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EM-BASED ENHANCMENT OF THE WIENER PILOT-AIDED CHANNEL ESTIMATION IN MIMO-OFDM SYSTEMS

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ABSTRACT

Pilot Aided Channel Estimation (PACE) in OFDM systems uses training sequences to estimate the channel in a subset of the frequency bins, followed by interpolation to recover the rest of the non-pilot sub-channels. Particularly, the Wiener interpolation method is based on the knowledge about channel statistics in order to find the optimal channel estimate in a frequency bin without pilot symbols in the linear MMSE sense. Nevertheless, the Wiener interpolation does not utilize the other available information at the receiver such as the received signals and the knowledge about the transmitted symbol alphabet. This paper presents a novel Expectation Maximization (EM) algorithm which optimally utilizes all available information at the receiver and thus enhances the Wiener interpolation. The proposed method is tested successfully on MIMO-OFDM systems in realistic channel conditions.

1. INTRODUCTION

The OFDM system has emerged as a good alternative to mitigate the effects of frequency selectivity in wideband mobile communication systems. The use of a Cyclic Prefix (CP) for preventing inter-block interference is known to be equivalent to multiple flat fading parallel transmission channels in the frequency domain [1]. Wideband communication systems with OFDM modulation can be combined with multiple transmit and receive antennas (MIMO) to achieve very high data rate transmission.

Typically, channel estimation within the MIMO-OFDM systems is based on the Pilot Aided Channel Estimation (PACE) methods where the pilot symbols are periodically transmitted along the time and/or frequency directions. The channel estimation in a bin with no pilot symbols is then obtained by interpolation. Common algorithms use Linear, Sinc or Cubic interpolators [2], [3]. The optimal solution in the sense of Minimum Mean Squared Error (MMSE) is the Wiener filter [4], [5] which relies on the autocorrelation function of the channel over time and/or frequency. Hence, the quality of the channel statistical information together with the signal to noise ratio (SNR) is crucial to the quality of the interpolation process. In addition, the Wiener approach, like all other interpolation approaches, does not exploit the other available information at the receiver such as the received signals in frequency bins without pilot symbols and the knowledge about the transmitted symbol alphabet. In the Wiener approach, the channel estimates at pilot frequency bins are obtained by a simple division of the received signal with the transmitted pilot symbol. Hence, the interpolation is compromised by the additive Gaussian channel noise.

This paper presents a novel method that enhances the channel estimate over all frequencies by exploiting all available information at the receiver. The channel estimation at the particular frequency bin with no pilots is posed herein as a Maximum Likelihood problem with missing information, i.e. the transmitted symbols. A suitable implementation of the Maximum Expectation (EM) algorithm is then derived in order to find the most likely channel estimates at the pilot bins.

The authors of [6], [7] and [8] also developed EM based semi-blind methods which estimate the channel not only based on the signals in pilot frequency bins, but all the received signals. The existing semi-blind approaches estimate the channel impulse response in time domain, although some of them present the final result in frequency domain by simply making a Fourier transform. As discussed in [9], these methods suffer from leakage problems. Another drawback of these methods is transforming between two domains which increases the computational complexity. In our approach, channel estimation and symbol detection are all done in frequency domain. The equations of the EM algorithm are derived based on the frequency correlations between the subcarriers. The correlation function is determined from the long term power delay profile of the channel. Thus, the leakage problems are avoided and the signal information from all the subchannels is used.

Another research direction has been proposed in [10],[11] where the EM approach has been used for estimating the OFDM channel in time varying condition. This approach is complementary to ours: while we use correlations in the frequency domain to improve the channel estimation, the algorithms in [10] and [11] are using the temporal evolution for each frequency bin. A combination of the two methods would be interesting to investigate. On the other hand, in [12] and [13] the EM algorithm is applied to estimate the channel model in MIMO-OFDM under the assumption that the spacing of the bins with pilot sequences does not satisfy the sampling theorem, which makes the standard Wiener approach infeasible.

Our approach in this paper deals with uncoded systems. In [14], the authors developed an iterative channel estimator where coded symbols are decoded at the receiver and sent back to the estimator to achieve a better estimation. Our method can be extended to such a coded system as well. The novelty of our scheme is in the estimator part where we enhance the Wiener solution by improving the channel estimates at the frequency bins with pilot symbols.

2. WIENER INTERPOLATION ENHACEMENT

The MSE performance of the Wiener interpolation directly depends on the estimation accuracy at pilot bins and is, hence, affected by the channel noise. In order to improve the MSE estimation performance, we want to improve the estimation at pilot bins. Before introducing the new methods, we shall first clarify again some basic assumptions.

The transmitted symbols are temporally white. As OFDM convert the serial transmitted symbols into parallel and transmit through parallel orthogonal sub-channels, the temporal whiteness is translated into the independency between the symbols transmitted in different sub-channels.

Transmitted symbols are chosen from a fixed symbols set $\{S\}=\{s_1,s_2\ldots s_M\}$, and the M-ary modulation scheme is known at the receiver side. The probability of each symbol realization is known and, for simplicity, we assume all symbols to be equally probable. In addition, in MIMO systems the transmitted data streams are statistically independent.

We first address SISO-OFDM systems with each OFDM sub-channel described by:

$$x_n = s_n h_n + \eta_n, n = 1\ldots N, \tag{1}$$

where n denotes the frequency bin index, and x, h, and η are the received signal, channel coefficient and noise realization.

In Wiener interpolation, the channel at non-pilot frequency bin is estimated by a weighted linear combination of the channel at pilot bins, so that, the channel coefficient h_n can be estimated as:

$$\hat{h}_n = \sum_{i=1}^{N_\mathrm{P}} w_n^i * \hat{h}_\mathrm{P}^i = \mathbf{w}_n^\mathrm{H} \hat{\mathbf{h}}_\mathrm{P} \tag{2}$$

where $\mathbf{w}_n = [w_n^i, \ldots w_n^{N_\mathrm{P}}]^\mathrm{T}$ are the Wiener coefficients and N_p is the number of pilot bins. $\hat{\mathbf{h}}_\mathrm{P} = [\hat{h}_\mathrm{p}^1, \ldots \hat{h}_\mathrm{p}^{N_\mathrm{P}}]^\mathrm{T}$ are the channels estimated at pilot bins.

In standard Wiener interpolation, $\hat{\mathbf{h}}_\mathrm{P}$ is estimated via ML approach with the likelihood being defined as:

$$L(\hat{\mathbf{h}}_\mathrm{P}) = \log p(\mathbf{x}_\mathrm{P} \mid \mathbf{s}_\mathrm{P}; \hat{\mathbf{h}}_\mathrm{P}) \tag{3}$$

With the assumption of independence of the pilot bins, the ML solution is simply given by:

$$\hat{\mathbf{h}}_\mathrm{P} = \left[\frac{x_\mathrm{P}^1}{s_\mathrm{P}^1}, \frac{x_\mathrm{P}^2}{s_\mathrm{P}^2}, \ldots \frac{x_\mathrm{P}^{N_\mathrm{P}}}{s_\mathrm{P}^{N_\mathrm{P}}} \right]^\mathrm{T} \tag{4}$$

where $s_\mathrm{P}{}^{n_\mathrm{p}}, n_\mathrm{p} = 1 \ldots N_\mathrm{P}$ are the pilot symbols.

Insert (2) into (1), we obtain:

$$x_n = s_n \mathbf{w}_n^\mathrm{H} \mathbf{h}_\mathrm{p} + \eta_n \tag{5}$$

For all the N sub-channels of we form the system:

$$\begin{cases} x_1 = s_1 \mathbf{w}_1^\mathrm{H} \mathbf{h}_\mathrm{p} + \eta_1 \\ \vdots \\ x_N = s_N \mathbf{w}_N^\mathrm{H} \mathbf{h}_\mathrm{p} + \eta_N \end{cases} \tag{6}$$

Hence, the optimization of $\mathbf{h}_\mathrm{P}$ can be posed as a Maximum Likelihood (ML) optimization with the missing data $\mathbf{s} = [s_1, s_2 \ldots s_N]^T$: here, the observation is the received signal, the hidden variable is the transmitted symbol, and the parameter is $\mathbf{h}_\mathrm{P}$. The likelihood in this case is:

$$L(\mathbf{h}_\mathrm{P}) = \log p(\mathbf{x}; \mathbf{h}_\mathrm{P}) \tag{7}$$

where $\mathbf{x} = [x_1, x_2 \ldots x_N]^T$ are the received signals at the transmitter at different frequency bins. By comparing (3) and (7), we can see that the likelihood in (7) corresponds to a joint probability of the received signal at all frequency bins, thus including the information at non-pilot bins.

Due to the independence of the transmitted symbols and the orthogonality of the sub-channels in OFDM, the received signals should also be independent which leads to:

$$\begin{aligned} L(\mathbf{h}_\mathrm{P}) &= \log \prod_{n=1}^{N} p(x_n; \mathbf{h}_\mathrm{P}) = \sum_{n=1}^{N} \log p(x_n; \mathbf{h}_\mathrm{P}) \\ &= \sum_{n=1}^{N} \log \left(\sum_{s_n \in \{s\}} p(x_n \mid s_n; \mathbf{h}_\mathrm{p}) p(s_n) \right) \end{aligned} \tag{8}$$

where the second sum goes over all symbols in the symbol alphabet and where, according to the prior assumptions, all prior symbol probabilities are equal.

The sum inside logarithm makes an analytical solution intractable. The EM algorithm [15] iteratively optimizes a lower bound instead of directly maximizing the likelihood, in two steps: the Expectation (E) step and the Maximization (M) step. Following the approach in [16] it can be shown that the solution of the maximization in the iteration i is:

$$\mathbf{h}_\mathrm{P}{}^{(i+1)} = \mathbf{R}_{ss(\mathbf{w})}^{(i)}{}^{-1} \mathbf{r}_{xs}^{(i)} \tag{9}$$

where:

$$\mathbf{R}_{ss(\mathbf{w})}^{(i)} = \sum_{n=1}^{N} \sum_{s_n \in \{s\}} p(s_n \mid x_n; \mathbf{h}_\mathrm{P}^{(i)}) s_n * s_n \mathbf{w}_n \mathbf{w}_n^\mathrm{H} \tag{10}$$

$$\mathbf{r}_{xs}^{(i)} = \sum_{n=1}^{N} \sum_{s_n \in \{s\}} p(s_n \mid x_n; \mathbf{h}_\mathrm{P}^{(i)}) s_n * x_n \mathbf{w}_n \tag{11}$$

Here $p(s_n \mid x_n; \mathbf{h}_{\mathrm{P}}^{(i)})$ is the posterior probability of the transmitted symbol s_n given the received symbol x_n at current estimation $\mathbf{h}_{\mathrm{P}}^{(i)}$, which can be calculated in the following way:

$$P(s_n \mid x_n; \mathbf{h}_{\mathrm{P}}^{(i)}) = \frac{P(s_n \mid x_n; \mathbf{h}_{\mathrm{P}}^{(i)})}{\sum_{s \in \{s\}} P(s_n \mid x_n; \mathbf{h}_{\mathrm{P}}^{(i)})} \quad (12)$$

If we use PSK modulation, i.e. the transmitted symbol has constant transmission power σ_s^2, then equation (10) can be simplified as:

$$\begin{aligned}\mathbf{R}_{ss(\mathbf{w})}^{(i)} &= \sigma_s^2 \sum_{n=1}^{N} \sum_{s \in \{s\}} P(s_n \mid x_n; \mathbf{h}_{\mathrm{P}}^{(i)}) \mathbf{w}_n \mathbf{w}_n^{\mathrm{H}} \\ &= \sigma_s^2 \sum_{n=1}^{N} \mathbf{w}_n \mathbf{w}_n^{\mathrm{H}}\end{aligned} \quad (13)$$

From (13), we can see $\mathbf{R}_{ss(\mathbf{w})}$ only depends on the Wiener weights, which can be pre-computed and stored. Only $\mathbf{r}_{xs}^{(i)}$ should be updated in each iteration.

The EM iteration starts from an initial estimation $\mathbf{h}_{\mathrm{P}}^{(0)}$ and iteratively update the estimation until it converges. A good initialization is essential for two reasons. It is well known that semi-blind/blind methods are insensitive to permutation and scaling of the signals, consequently the cost functions have several maxima. The initialization of the recursions must be made close enough to the correct solution so that the algorithm will converge to it. Secondly, a good initialization speeds up significantly the convergence of the algorithm. A suitable initialization is the standard Wiener solution presented in (3). In our simulations with pilot bins satisfying the sampling theorem we have not observed cases where the algorithm converged to a wrong solution. In the case that the sampling theorem is not satisfied, the Wiener interpolation can not be used and the methods developed in [12] and [13] should be applied. Regarding the convergence speed, only a few iterations, 2-3, were sufficient to achieve the solution.

The EM also gives the posterior probability $P(s_n \mid x_n; \mathbf{h}_{\mathrm{P}}^{(i)})$ of the transmitted symbol after each iteration, which can be used in symbol detection in the Maximum a posterior Probability method.

The extension of the above scheme to MIMO-OFDM systems is possible. In a MIMO system with M_t transmitting antennas and M_r receiving antennas, by using OFDM, we divide the multipath channel between each antenna pair into N orthogonal sub-channels. Then, for each sub-channel, their $M_r \times M_r$ channel coefficients can be written in a matrix $\mathbf{H}$:

$$\mathbf{H}_n = \begin{bmatrix} h_{(1,1),n} & \cdots & h_{(1,M_t),n} \\ \vdots & \ddots & \vdots \\ h_{(M_r1),n} & \cdots & h_{(M_rM_t),n} \end{bmatrix} \quad (14)$$

Here n is the index denoting the nth sub-channel. Each antenna pair can be regarded as a SISO OFDM system. Wiener interpolation for the channel between antenna pair (m_t,m_r) can be expressed as:

$$\hat{h}_{(m_t,m_r),n} = \sum_{i=1}^{N_{Pilots}} w_{(m_t,m_r),n}^{i} h_{(m_t,m_r),\mathrm{Pilot}}^{i} = \mathbf{w}_{(m_t,m_r),n}^{\mathrm{H}} \mathbf{h}_{\mathrm{Pilot}} \quad (15)$$

A common assumption is that for close together antennas all the antenna pairs have the same channel statistics. Then, we use the same Wiener coefficients, $\mathbf{w}_n$, for every antenna pairs:

$$\begin{aligned}\mathbf{H}_n &= \begin{bmatrix} h_{(1,1),n} & \cdots & h_{(1,M_t),n} \\ \vdots & \ddots & \vdots \\ h_{(M_r,1),n} & \cdots & h_{(M_r,M_t),n} \end{bmatrix} \\ &= \sum_{i=1}^{N_{Pilots}} w_n^i \begin{bmatrix} h_{(1,1),\mathrm{Pilot}}^i & \cdots & h_{(1,M_t),\mathrm{Pilot}}^i \\ \vdots & \ddots & \vdots \\ h_{(M_r,1),\mathrm{Pilot}}^i & \cdots & h_{(M_r,M_t),\mathrm{Pilot}}^i \end{bmatrix}\end{aligned} \quad (16)$$

The received signal at m_r^{th} receiving antenna can be computed as:

$$\begin{aligned}x_{m_r,n} &= \left[h_{(m_r,1),n}, \ldots h_{(m_r,M_t),n}\right] \begin{bmatrix} s_{(m_r,1),n} \\ \vdots \\ s_{(m_r,M_t),n} \end{bmatrix} + \eta_{m_r,n} \\ &= \sum_{i=1}^{N_{\mathrm{P}}} w_n^i \left[h_{(m_r,1),\mathrm{P}}^i, \ldots h_{(m_r,M_t),\mathrm{P}}^i\right] \begin{bmatrix} s_{(m_r,1),n} \\ \vdots \\ s_{(m_r,M_t),n} \end{bmatrix} + \eta_{m_r,n} \\ &= \left[h_{(m_r,1),\mathrm{P}}^1 \ldots h_{(m_r,M_t),\mathrm{P}}^1, \ldots, h_{(m_r,1),\mathrm{P}}^{N_{\mathrm{P}}}, \ldots h_{(m_r,M_t),\mathrm{P}}^{N_{\mathrm{P}}}\right] \cdot \\ &\quad \cdot \begin{bmatrix} w_n^1 \mathbf{I}_{M_t \times M_t} \\ \vdots \\ w_n^{N_{\mathrm{P}}} \mathbf{I}_{M_t \times M_t} \end{bmatrix} \begin{bmatrix} s_{(m_r,1),n} \\ \vdots \\ s_{(m_r,M_t),n} \end{bmatrix} + \eta_{m_r,n}\end{aligned} \quad (17)$$

Denote:

$$\mathbf{h}_{m_r,\mathrm{P}} = \left[h_{(m_r,1),\mathrm{P}}^1 \ldots h_{(m_r,M_t),\mathrm{P}}^1, \ldots, h_{(m_r,1),\mathrm{P}}^{N_{\mathrm{P}}}, \ldots h_{(m_r,M_t),\mathrm{P}}^{N_{\mathrm{P}}}\right]^{\mathrm{T}}$$

$$\mathbf{W}_n = \left[w_n^1 \mathbf{I}_{M_t \times M_t} \ldots w_n^{N_{\mathrm{P}}} \mathbf{I}_{M_t \times M_t}\right]^{\mathrm{T}}$$

$$\mathbf{s}_{m_r,n} = \left[s_{(m_r,1),n} \ldots s_{(m_r,M_t),n}\right]^{\mathrm{T}}$$

Then (17) can be represented as:

$$x_{m_r,n} = \mathbf{h}_{m_r,\mathrm{Pilot}}{}^{\mathrm{T}} \mathbf{W}_n \mathbf{s}_{m_r,n} + \eta_{m_r,n} \quad (18)$$

Equation (18) has a very similar form as (5). The definition of likelihood and the derivation of EM algorithm can just follow (7) to (8). Here, we won't repeat them, but just give the updating rule for the estimation:

$$\mathbf{h}_{m_r,\mathrm{P}}^{(i+1)} = \mathbf{r}_{xs}^{(i)} \mathbf{R}_{\mathbf{ss(W)}}^{(i)}{}^{-1} \quad (19)$$

with:

$$\begin{aligned}\mathbf{R}_{\mathbf{ss(W)}}^{(i)} &= \sum_{n=1}^{N} \sum_{\{\mathrm{S}\}} P(\mathbf{s}_{m_r,n} \mid x_{m_r,n}; \mathbf{h}_{m_r,\mathrm{P}}^{(\mathrm{i})}) \mathbf{W}_n \mathbf{s}_{m_r,n} (\mathbf{W}_n \mathbf{s}_{m_r,n})^{\mathrm{H}} \\ \mathbf{r}_{x\mathbf{s}(\mathbf{W})}^{(i)} &= \sum_{n=1}^{N} \sum_{\{\mathrm{S}\}} P(\mathbf{s}_{m_r,n} \mid x_{m_r,n}; \mathbf{h}_{m_r,\mathrm{P}}^{(\mathrm{i})}) x_{m_r,n} (\mathbf{W}_n \mathbf{s}_{m_r,n})^{\mathrm{H}}\end{aligned} \quad (20)$$

3. SIMULATION RESULTS

For testing the proposed SISO and MIMO OFDM channel estimation and equalization we have used a realistic frequency selective channel developed by the 3GPP-SCM group [17]. The main parameters of the system and channel are summarized in Table 1. The coherence bandwidth of the channel is approximately 1/1.2sμs = 833kHz. The pilot bins are assigned that two pilot bins are inside the coherence band width, i.e. the spacing of pilot bins is approximately 416kHz.

Table 1. Main system and SCM channel parameters:

Environment	Micro-cellular	
Antenna configuration (only for MIMO)	4x4	
Bandwidth	20MHz	
Maximum delay	1.2μs	
Coherence bandwidth	833kHz	
Number of sub-channels	1024	256
Spacing of pilot bins	22bins	5bins

Figure 1 shows the MSE performance of Wiener Interpolation and EM Based Improvement of Wiener Interpolation.

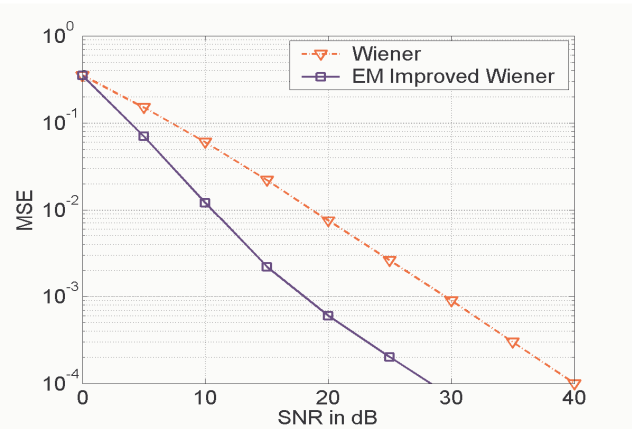

Figure 1: Wiener vs EM Improved Wiener in SISO

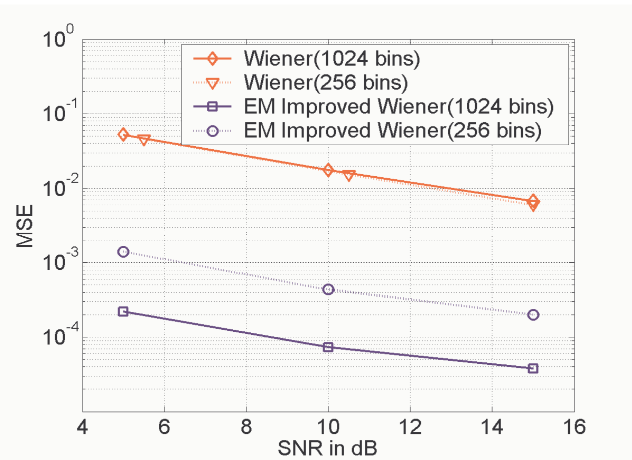

Figure 2: MSE performance: Wiener vs EM improved Wiener in MIMO

We can see that by using EM, the estimation performance is improved significantly. The advantage of EM improved Wiener estimation scheme comes from the fact that the pilot bins are estimated together and the statistical information from the other bins where no pilots are sent is used. In this way the noise is averaged and the estimation is more accurate than in the case of the standard Wiener.

Further results for the MIMO case, where the traditional Wiener and the EM improved Wiener interpolations are compared, are depicted in Figure 2.

The two curves for the standard Wiener interpolation are almost overlapping because the pilot bins spacing is the same in both cases and accordingly the performance of the Wiener interpolation is the same.

We can observe that the EM improved Wiener interpolation works better in OFDM with 1024 bins, i.e. it makes much more improvement over standard Wiener interpolation when more bins are available. This is reasonable as standard Wiener interpolation only uses the input and the output at the pilot bins, the numbers of which in both cases are the same, while EM exploits the information at non-pilot bins. In OFDM with 1024 bins, there are relatively more non-pilot bins available and the noise can be therefore better reduced.

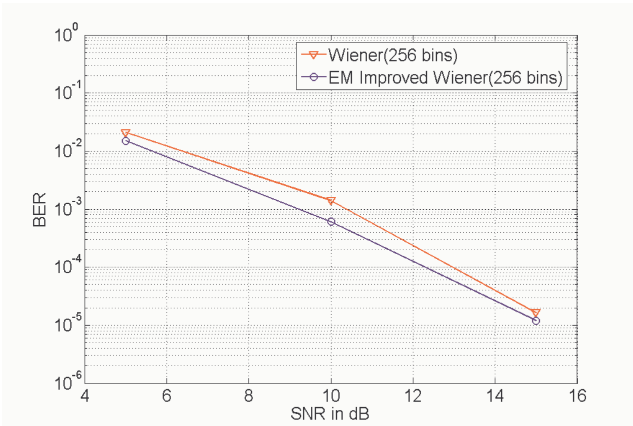

Figure 3: BER performance, Wiener vs EM Improved Wiener in MIMO. 256 frequency bins.

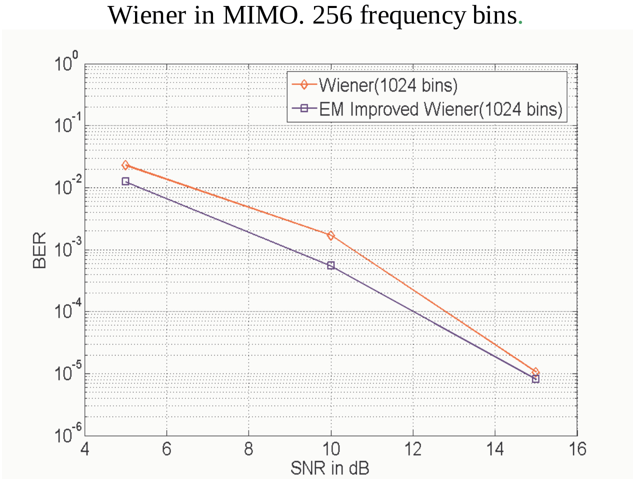

Figure 4: BER performance, Wiener vs EM Improved Wiener in MIMO 1024 frequency bins.

The uncoded bit error rate performance of Wiener interpolation and the EM improved Wiener method has been compared with the results shown in Figure 3 and 4. The modulation alphabet is QPSK.

In both cases we see a gain due to the improved channel estimation, gain up to 2dB in signal to noise ratio. This gain is most significant in the SNR rage important for mobile communications, namely between 5 and 10dB.

4. CONCLUSION

In standard Wiener interpolation, only the transmitted symbols and received signals at pilot bins are used for the channel estimation. By including the information at non-pilot bins, the estimation performance is largely improved. The knowledge of transmitted symbol alphabet and the received signal at non-pilot bins is exploited with a maximum likelihood approach. Analytical solution could not be directly found. Suitable expression has been derived to apply the EM algorithm.

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