IMPLEMENTATION OF A BLIND ADAPTIVE DECISION FEEDBACK EQUALIZER

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ABSTRACT

This paper proposes an efficient implementation of the blind Self-Adaptive Decision Feedback Equalizer (SA-DFE) presented in [1]. This innovative low-complexity equalizer has the particularity of adjusting its structure with the difficulty of the channel. The equalizer switches to a linear recursive equalizer when the channel is very dispersive and to a decision-directed DFE when the output of the equalizer presents reliable decisions. This paper gives details about the two structures of the SA-DFE and justifies why these structures are relevant. However, this paper focuses more particularly on the transition between the two different modes of the SA-DFE. Indeed, simulations show that if the transition from one mode to another is not well-implemented, the equalizer can oscillate between the two structures involving a major loss of convergence rate. Several means to obtain smooth transitions are proposed and simulation results are presented and analyzed. This unsupervised self-adaptive equalizer has been widely tested in the case of underwater communications and shows good ability in practice to deal with severe frequency-selective and quickly time-varying channel.

1. INTRODUCTION

Data transmission over frequency-selective channels requires equalization techniques to remove intersymbol interference (ISI). When the channel is characterized by strong ISI and high variations in time, adaptive equalization is commonly used in order to face high computational complexity. Most of the adaptive equalizers commonly implemented use the criterion of the Minimum Mean Square Error (MMSE) between the equalizer output and the transmitted data sequence in order to adjust its filter coefficients. During a learning phase, a data sequence appropriately chosen and known by the receiver is transmitted in order to initialize the filter coefficients. This learning phase creates a loss of data rate that can considerably slow down data transmission.

Several methods have been designed to recover the signal without using a training sequence. One of the simplest and most effective blind equalization algorithms, called the Constant Modulus Algorithm (CMA) [2], [3] employs the high order statistical properties of the signal to recover the transmitted data.

A self-adaptive equalizer has been proposed in [1], [15] which is able, using blind algorithms, to deal with severe quickly time-varying and high-order frequency selective channels with an adaptive and reversible structure. The main characteristic of the unsupervised equalizer presented in [1] is its ability to modify its structure on its own when the channel becomes too constraining or, on the contrary, to improve the performance of the equalization using a decision structure in the feedback loop when the output of the equalizer presents reliable decisions. This adaptative structure provides this equalizer with a fast convergence rate and low residual Mean Square Error (MSE). The Self-Adaptative equalizer (SA-DFE) has two distinct modes in order to fight ISI: the starting-up or convergence mode where the structure of the equalizer is recursive linear, and the tracking mode where the structure of the equalizer is a DFE. This modification of the structure is possible because the filters of the equalizer are similar in the two modes when the SA-DFE presents a sufficiently low BER at its output. However, the change of structure can be done by evaluating some measure of performance such as the estimated MSE reaching an appropriately chosen threshold J_0. When the threshold is reached, we note that a badly-implemented SA-DFE equalizer can oscillate between the two structures for a long time before reaching a steady state. The aim of this paper is to show how to avoid these undesired oscillations by proposing a apppropriate implementation of the SA-DFE equalizer.

In the second section, we will introduce the context and present the SA-DFE. We will consider a baseband communication, assuming that time synchronization is accomplished perfectly. In the third section, we will describe how to make the transition between the two structure of equalizer without creating undesired oscillation and without using an hysteresis on the decision threshold. Finally, performance of various implementations of the SA-DFE equalizer will be shown on frequency selective channels.

2. CONTEXT AND PRESENTATION OF THE SA-DFE

We consider here only the Single Input-Single Output (SISO) context, in discrete-time baseband communications with complex symbols d_k being independent, identically distributed (i.i.d) with zero-mean and transmitted on time-invariant channels. The signal received by the equalizer can be approximated by

$$s[k] = \sum_{p=0}^{P} h[p]d[k-p] + \eta[k], \qquad (1)$$

where $d[k]$ is the transmitted data and $\eta[k]$ is a zero-mean, white Gaussian random process with variance σ_η^2. Symbols are sent through baseband QPSK modulation, through a discrete transmission channel $H(z)$:

$$H(z) = \sum_{p=0}^{P} h[p]z^{-p}. \qquad (2)$$

As mentionned above, the equalizer has two structures, depending on the difficulty of the channel. When the equalizer is in convergence mode, the structure is linear recursive, and when the equalizer is in tracking mode, the structure is a DFE. A detailed description of the two structures and a discussion of the choice of this adaptive structure is given here.

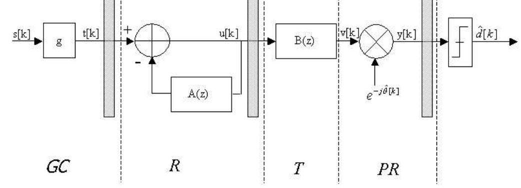

Figure 1: SA-DFE in Convergence (or Starting-up) Mode: Recursive Linear Equalizer

2.1 Convergence or Starting-up mode

Figure 1 shows the structure of the equalizer in the convergence mode. This equalizer is made up of four main parts, indicated in figure 1: the Gain Control (*GC*), the Recursive Part (*R*) , the Transversal Part (*T*)and the Phase Rotator (*PR*).
The notations are given for the signal at the various places in the cascade. Buffers **T**, **U** are of length $L+1$ and buffer **Y** is of length N.
These buffers are the practical implementation of the vectors $\mathbf{U}_N$, $\mathbf{U}_{L+1}$, **T** and **Y** used for the adaptation of the filter coefficients. At each symbol time k, we have:

$$\begin{aligned}
\mathbf{A} &= [a_1, \ldots, a_{N-1}, a_N]^T && (3)\\
\mathbf{B} &= [b_0, \ldots, b_{L-1}, b_L]^T && (4)\\
\mathbf{T}[k] &= [t[k], \ldots, t[k-L]]^T && (5)\\
\mathbf{U}_N[k-1] &= [u[k-1], \ldots, u[k-N]]^T && (6)\\
\mathbf{U}_{L+1}[k] &= [u[k], u[k-1], \ldots, u[k-L]]^T && (7)\\
\mathbf{Y}[k] &= [y[k-1], \ldots, y[k-N]]^T, && (8)
\end{aligned}$$

with $N, L > 0$ and N roughly equal or greater than the length of the channel. According to the notations of figure 1, we have:

$$\begin{aligned}
t[k] &= gs(k) && (9)\\
u[k] &= t[k] - \mathbf{A}^T \mathbf{U}_N[k-1] && (10)\\
y[k] &= \mathbf{B}^T \mathbf{U}_{L+1}[k]\, e^{-j\hat{\theta}[k]}. && (11)
\end{aligned}$$

where **A**, **B** are the vectors of the adaptive filters and g is a factor used to operate a gain control on the signal $s[k]$. We note that only the buffer **U** is useful in the convergence mode of the equalizer. However, buffers **T** and **Y** need to be updated at each symbol time in order to be ready for a potential transition from the convergence mode to the tracking mode. This is a necessary and important condition to have smooth transitions from one structure to another. We suggest in the following sections blind criteria of optimality that can be used for the four devices *GC, R, T, PR* and we give a brief justification why these criteria are relevant. More details about these criteria can be found in [1].

2.1.1 Gain Control

We set the power level $u(k)$ at a particular value, for instance σ_d^2. An unsupervised criterion of optimality for *GC* is:

$$J_{GC}(g) = E\left\{|u[k]|^2\right\} = \sigma_d^2 \qquad (12)$$

In a simulation context, σ_d^2 is often set to 1. For practical implementation of the equalizer where the mathematical operations do not exeed 1, σ_d^2 will be set to a value lower than 1, for example σ_d^2 = 0.25.

2.1.2 Recursive Part

It is known [7], [1] that for an infinite-length equalizer the recursive part *R* of the recursive linear filter (figure 1) is precisely what the DFE structure (figure 2) needs in its feedback parts, which is the structure used by the SA-DFE equalizer for the tracking mode. We note that this result is proven for equalizers of infinite length, which is not the case of the SA-DFE equalizer. It is known [8] that for a finite length DFE, assuming filters of sufficient length, the ideal solution of the infinite-length equalizer can be almost reached.
In order to satisfy this condition, the recursive part *R* must be a whitening filter, meaning that the spectral density of the recursive filter output is constant. Such a filter is obtained by minimizing $E\left\{|u[k]|^2\right\}$and requires *R* to be placed before the transversal part *T*.
Thus, the optimal prediction vector minimizes the function:

$$J_{RP}(\mathbf{A}) = E\left\{|u[k]|^2\right\} \qquad (13)$$

Using an adaptive recursive filter at the entry of an equalizer has been often criticized [4], [5] for its possible instability or wrong convergence. Effectively, if the zeros of $((1+A(z))$ are close to the unit circle, or the step size is too large, or the length of the recursive filter is badly estimated, the adaptive recursive filter can become unstable or converge to a local minimum. Nevertheless, the power of the noise σ_w^2 is never equal to zero and consequently the zeros of $(1+A(z))$ are fairly distant from the unit circle, which gives good robustness to the adaptive recursive filter. Besides, a step size lower than 0.003 and a number of coefficients greater than P-1 gives excellent results. Since the maximum delay spread of the channel is roughly known for a given transmission environment, the latter constraint can easily be respected. This blind adaptive recursive filter was widely tested with success in real and simulated time variant contexts without visible instability.

2.1.3 Transversal Part

The Godard criterion [3] or CMA [2] allows blind deconvolution of nonminimum phase channels. It is proven in [9], [10], [11] that a finite-length equalizer with the CMA criterion converges towards a solution close to the Decision-Directed MMSE (DD-MMSE) solution up to a complex gain factor. Thus, the adaptation criterion of the transversal part minimizes the function:

$$J_G(\mathbf{B}) = E\left\{\left[|v[k]|^p - R_p\right]^2\right\}, R_p = \frac{E\left\{|d[k]|^{2p}\right\}}{E\left\{|d[k]|^2\right\}}. \qquad (14)$$

We only investigate the classical case where $p=2$. We notice that the signal $u[k]$ at the input of the transversal part *T* is uncorrelated by the recursive part *R*. As a result, the autocorrelation matrix of the signal $u[k]$ is well-conditionned. This property improves the convergence rate of *T*. We observe that the CMA normalizes the modulus of the signal at the output of *T* but does not correct its phase. Consequently, the phase rotator part *PR* has to be cascaded at the output of *T*.

2.1.4 Phase Rotator

In order to correct the phase error introduced by the channel and the demodulator, a phase rotation $e^{-j\hat{\theta}[k]}$ is required, where $\hat{\theta}[k]$ is an estimate of the phase error between the modulating and the demodulating carrier waves. To adjust $\hat{\theta}[k]$, a possible criterion is to minimize the DD-MMSE:

$$J_{PR}(\hat{\theta}) = E\left\{\left|v[k]e^{-j\hat{\theta}[k]} - \hat{d}_k\right|^2\right\}. \qquad (15)$$

This criterion is associated with a Phase Locked Loop (PLL) of order 2. We note that, since the statistics of QAM signals are unaffected by any particular rotation, there is still a phase ambiguity in an unsupervised approach. This problem is classically solved by differential encoding.

2.2 Tracking mode

Figure 2 shows the structure of the equalizer in the tracking mode. The structure used here is the classical DFE equalizer [14], [7] with a gain control at the entry. The equalizer is also divided into four parts: *GC, T, PR, R*. The criterion of optimality used here for the adaption of the recursive and the transversal filter is DD-MMSE (15). In this configuration, *GC* and *T* are redundant, so the coefficient $g[k]$ can be held at a fixed value g.
We note that, contrary to figure 1, the buffer **U** is not represented in

figure 2. This is because $\mathbf{U}$ is not used in this mode. The buffers $\mathbf{U}_{L+1}$ and $\mathbf{U}_N$ are filled progressively at each symbol time with a zero sample. The purpose of this operation is to erase the oldest memory of the buffer that is no more useful to the adaptive filter and to keep updated the other coefficients in order to be prepared for an eventual transition from the tracking mode to the convergence mode. In the simulation part, computations were made to compare this implementation with the case where the buffer $\mathbf{U}_{L+1}$ is not refreshed.

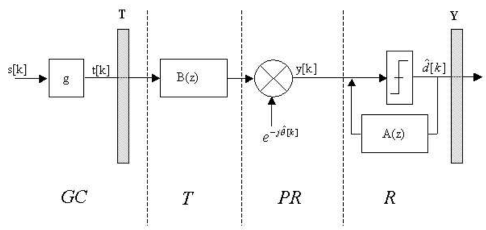

Figure 2: SA-DFE in Tracking Mode: DFE

We propose here a solution for the update of the buffers when the equalizer is in tracking mode. For the purpose of the illustration, we assume here that the equalizer was in convergence mode from time 0 to $k-1$ and has a transition to the tracking mode at the time k. The following equations show how the buffers are filled at symbol time k:

$$\mathbf{T}[k] = [t[k], \ldots, t[k-L]]^T \tag{16}$$

$$\mathbf{U}_N[k-1] = [0, u[k-2], \ldots, u[k-N]]^T \tag{17}$$

$$\mathbf{U}_{L+1}[k] = [0, u[k-1], u[k-1], \ldots, u[k-L]]^T \tag{18}$$

$$\mathbf{Y}[k] = \left[\hat{d}[k-1], y[k-2], \ldots, y[k-N]\right]^T. \tag{19}$$

Until the equalizer was in convergence mode, the buffer $\mathbf{Y}$ was filled with the estimated data $y[k]$ (8). But when the equalizer is in tracking mode, the estimated data are assumed to be sufficiently reliable and it is possible to work with the decided data. Buffer $\mathbf{Y}$ is then filled with the decided data $\hat{d}[k]$.

2.3 Transition between convergence and tracking mode

Following the notations of figure 2, we have:

$$t[k] = gs[k] \tag{20}$$

$$y[k] = (\mathbf{B}^T\mathbf{T}[k])e^{-j\hat{\theta}[k]} \tag{21}$$

$$w[k] = y[k] - \mathbf{A}^T\mathbf{Y}[k] \tag{22}$$

2.3.1 Switching Rule

To control the running mode (convergence or tracking mode), some performance measures are required. The decision of the mode of the equalizer at each symbol time k could be performed by comparing the BER (Bit Error Rate) to a threshold previously chosen. When the equalizer reaches the threshold, the estimated data are considered as sufficiently reliable. An exact value of the BER is not available at the output of the equalizer each symbol time k, but we note that this value has connection with the MSE. We can have an estimation of the MSE that will be re-estimated each symbol time from the estimated data. We call the estimate of the MSE the Decision-Directed MSE (DDMSE).

When the MSE is low, DDMSE and MSE are mainly the same. When the MSE is high, the MSE and DDMSE are different but the MSE tends to be higher than the DDMSE. We define $J_{DD}(k)$ as the DDMSE of the system at time k. We have:

$$J_{DD}(k) = \lambda J_{DD}(k-1) + (1-\lambda)\left|\hat{d}(k) - w(k)\right|^2 \tag{23}$$

where λ is the forgetting factor of the MSE estimator (DDMSE). The parameter λ is important for the design of the self-adaptive equalizer. If the reactivity of the MSE estimator is too fast, the equalizer could switch unnecessarily between the two modes of the equalizer On the contrary, if the estimator responds slowly, the convergence time to reach the tracking mode will increase. A good behavior is obtained for $\lambda = 0.99$.

Finally, the commutation rule will be given by:

- $J_{DD}(k) > J_0$: convergence mode
- $J_{DD}(k) < J_0$: tracking mode

where J_0 is the threshold of the system. The choice of J_0 appears to be a crucial point when designing the SA-DFE equalizer. It is important for the threshold to match with a sufficient low BER (Bit Error Rate) in order to avoid a pathological behavior of the DFE: the threshold must be small enough to ensure that the estimated MSE and the true MSE are roughly the same. For example, J_0 = -6 dB appears to be a good choice in practice for the QPSK case.

2.3.2 How to have smooth transitions

Observing the behavior of the equalizer, we note the presence of oscillations between the two structures. These oscillations are common and do not affect the convergence rate of the SA-DFE when the program is well implemented. However it can slow down the convergence rate of the equalizer considerably when the transitions are not well managed and in particular, as indicated in what follows, when the internal buffers of the equalizer are not updated in an appropriate way.

A possible and immediate solution to this problem is to compute in parallel the output of both modes of the equalizer at each symbol time. Then, a decision is made to take the output of one mode or the other, depending on the value of the current estimated MSE. However, it is not always possible to compute both structures of the SA-DFE equalizer in parallel in the context of a real-time communication on an embedded system, due to its computational complexity. Another solution is to implement hysteresis on the threshold J_0. For example it is possible to have a threshold of J_1= -5dB for the transition of the tracking mode to the convergence mode and a threshold J_2= -6dB for the transition of the convergence mode to the tracking mode. As will be seen in the simulation sections, this solution can give mediocre results in practice, especially on real channels.

In order to have a reliable and stable equalizer, attention has to be paid to how to manage the internal buffers of the equalizer. We consider the three buffers successively and we propose different configurations for each of them, including the one proposed previously.

- Buffer $\mathbf{T}$
  The solution proposed in Eq. (5) and (16) in this paper is to inject $t[k]$ in buffer $\mathbf{T}$ at each symbol time k for the convergence and the tracking mode. (*Configuration T.1*)
  A worse solution would be to neglect to update the buffer at each symbol time when the equalizer is in convergence mode, and update it only in the tracking mode. (*Configuration T.2*)
- Buffer $\mathbf{U}$
  The solution previously proposed in Eq. (6), (7), (17) and (18) is to update the buffer $\mathbf{U}$ when the equalizer is in tracking mode by shifting the data at each sample time and filling the buffer with zero. At each iteration, we suppose $u[k] = 0$, which completely erases the memory of buffer $\mathbf{U}$ in N iterations. *(Configuration U.1)*
  A worse solution proposed here is to leave the buffer $\mathbf{U}$ in the same state when the equalizer is in tracking mode. When the equalizer performs a transition from the tracking mode to the convergence mode, the samples in buffer $\mathbf{U}$ come from the previous transition, and are not in the actual context of the equalizer.*(Configuration U.2)*
- Buffer $\mathbf{Y}$
  The solution previously proposed in Eq. (8) and (19) is to update the buffer at each symbol time, filling the buffer with $y[k]$ when the equalizer is in convergence mode and with $\hat{d}[k]$ when the equalizer is in tracking mode. (*Configuration Y.1*)

A worse solution that may occur when the equalizer is not well-implemented would be to omit to update buffer **Y** when the equalizer is in convergence mode and to adapt the coefficients of filter **A** using the vector **Y**, filled with the decided data only when the equalizer is in convergence mode.(*Configuration Y.2*)

3. SIMULATION RESULTS

3.1 Simulation over time invariant channels

Simulation results are given for the MSE on the time-invariant channel: $\mathbf{h} = [2-0.4j, 1.5+1.8j, 1, 1.2-1.3j, 0.8+1.6j]^T$.
For the simulation context, we have $J_0 = -6\,dB$, $L = 21$, $N = 10$, $E_S/N_0 = 15\,dB$, $\sigma_d^2 = 1$.
The step size used for the adaptation algorithms are set at the value: $\mu_{\mathbf{A}} = \mu_{\mathbf{B}} = 0.003$, $\mu_G = 0.01$.
Because of the unsupervised character of the equalizer, the initialization of the parameters are crucial for a good convergence of the equalizer. We have here:
$\mathbf{A} = [a_1, \ldots, a_N]^T$, with $a_i = 0 \;\; 0 \leq i \leq L$ and:
$\mathbf{B} = [b_0, \ldots, b_R, \ldots, b_L]^T$ with $b_R = 1, R = 16, b_i = 0, \;\; 0 \leq i \leq L, i \neq R$.
We note that the internal buffers are initialized before each simulation, and are never reinitialized during a simulation. As a matter of comparison, the curve of the Data Aided DFE and of the blind Linear Equalizer (LE) were reported on the same figure. The simulation context is the same for the two curves exept that the linear equalizer is composed of only a transversal filter with 31 coefficients. The estimated MSE (23) of the SA-DFE equalizer is computed for different implementations of the buffers, described previously. For each of the 3 buffers, 2 implementations are proposed, the first is the optimal one, the second is the non-optimal configuration. For each configuration of the SA-DFE equalizer, Monte-Carlo simulations were performed, using 100 different runs.
Firstly, we note that for a well-designed SA-DFE (i.e configuration T.1 & U.1 & Y.1), the convergence rate is very fast and the residual MSE is roughly the same as the supervised data-aided DFE.
Using configuration U.2 (resp. Y.2) rather U.1 (resp Y.1) does not change the convergence rate of the equalizer significantly. But a worse implementation of buffer **T** can drastically slow down the convergence rate of the SA-DFE. We can see that the line slope is very smooth compared to the optimal configuration, meaning that oscillations between the two modes last a long time.

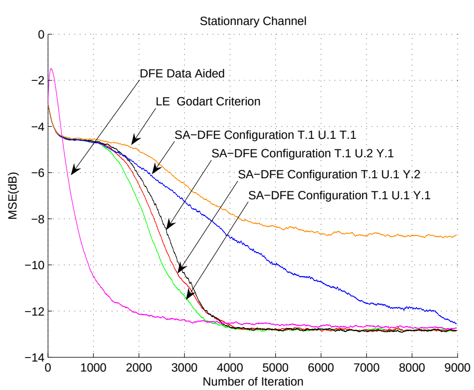

Figure 3: Plot of the MSE versus symbol number over a time-invariant channel. The figure shows results for the optimal implementation of the SA-DFE (T.1 U.1 Y.1) and the three other implementations proposed. The curves of the conventional DFE Data-Aided equalizer optimized with the MMSE criterion and the linear blind equalizer optimized with the Godard criterion are plotted as a reference.

3.2 Simulation over real channels

We dedicate the second part of the simulation to the underwater acoustic channel. The transmission simulated here is a sample of a large database collected by GESMA in collaboration with SERCEL and ENST Bretagne, during a series of measures in a sea environement. The channel chosen here is highly frequency-selective and is subject to large variations in time. The communication flow is approximately $10kbps$. [12].
The SA-DFE used here is a generalization of the SA-DFE equalizer to Single Input-Multiple Output (SIMO) channels, which introduces spatial diversity. A detailled description of this equalizer is made in [13]. The filters of the SA-DFE equalizer are configured as follows: L= 25, N= 25, used with 4 different sensors. The initialization of the filters of the SA-DFE are the same as for the time invariant channel. The step sizes of the filters are the same as in Section $III.A$.
Two different implementations of the equalizers were tested over this channel: *configuration T.1 U.1 Y.1* which is the implementation proposed throughout this paper, and *configuration T.2 U.2 Y.2*, which is the worst implementation, described above. For each configuration, simulations were performed on equalizers configured either with hysteresis (J_1 = -5 dB, J_2 = -6 dB), or without hysteresis (J_0 = -6 dB).

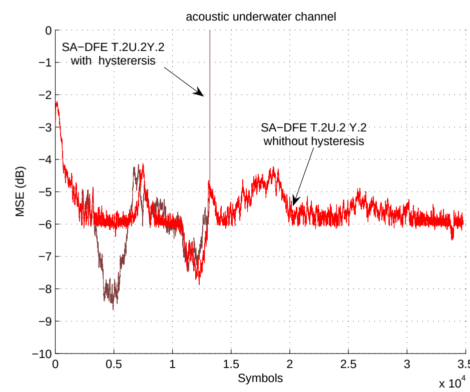

Figure 4: Mean Square Error - Underwater Acoustic Channel

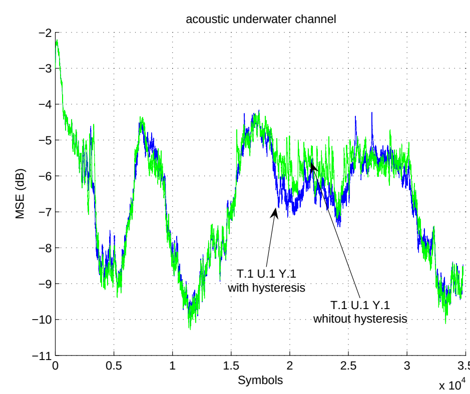

Figure 5: Mean Square Error - Underwater Acoustic Channel

The results of the simulations on time-varying channels show (figure 4) that adding hysteresis to a badly-implemented SA-DFE equalizer (T2,U2, Y2) will introduce instability: in our example, the transversal equalizer coefficients go to infinity. We can also note that a badly implemented equalizer does not manage to go into tracking mode, because of the difficulty of the channel.
Results given with well-implemented equalizer (T1, U1, Y1) on the same channel will give far better results than those given by

an equalizer badly implemented. On the other hand, implementing additional hysteresis will not improve the performance of the SA-DFE equalizer notably (figure 5).

The simulation example shows here that implementing hysteresis is not a necessary solution for a device that will be implemented in a real system. Moreover, it shows the importance of having a well-implemented equalizer for the case of difficult and non-time invariant channels.

We note that the transition from tracking mode to convergence mode can introduce a modification in the equalizer delay and consequently a loss of symbols in the case of particularly severe frequency selective channels, typically the channel that is presented here. This loss of one or several symbols can seriously compromise the performance of the whole communication system, and more particularly when the system includes channel coding. Further investigations have to be done to solve this issue.

4. CONCLUSION

In the present paper a presentation of a fast converging Self-Adaptive DFE is given and an efficient implementation of this equalizer is proposed. A justification of the self adaptive structures is proposed and several implementations of the equalizers are evaluated and compared to the initial one. This paper should clarify some details of the implementation of the SA-DFE equalizer necessary to obtain a reliable device. Performance of the SA-DFE is superior to that of the classical blind linear equalizer optimized with the Godart criterion, in particular regarding its convergence rate.

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