpha_i}{2}\frac{d\mathrm{sc}^2(f)}{df}\right),$$

where $\mathrm{sc}(f) = \mathrm{sinc}\,(fT_bN)$.

For performance comparison, $J = 200$ Monte Carlo simulations were realized for channels with 10 paths, i.e. variances were computed from 2000 values at each SNR. The frequency separation between two filters of the filter bank was set to 0.5Hz. The amplitudes were drawn with uniform random phase and the CRLBs were averaged over the uniform distribution of the phase. Figure 5 presents variance performance for the amplitude and the Doppler frequency estimated with the procedures described in section 4 together with the corresponding CRLBs. Clearly when interferences among paths with distinct delays are not taken into account, performance is sligthly degraded at large SNRs: the variance curve diverges from a straight line. This effect is removed with the interference cancellation method.

6. CONCLUSION

In this paper we have proposed a method for multipath channel estimation in the presence of multipath Doppler shifts. Based on the properties of the PN sequence ambiguity function, it consists of estimating first the time delay parameters by means of a Bernoulli-Gaussian deconvolution procedure applied at the matched filter bank outputs. Then amplitude and Doppler shift estimation is carried out for each estimated time delay by applying a Levenberg-Marquardt algorithm. The possible presence of simultaneous paths is taken into account via the introduction of a poisson prior upon the number of paths. Performance is compared with the Cramer-Rao Lower Bounds and it is shown that a further interference cancellation step is required in order to ensure good results at high SNR.

REFERENCES

[1] R.A. Iltis and A.W. Fuxjaeger, "A Digital DS Spread-Spectrum Receiver with Joint Channel and Doppler Shift Estimation," *IEEE Transactions on Communications*, vol. 39, No.8, pp. 1255–1267, August 1991.

[2] T.H. Eggen, A.B. Baggeroer and J.C. Creisig, "Communication over Doppler Spread Channels - Part I: Channel and Receiver Presentation," *IEEE Journal of Oceanic Engineering*, vol. 25, No.1, pp. 62–71, January 2000.

[3] J.M. Kelly and M.S. Braasch, "Validation of Theoretical GPS Multipath Bias Characteristics, " in *Proc. of the Aerospace Conf.*, vol. 3, pp. 1317–1325, March 2001.

[4] N. Levanon, *Radar principles*. John Wiley and Sons, 1988.

[5] P.M. Woodward, *Probability and information theory, with applications to radar*. 2nd edition Pergamon Press, Oxford, 1964.

[6] O. Rabaste and T. Chonavel, "Multipath channel estimation via the MPM algorithm," in *Proc. EUSIPCO 2005*, Antalya, Turkey, September 2005.

[7] B. Chalmond, "An iterative gibbsian technique for reconstruction of m-ary images," *Pattern Recognition*, vol. 22, No.6, pp. 747–761, 1989.

[8] C.P. Robert, *The Bayesian Choice. A decision-theoretic motivation*. Springer-Verlag, 1994.

[9] O. Rosec, J.M. Boucher, B. Nsiri and T. chonavel, "Blind Marine Seismic Deconvolution Using Statistical MCMC Methods," *IEEE Journal of Oceanic Engineering*, vol. 28, No.3, pp. 502–512, July 2003.

[10] K. Levenberg, "A method for the solution of certain problems in least squares," *Quart. Appl. Math.*, vol. 2, pp. 164–168, 1944.

[11] D. Marquardt, "A method for least-square estimation of nonlinear parameters," *SIAM J. Appl. Math.*, vol. 11, pp. 431–441, 1963.

[12] A. Papoulis, *Probability, random variables and stochastic processes. Third edition*. McGraw-Hill, 1991.

[13] M. Feder and E. Weinstein, "Parameter estimation of superimposed signals using the EM algorithm," *IEEE trans. on Acoustics, Speech and Signal Processing*, vol. 39, pp. 1–38, 1977.

[14] S. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. Prentice Hall, 1993.