ON THE INTERPLAY BETWEEN SCHEDULING, USER DISTRIBUTION, CSI, AND PERFORMANCE MEASURES IN CELLULAR DOWNLINK

Eduard A. Jorswieck, Mats Bengtsson, and Björn Ottersten

Signal Processing, S3, Royal Institute of Technology (KTH)
SE-100 44 Stockholm, Sweden.
{Eduard.Jorswieck,Mats.Bengtsson,Bjorn.Ottersten}@ee.kth.se

ABSTRACT

The cross-layer design of future communication systems jointly optimizes multiple network layers with the goal of boosting the system wide performance. This trend brings together the physical and the medium access layers. For the joint optimization of these two lowest layers, it is necessary to understand and relate their terms and concepts. In this paper, we study the interplay between four terms, namely channel state information from link-level, scheduling and user distribution from system level, and different performance measures from both levels. The envisaged scenario is the cellular downlink transmission.

The average sum rate describes the long-term performance from a system perspective. The optimal scheduling policy as well as the impact of the user distribution can be clearly characterized as a function of the channel state information (CSI). In contrast, the short-term system performance which is described by the outage sum rate, shows a varying behavior in terms of the optimal scheduling policy and as a function of the user distribution.

The analysis is performed by employing Majorization theory for comparing different user distributions. Three different CSI scenarios, namely the uninformed base, the perfectly informed base, and the base with covariance knowledge are studied. Finally, the extension to two less well known performance measures, the maximum throughput and the delay-limited sum rate is addressed.

1. INTRODUCTION

It was recently argued in [1] that the optimization of scheduling schemes in future communication systems requires so-called cross-layer design. This lead to the development of scheduling algorithms which take link layer as well as physical layer parameters into account [2]. E.g. in [3], the optimal power allocation and scheduling algorithm were derived for stabilizing a number of queues for fading channel which fulfill the Markovian assumption. The connection between the stability region and the ergodic capacity region in multiple antenna multiuser multiple access channels is studied in great detail in [4] from a geometric point of view.

In [5] it was shown that the optimum strategy for maximizing the sum capacity with perfect channel state information (CSI) of a cellular single-input single-output (SISO) multiple access channel (MAC) is to allow only the best user to transmit at each time slot. The result in [5] has induced the notion of multiuser diversity, i.e. the achievable capacity of the system increases with the number of users. In addition to this, the result in [5] has led to the development of opportunistic downlink scheduling algorithms [6] for the broadcast channel (BC). In [7], the average sum rate of the SISO MAC with successive interference cancellation (SIC) under a sum transmit power constraints was studied for different types of CSI. Recently, the downlink case was analyzed in [8]. It turned out that the optimal *scheduling* depends strongly on the *CSI* at the transmitter.

The average sum rate describes the long-term system throughput. This performance measure can be used by the system operator to optimize his overall throughput. The short-term system throughput is measured by the outage sum rate and its corresponding outage probability [9]. It describes the probability that an outage occurs during the next transmission block. The properties of the outage probability with respect to the optimal transmit strategy and the channel statistics (e.g. the user distribution) are different to the average sum rate [10]. There are two further *performance measures*, namely the delay limited sum rate [11, 12] and the maximum throughput [13, 14], that describe the guaranteed performance and the goodput of the system.

Recently, the scaling laws of wireless networks were analyzed under simplified assumptions, e.g. the fading variances of the participating users are equal (e.g. all users are located on a unit circle around the base), or for SNR approaching infinity. In [7, 8], different *user distributions* are compared using Majorization theory and their impact on the average sum rate was characterized. For perfect and long-term CSI, the sum rate was shown to be Schur-convex with respect to the user distribution and for an uninformed base station, the sum rate is Schur-concave. Also, the asymptotic sum rate loss between the best case and the worst case user distribution, was derived.

In this paper, we shed light on the interplay between these four terms: CSI, scheduling, performance measure, and the user distribution. With respect to the four performance measures average sum rate, outage sum rate, maximum throughput and delay limited sum rate, our contributions are the following: We derive the optimal transmit and scheduling strategies as a function of the available CSI, namely for perfect, covariance, and no CSI. We analyze the impact of the user distribution on the performance for these different types of CSI and compare different scenarios in terms of performance.

The paper is organized as follows. In the next section 2, the signal and channel model, the performance measures that will be used, and the measure for the user distribution are introduced. In section 3, the main results are collected and illustrated. The results are ordered according to their performance measure. Finally, in section 4, the paper is concluded.

2. PRELIMINARIES

2.1 Signal and channel model

In the signal model, there are K mobile users who are going to receive data from one base station. The single-antenna quasi-static block flat-fading channels $h_1,\ldots,h_K$ between the mobiles and the base are modeled as constant for a block of coherence length T and from block to block as zero-mean independent complex Gaussian distributed ($\mathrm{CN}(0,c_k)$). The variance is $c_i = \mathbb{E}\left(h_i^* h_i\right)$ for $1 \leq i \leq K$. The additive zero-mean white Gaussian noise $n_k(t)$ at the each receiver is independent identically distributed (iid) and has variance σ_n^2. Furthermore, we assume that the sum transmit power is constrained to be P. The SNR is given by $\rho = \frac{P}{\sigma_n^2}$. The received signal at mobile k at time t is

$$y_k(t) = h_k \sum_{l=1}^{K} x_l(t) + n_k(t)$$

We omit the time index for convenience. The statistics of the fading channel coefficients h_i are completely characterized by c_i. The transmit power directly corresponds to the variance of the transmit signals $p_i = \mathbb{E}\left(x_i^* x_i\right)$ for $1 \le i \le K$. The l_1-norm of the power allocation vector $\mathbf{p} = [p_1, \ldots, p_K]$ is constrained to be one $||\mathbf{p}|| = \sum_{k=1}^{K} p_k = P = 1$. For $1 \le k \le K$ define w_k by $||h_k||^2 = c_k w_k$, i.e. w_k are iid standard exponential distributed random variables. We assume that the receivers have perfect CSI. Further on, we collect the channel states in a vector $\mathbf{h} = [h_1, \ldots, h_K]$.

2.2 Performance measures

Consider the instantaneous sum rate with scheduling policy $\mathbf{p}(\mathbf{h})$

$$C(\alpha) = C(\mathbf{h}, \text{SNR}) = \log\left(1 + \text{SNR} \sum_{k=1}^{K} p_k(\mathbf{h})||h_k||^2\right) \tag{1}$$

The instantaneous sum rate depends on the deterministic SNR and on the channel which is a random variable. That means the instantaneous sum rate is also a random variable (indicated by α). In the block fading model, the channel is constant for the coherence time T. It is assumed that the coherence time T is large enough to code over many blocks in order to achieve almost the mutual information. Then the mutual information in (1) has its usual meaning as the instantaneous capacity [9].

Since the scheduling policy depends on the channel state, it could also vary randomly from fading block to fading block. As a result, the instantaneous capacity itself is a random variable and has a probability density function (pdf) $p_C(\alpha)$. The average of the random variable

$$\mathbb{E}[C(\alpha)] = \int_0^\infty \alpha p_C(\alpha) d\alpha$$

is the average sum rate. For single user systems with perfect CSI it is called ergodic capacity [15]. In multiuser systems with perfect CSI we can call it ergodic sum capacity and it describes the overall performance of the system in average. For finer analysis, the cumulative distribution function (cdf) of C is important. It is the outage probability of the channel, i.e.

$$\Pr[C(\alpha) < R] = \int_0^R p_C(\alpha) d\alpha.$$

The outage probability gives the probability that a certain sum rate R cannot be achieved for a channel state. The system is in an outage means we cannot guarantee to successfully deliver any information at the sum rate R during this channel state.

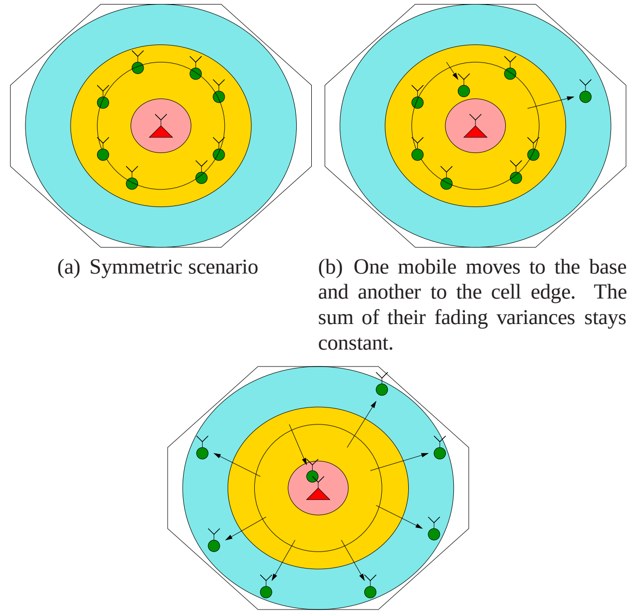
The feasible delay can be exploited either by increasing the length of one codeword or by introducing some kind of automatic repeat request (ARQ). If the block length of the codeword is increased, the outage probability for this codeword is reduced. Here, following [13], we consider the 'Maximum Zero-Outage Throughput'. The receiver requests a retransmission as long as outages occur until the codeword is successfully decoded. Therefore, the complete information is reliable transmitted. The maximum throughput for this simple retransmission scheme is given by

$$T(\text{SNR}) = \max_{R \ge 0} R\left(1 - \Pr\left[C(\mathbf{h}, \text{SNR}) \le R\right]\right). \tag{2}$$

In [13] the quantity in (2) is called 'Maximum Zero-Outage Throughput' (compare to [14]).

For the delay-constraint analysis, the ergodic capacity as well as the outage probability and the maximum throughput are not suitable. Both approaches do not guarantee the successful transmission of information in a finite number of blocks. Therefore, we restrict the delay to one fading block and fix the outage probability to some ε, i.e. $\Pr[C(\alpha) < R] = \varepsilon$. Then we solve this for R. In order to avoid outages at all, we set $\varepsilon = 0$ and obtain the zero-outage sum rate, or the delay-limited sum rate R^* with

$$\Pr[C(\alpha) < R^*] = 0.$$



(a) Symmetric scenario

(b) One mobile moves to the base and another to the cell edge. The sum of their fading variances stays constant.

(c) All but one mobile is at the cell edge

Figure 1: Fair comparison of user distributions

From an information theoretic point of view, the notion of delay-limited sum rate and outage sum rate is somewhat problematic, since the code that achieves capacity requires a long block length, but a block fading channel model is assumed. However, following the arguments in [16], the outage probability predicts surprisingly well the error probability of actual codes for practical values of block length [17].

2.3 Measure for user distributions

In order to guarantee a fair comparison between different user distributions, we constrain the sum variance to be equal to the number of users, i.e. $\sum_{k=1}^{K} c_k = K$. In figure 1, the implications of this constraint are shown. Starting from the symmetric scenario $c_1 = c_2 = \ldots = c_K = 1$, one mobile moves towards the base while another moves to the cell edge. The other extreme scenario occurs when all but one user have very small fading variances c_k. Under the normalization above, this leads to the variances $c_2 = c_3 = \ldots = c_K = 0$ and $c_1 = K$.

Without loss of generality, we order the users in a decreasing way according to their fading variances, i.e. $c_1 \ge c_2 \ge \ldots \ge c_K$. The constraint regarding the sum of the fading variances verifies that we compare scenarios in which the channel carries the same average sum power. We need the following definitions [18]:

Definition 1. *For two vectors $\mathbf{x}, \mathbf{y} \in R^n$ one says that the vector $\mathbf{x}$ majorizes the vector $\mathbf{y}$ and writes $\mathbf{x} \succ \mathbf{y}$ if $\sum_{k=1}^{m} x_k \ge \sum_{k=1}^{m} y_k$ for $m = 1, \ldots, n-1$ and $\sum_{k=1}^{n} x_k = \sum_{k=1}^{n} y_k$.* [1]

The next definition describes a function Φ which is applied to the vectors $\mathbf{x}$ and $\mathbf{y}$ with $\mathbf{x} \succ \mathbf{y}$:

Definition 2. *A real-valued function Φ defined on $\mathscr{A} \subset R^n$ is said to be Schur-convex on $\mathscr{A}$ if from $\mathbf{x} \succ \mathbf{y}$ on $\mathscr{A}$ follows $\Phi(\mathbf{x}) \ge \Phi(\mathbf{y})$. Similarly, Φ is said to be Schur-concave on $\mathscr{A}$ if from $\mathbf{x} \succ \mathbf{y}$ on $\mathscr{A}$ follows $\Phi(\mathbf{x}) \le \Phi(\mathbf{y})$.*

[1] Note that sometimes majorization is defined by the sum of the *smallest* m components [19]

The definition of Schur-convex and Schur-concave corresponds with our understanding of less and more spread out. The most spread out vector has equal entries, while the less spread out vector has only one entry which is equal to K in our case. It is worth mentioning that majorization induces only a partial order on vectors with more than two components. This is due to the fact that vectors with more than two components cannot be totally ordered. At least the extreme cases can be used for comparison with any other vector.

3. MAIN RESULTS

3.1 Average sum rate analysis

The next three lemmata yield the average sum capacity and average sum rate expressions for the three CSI scenarios considered. The proofs can be found in [8].

Lemma 1. *The sum rate with perfect CSI at the base station is achieved by TDMA. The optimal power allocation is to transmit into direction of the best user l with $||h_l||^2 > ||h_k||^2$ for all $1 \le k \le K$ and $l \ne k$. The ergodic sum capacity is then given by*

$$C_{pCSI}(\rho, c) = \mathbb{E}\left(\log\left[1 + \rho \max\left(||h_1||^2, ..., ||h_K||^2\right)\right]\right). \qquad (3)$$

Lemma 2. *The optimal transmit strategy to achieve the average sum capacity with long-term CSI is TDMA. Only the user with highest channel variance c_k is allowed to transmit. The achievable average sum capacity is given by*

$$C_{cCSI}(\rho, \mathbf{c}) = \mathbb{E}\log(1 + \rho c_1 w_1) \qquad (4)$$

Lemma 3. *For no CSI at the base, the most robust transmit strategy against worst case user distribution is equal power allocation and the ergodic sum rate[2] is given by*

$$C_{noCSI}(\rho, \mathbf{c}) = \mathbb{E}\log\left(1 + \rho\sum_{k=1}^{K} c_k w_k\right). \qquad (5)$$

Next, let us characterize the impact of the spread of the fading variances on the ergodic sum capacity for the cases with perfect, covariance and on the ergodic sum rate with no CSI at the base.

Theorem 1. *Assume perfect CSI at the mobiles. For perfect CSI at the base, the ergodic sum capacity in (3) is a Schur-convex function w.r.t. the fading variance vector c. For a base which knows the fading variances, the ergodic sum capacity in (4) is a Schur-convex function w.r.t. the fading variance vector c. For an uninformed base station, the ergodic sum rate in (5) is a Schur-concave function w.r.t. the fading variance vector c.*

The proof can be found in [8].

For illustration the ergodic sum rate comparison is shown in figure (2) for 10 dB SNR. In the interesting range $c_1 > c_2$, i.e. $c_1 = [1..2]$, the three curves show the predicted behavior. The $(+)$ curve with perfect CSI increases with increasing correlation (Schur-convex). The (0) curve with long-term CSI increases with increasing correlation (Schur-convex) and the $(\square)$ curve without CSI decreases with increasing correlation (Schur-concave). The minimax - gap can be observed at the point $c = 1$. It is the difference between the curve for long-term and no CSI. The gap increases with increasing SNR. The gap is the price to pay for the robustness against worst case user distribution.

[2] Since the optimal transmit strategy for no CSI is motivated by a compound channel approach, we cannot talk about the sum capacity. Instead we use the term sum rate.

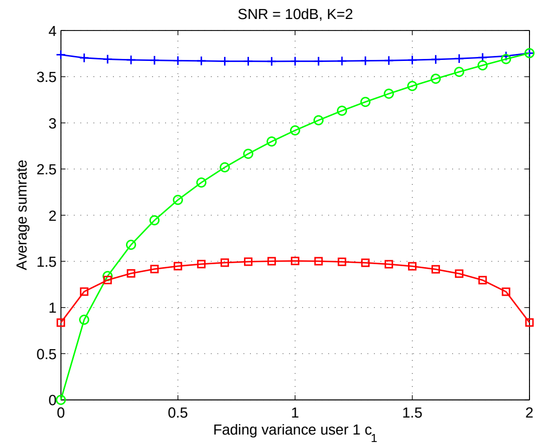


Figure 2: Average sum rate of two user SISO MAC for different types of CSI (perfect $+$, long-term $\circ$ and without $\square$) over fading variance of user one at 10dB SNR

3.2 Outage sum rate analysis

The optimization problem with respect to outage sum rate is given by

$$\min_{p_k} \Pr\left[\sum_{k=1}^{K} p_k c_k w_k \le z\right] \quad \text{s.t.} \quad \sum_{k=1}^{K} p_k \le P, p_k \ge 0 \qquad (6)$$

with $z = \frac{2^R - 1}{\rho}$. It turns out that the optimal scheduling policy depends on the sum rate R and SNR ρ.

Theorem 2. *For an uninformed base station, K equally distributed users, and fixed sum rate R, then there are $K-1$ SNR values $\rho_1 < \rho_2 < ... < \rho_{n_T-1}$, such that for all $\rho \in (\rho_l, \rho_{l+1})$ only one optimal scheduling policy exists. The optimal policy given by*

$$p_1 = ... = p_{l+1} = \frac{1}{l+1} \quad \textit{and} \quad p_{l+2} = ... = p_K = 0. \qquad (7)$$

For $\rho = \rho_l$ exist two optimal power allocation. These are to equally allocate over l or $l+1$ users. For all SNR values greater than $\overline{\rho}$ given by

$$\overline{\rho} = 2^R - 1$$

equal power allocation across over all users is optimal. Allocate power to a single user (TDMA) is optimal if the SNR is smaller than or equal to

$$\hat{\rho} = \frac{2(2^R - 1)}{-2\mathcal{L}_w(-1, -1/2\exp(-1/2)) - 1}. \qquad (8)$$

$\mathcal{L}_w$ is the Lambert W function[3]. The Lambert W-function, also called the omega function, is the inverse function of $f(W) = W\exp(W)$ [20].

Due to space constraints, the proof is omitted. It is based on the results for outage probability minimization in multiple antenna systems [21].

If the users are not equally distributed and the information is not available at the base station, it can be shown by a compound channel approach that equal power allocation across all users is optimal. Further on, the impact of the user distribution on the outage sum rate is characterized in the following theorem.

[3] The first parameter describes the branch whereas the second is the actual argument.

Theorem 3. *Assume that the base station is uninformed and the user distribution is according to* $\mathbf{c}$. *For fixed transmission rate* R *and for SNR* $\rho < \underline{\rho} = \frac{2^R-1}{2}$, *the sum outage probability is a Schur-concave function of the user distribution* $c_1,\ldots,c_K$, *i.e. a less equal distribution of users decreases the sum outage probability. For SNR* $\rho > \overline{\rho} = 2^R - 1$, *the sum outage probability is a Schur-convex function of the user distribution* $c_1,\ldots,c_K$, *i.e. a less equal distribution of users increases the sum outage probability.*

Next, we proceed directly to the perfectly informed base station.

Theorem 4. *With perfect CSI at the base, the optimal scheduling is TDMA and the outage probability is given by*

$$Pr\left[\max[||h_1||^2,\ldots,||h_K||^2 \le z\right]. \tag{9}$$

For fixed sum rate R *and SNR*

$$\rho \le \hat{\xi} = \frac{2^R-1}{(\mathscr{L}_W(-2\exp(-2))+2)K} \tag{10}$$

the sum outage probability is Schur-concave with respect to $\mathbf{c}$ *and for SNR*

$$\rho \ge \bar{\xi} = \frac{2^R-1}{(\mathscr{L}_W(-2\exp(-2))+2)\min\limits_{1\le k\le K} c_k} \tag{11}$$

the sum outage probability is Schur-convex.

Proof. Due to the space constraints, we give here only the sketch of the proof. In order to verify Schur's condition, note that the outage probability can be written as

$$P_{out}^{pCSI}(\mathbf{c}) = \prod_{k=1}^{K}(1-\exp(-z/c_k))$$

which is obviously a symmetric function[4] with respect to $\mathbf{c}$. The difference of the first derivatives of $P_{out}^{pCSI}(\mathbf{c})$ with respect to c_1 and c_2 is given by

$$\begin{aligned}\Delta(\mathbf{c}) \quad = \quad & \prod_{k=3}^{K}(1-e^{-z/c_k})e^{-z/c_1-z/c_2}\frac{1}{c_1^2c_2^2} \\ & \cdot\left[(1-e^{-z/c_1}e^{z/c_1}c_1^2-(1-e^{-z/c_2}e^{z/c_2}c_2^2\right].\end{aligned}$$

The sign of $\Delta(\mathbf{c})$ depends on the sign of the difference in the second line. The monotony properties of the function $(1-\exp(-z/c))\exp(z/c)c^2$ lead to the inequalities in (11) and (10).

In figure (3), the impact of the user distribution on the outage sum probability for perfect CSI and no CSI is shown. Of course, the uninformed base has higher sum outage probability than the perfectly informed base. For both surfaces, there are three ranges for z. For small z, the function is Schur-convex with respect to $\mathbf{c}$. Then there is an area for intermediate z in which the function has local maxima and minima and finally for high z the function is Schur-concave as predicted.

For the intermediate case with long-term CSI, the optimal scheduling policy cannot be given in closed form. The corresponding optimization problem, i.e. the minimization of (6) with knowledge of $c_1,\ldots,c_K$ is a nonconvex optimization problem. Therefore, only necessary conditions were stated in [21]. We omit the analysis of this case but include it in the illustrations. In figure (4) and figure (5), the sum outage probability of the two user SISO MAC for no CSI, long-term CSI, and perfect CSI at the base is shown over the fading variance of user one c_1 for $z = 0.02$ and $z = 1$. From the figures (4) and (5) the chameleonic behavior of the sum outage rate can be observed that is predicted in Theorem 3 and 4.

[4] A function is called symmetric if the argument vector can be arbitrarily permuted without changing the value of the function.

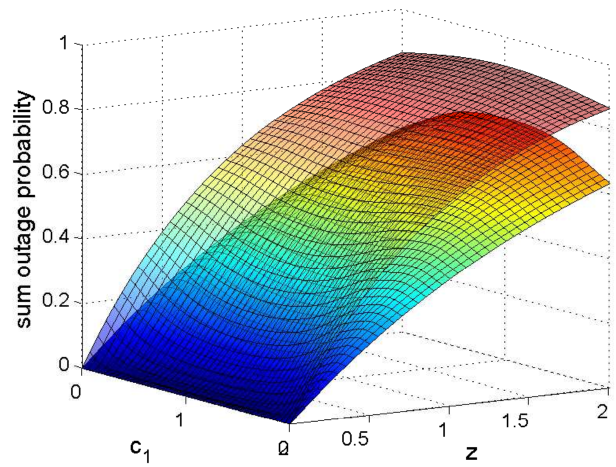


Figure 3: Outage probability for sum rate of two user SISO MAC for no (upper curve) and perfect (lower curve) CSI at the base over fading variance of user one c_1 and over $z = \frac{2^R-1}{\rho}$

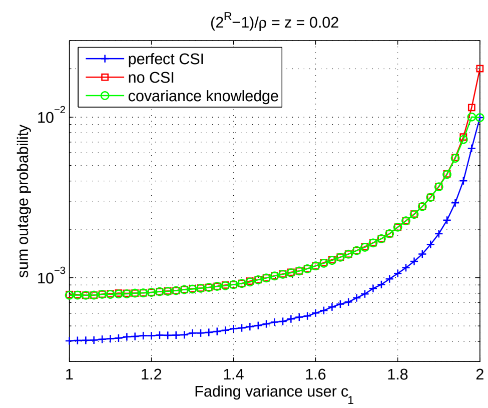


Figure 4: Outage probability for sum rate of two user SISO MAC for no, covariance, and perfect CSI at the base over fading variance of user one c_1 and $z = 0.02$

3.3 Further performance metrics

In this section, we briefly point out the extension to other performance measures. The maximum throughput is closely related to the outage probability and it is defined as

$$R_{MZT} = \arg\max_{R>0} R\cdot(1-\Pr_{out}(R)) \tag{12}$$

with outage probability $\Pr_{out}(R)$. Even in the simplest setting the optimization problem in (12) leads to a complicated solution containing again the Lambert-W function [14]. Table 1 shows the maximum throughput for equally distributed users $\mathbf{c} = \mathbf{1}$ for perfect CSI and no CSI. The multiuser diversity that stems from the fact that the best user is exclusively scheduled can clearly be observed. At this point, we leave the maximum throughput for future research.

The delay-limited sum rate R_0 is defined by

$$\Pr\left[p(\mathbf{h})\max_{1\le k\le K}||h_k||^2 \le z\right] = 0. \tag{13}$$

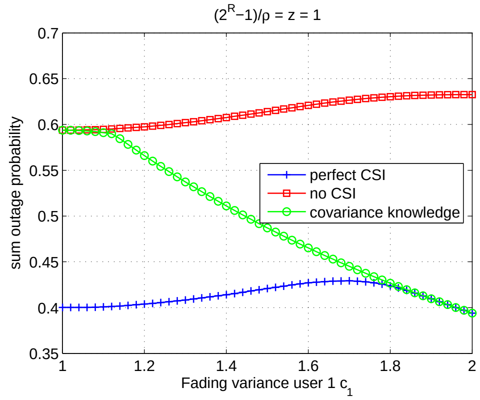


Figure 5: Outage probability for sum rate of two user SISO MAC for no, covariance, and perfect CSI at the base over fading variance of user one c_1 and $z = 1$

CSI \ M	1	2	3	5	10	20
pCSI	0.264	0.523	0.753	1.132	1.785	2.5265
noCSI	0.264	0.417	0.521	0.664	0.861	1.0521

Table 1: Scaling of maximum throughput with perfect and no CSI at the base at SNR 0 dB.

The delay-limited sum rate R_0 is given by

$$R_0 = \log\left(1 + \frac{\rho}{\beta}\right)$$

with $\beta = \mathbb{E}\left[\frac{1}{\max_{1\leq k\leq K}||h_k||^2}\right]$. The expectation as a function of the user distribution $\mathbf{c}$ is simplified to

$$\beta(\mathbf{c}) = \int_0^\infty \frac{1}{t^2}\prod_{k=1}^{K}\left(1 - \exp\left(-\frac{t}{c_k}\right)\right)dt.$$

Again, we omit further analysis at this point. The properties of $\beta(\mathbf{c})$ are still to be investigated.

4. CONCLUSION

The connection between CSI, scheduling, different performance measures, and the user distribution was studied. Depending on the CSI and the applied performance measure, the optimal scheduling policy and the impact of the user distribution were derived. In order to compare different user distributions, a Majorization theory framework was used.

For the average sum rate, the optimal scheduling policy is TDMA if perfect and long-term CSI is available at the transmitter. If no CSI is available the most robust policy against the worst case user distribution is equal power allocation. The impact of the user distribution matched the intuition, i.e. for perfect and long-term CSI, the average sum rate is Schur-convex while it is Schur-concave for no CSI.

In contrast, the outage sum rate behaves differently and it turns out that the optimal scheduling policy as well as the impact of the user distribution depends on the working point. For no CSI and high outage probability, TDMA is the optimal scheduling policy - for smaller outage probabilities, more users are simultaneously active until equal power allocation is optimal. The same held for the impact of the user distribution for equal power allocation. For perfect CSI, a similar chameleonic behavior was observed.

Finally, open problems with respect to the maximum throughput and the delay-limited sum rate were motivated and briefly discussed. The extension to the multiple antenna case are currently studied.

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