Impact of ballistic target model uncertainty on IMM-UKF and IMM-EKF tracking accuracies

A. Farina
Selex Sistemi Integrati
Roma, Italy
a.farina@selex-si.com

S. Immediata
Selex Sistemi Integrati
Roma, Italy
simmediata@selex-si.com

L. Timmoneri
Selex Sistemi Integrati
Roma, Italy
ltimmoneri@selex-si.com

Abstract. Some aspects of tracking of ballistic targets (BT) are analysed in this paper. In particular, the uncertainty in the kinematic model of the ballistic target is introduced and the impact on the tracking accuracies and robustness is detailed. Two architectures have been selected for comparison purposes: (i) an Interacting Multiple Model (IMM) constituted by a number of Extended Kalman Filters (EKFs) matched to the BT dynamics, each filter having the capability of on-line estimation of the BT characteristics (for instance the ballistic coefficient) and (ii) an IMM constituted by a number of Unscented Kalman Filters (UKFs). The performance evaluations of the designed IMM tracking algorithms are obtained via Monte Carlo simulation.

Keywords: Target tracking, estimation, Unscented Kalman filter, Extended Kalman filter, Ballistic Target Model

1. Summary

This paper presents mathematical procedures and simulation results for estimating the accuracy of a target track obtained by fusing two sequences of radar plots provided by two sensors in presence of a ballistic target (BT). It is assumed that the two sensors could have a detection probabilities Pd less than 1.

The novelty of the paper consists in the evaluation of performance of two different tracking architectures based on: (i) multiple model approach, and (ii) design of filters matched to the BT kinematics; the comparison is performed including in the trajectory generator the effect of uncertainty in the knowledge of the BT model by adding process noise in the BT motion equations.

2. Introduction

The problem of BT tracking has been described in the specialized technical literature, see for instance [1-4]. One of the fundamental information required for designing a successful tracking architecture is the development of a BT kinematics model: it has to be noted that the BT motion equations are highly non linear so that an optimum tracking filter cannot be found in principle; thus sub-optimum filters are the usual current practice. In addition to this, it has also to be considered that in the real world the behavior of the BT could be different with respect to the model prediction; to counteract this problem, process noise has to be added to the BT model during the trajectories generation[1-4] and the tracker has to be tested against these trajectories to acquire the necessary confidence on the achievable tracking accuracies.

This paper deals with the problems of designing an effective tracking architecture in presence of BT. Two architectures have been selected for comparison purposes: (i) an Interacting Multiple Model (IMM) constituted by a number of Extended Kalman Filters (EKFs) matched to the BT dynamics (see sections 3 and 4) each filter having the capability of on-line estimation of the BT characteristics (for instance the drag coefficient β) and (ii) an IMM constituted by a number of Unscented Kalman Filters (UKFs). For a complete description of the two trackers, see [3],[10]. The IMM-EKF and IMM-UKF are tested against BT trajectories generated with a BT model with a proper level of process noise (see section 5).

The paper is organized as follows: next section 3 contains a brief description of the BT kinematics and radar measures. Section 4 recalls the motivation for the choice of the IMM-based tracking architectures and the technique used for fusing the plots received from two radars. Section 5 presents the achieved results; conclusions and references are contained in sections 6 and 7 respectively.

3. Models of BT dynamic and radar measures

Three main forces affect the BT motion: thrust, drag and gravity. For the sake of this paper, it is assumed that the BT is in the cruise phase during the BT state vector estimation while drag and gravity are acting on the target body during the re-entry phase.

The drag acceleration expression is [1-4].

$$\mathbf{a}_{drag}=\begin{bmatrix} a_{dragx} \\ a_{dragy} \\ a_{dragz} \end{bmatrix}=-\frac{1}{2}\frac{\rho(z)\cdot g_0}{\beta}\sqrt{\dot{x}^2+\dot{y}^2+\dot{z}^2}\cdot\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} \tag{1}$$

where β is the ballistic coefficient (N/m^2), $\rho(z)$ is the air density function of the height:

$$\rho(z) = 1.21907 \cdot e^{-z/9146.64} \tag{2}$$

$\dot{x}, \dot{y}, \dot{z}$ are the velocity components of the BT along the three axes of a Cartesian reference system. The gravity acceleration is considered constant, g_0=9.8m/s^2 and directed along the z-axis. Process noise has to be added in the modeling of BT dynamic to account for all forces that have not been considered in the model and possible deviations of the model from the reality.

Process noise is modeled as a zero-mean white Gaussian process with non-singular covariance matrix:

$$\mathbf{Q} = q\begin{bmatrix} \boldsymbol{\theta} & \underline{0} \\ \underline{0} & \boldsymbol{\theta} \end{bmatrix}, \tag{3}$$

with

$$\boldsymbol{\theta} = \begin{bmatrix} T^3/3 & T^2/2 \\ T^2/2 & T \end{bmatrix} \tag{4}$$

where q is a parameter related to process noise intensity and T is the BT trajectory sampling time.

The measurements, collected by the radar for target tracking, are the range r, elevation ε and azimuth ϑ.

The error standard deviations of these measurements are denoted as σ_r (for range), σ_ε (for elevation) and σ_ϑ (for azimuth). Radar measurements are transformed to the Cartesian coordinates so that the measurement equation is linear:

$$\mathbf{z}_k = \mathbf{H}\mathbf{s}_k + \mathbf{v}_k \tag{5}$$

where $\mathbf{v}_k$ is the noise on the measured Cartesian co-ordinates; it is zero-mean white Gaussian with covariance matrix $\mathbf{R}_k$ [1] while the definition of $\mathbf{H}$ is

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The definition of $\mathbf{s}_k$ is reported in next section 5.

Mathematical details on the EKF and UKF filter derivation for the BT tracking application can be found in [3] and [10].

For all practical purposes this is a good approximation, which greatly simplifies the tracking algorithm; otherwise one would also have to take into consideration the non-linearity of the measurement equation.

4. IMM-UKF and IMM-EKF architecture

The theory and the application of the IMM have been the subject of many publications, see for instance 4. The rationale for choosing the IMM approach for the tracking of a potential BT is essentially due to the fact that the BT characteristics are not generally “a priori” known, thus it is required to “on-line” estimate the BT parameters to maximize the tracker accuracy. The IMM offers the possibility of mixing the output of different filters designed for different BTs, each one having the possibility of adapting its parameters to the target to be tracked, thus permitting the correct tracking of BTs pertaining to different “classes”. In addition to this, the probability of selecting one of the filters existing in the bank of the IMM gives a clear indication of the confidence of the tracker on the type of target under analysis; this is an intrinsic capability of non co-operative target classification, available “for free” by the IMM.

The two tracking architectures compared in this paper (IMM-EKF and IMM-UKF) have been already described in [3] and [9] and their performance analyzed in absence of process noise in the trajectories generation showing practically the same performance. In this paper, we extend the results of papers [3 - 9] by including the BT model uncertainty and we obtain partially different conclusions. In fact, the presence of process noise in the trajectory generation enhances the essential difference between the two approaches. The prediction of the EKF covariance matrix is performed by first order Taylor expansion of the BT motion equations; instead, for the UFK, the same propagation is achieved via propagation of properly selected particles [7] thus resulting in an improvement of the filter accuracies as already stated in [1].

The implemented IMM architectures are the following :

IMM-EKF. The bank is constituted by four filters ; the first two are Kalman Filter (KF) matched to maneuvering and not maneuvering ABT. The remaining filters are two EKFs matched to BT cruise and re-entry phases with different initial beta values.

IMM-UKF. It has the same IMM-EKF number of filters. The KFs are not changed while the BT filters are based on the UKF theory.

4.1 Plots fusion approach.

The two sequences of plots provided by the two non-colocated radars are interleaved according to their time of arrival. Consequently, just one sequence of plots is formed. The time interval between the plots might not be constant; the measurement accuracy is related to the radar which has provided the plot, to the distance of the target from the radar and to the scan - off angle with which the radar beam has looked at the target. The co-ordinate reference system is located on the first radar; thus, also the plots of the second radar are referenced to this system co-ordinate. Next figure 4.1 gives a pictorial view of the plots fusion concept.

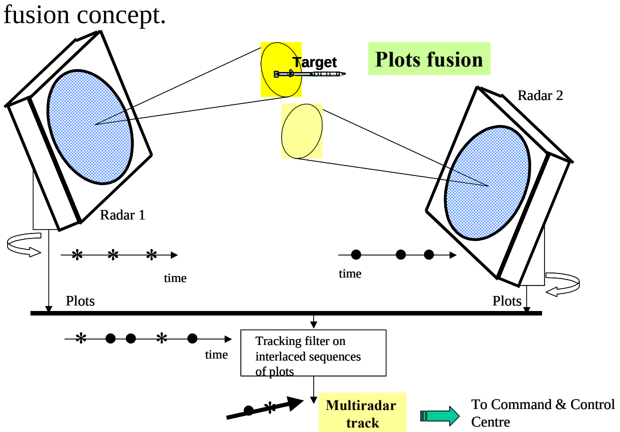

Figure 4.1 Plots fusion approach.

5. Achieved Results

The study case selected for the IMM-EKF & IMM-UKF comparison concerns a simulated BT with the following characteristics: single stage, linear consumption of propellant vs. flight time, drag coefficient=50000 N/m^2, radar detection range=150 km, target radar cross section=1 m^2. Two radars are considered in the simulated scenario at a distance of 9 km; the two radars are named “Radar Master” (the one performing the plots fusion) and “Radar Slave” in the following. The parameters of these two notional radars are: range accuracy=25m, azimuth accuracy=0.1°, elevation accuracy =0.1°, data rate=6 seconds, either Pd=1 or Pd=0.9 constant with target range. The BT- fused radars geometry is depicted in figure 5.1.

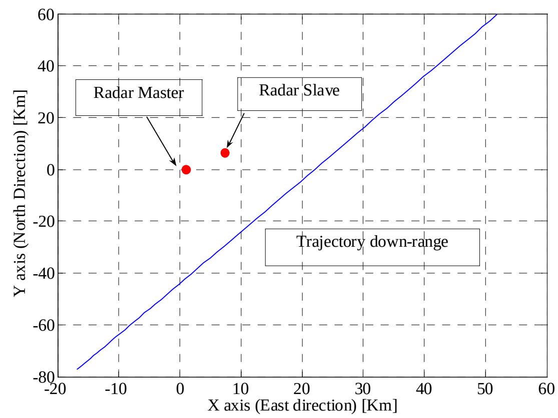

Figure 5.1 BT-two radars geometry.

The IMM architectures are constituted by a number of EKFs or UKFs matched to the BT dynamics (see section 3) each filter having the capability of on-line estimation of the BT characteristics (for instance the drag coefficient; see [1] for more details).

At the k-th time instant the state vector $\mathbf{s}_k$ contains the position, the speed components of target with respect to the Cartesian axes, and the ballistic coefficient:

$$\mathbf{s}_k = \begin{bmatrix} x_k & \dot{x}_k & y_k & \dot{y}_k & z_k & \dot{z}_k & t_{Nk} & \beta_k \end{bmatrix}^T$$

Given the state $\hat{\mathbf{s}}_{k/k}$ estimated at the *k*-th time instant it is possible to compute the corresponding estimation covariance matrix $\mathbf{P}_{k/k}$ as reported in [3],[10].

The tracker performance are expressed via the volume of uncertainty (one sigma) ellipsoid of the target track (see [11])computed in two ways: (i) use the estimate of the tracking architecture covariance matrix (referred as $P_{k/k}$), (ii) use the covariance matrix of the estimated state vector P_{k/k_calc} which is defined as the tracking accuracy covariance matrix averaged over the Monte Carlo trials.

Note that, in absence of uncertainty in the BT model, the P_{kk} and the P_{kk_calc} are coincident for both the IMM-UKF and IMM-EKF. In presence of the model uncertainty, the P_{kk} and the P_{kk_calc} are still coincident for the IMM-UKF, while the IMM-EKF "under estimates" the P_{kk} and the two matrices are different. The real filter capabilities are derived from the P_{kk_calc} and the achieved results are reported in the following figures showing the superior performance of the IMM-UKF approach.

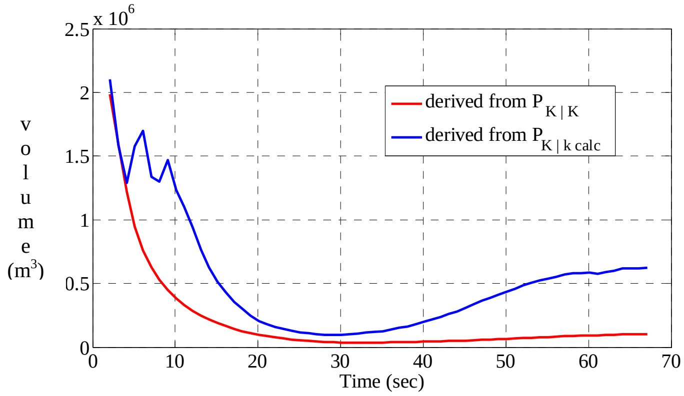

Figure 5.2. Comparison of IMM-EKF uncertainty volume computed from the P_{kk} and the P_{kk_calc} in presence of uncertainty in the BT model.

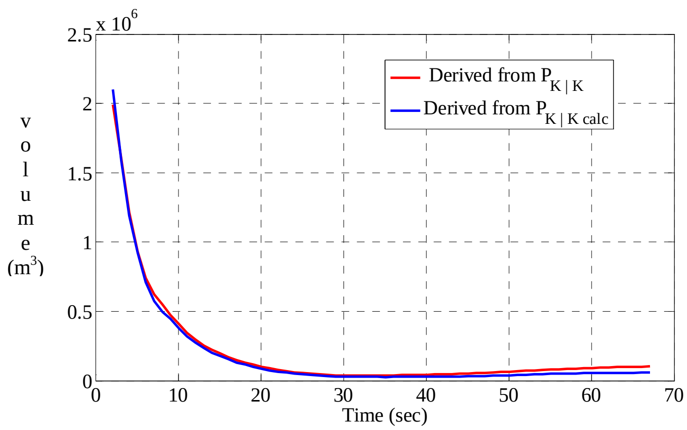

Figure 5.3. Comparison of IMM-UKF uncertainty volume computed from the Pkk and the Pkk_calc in presence of uncertainty in the BT model.

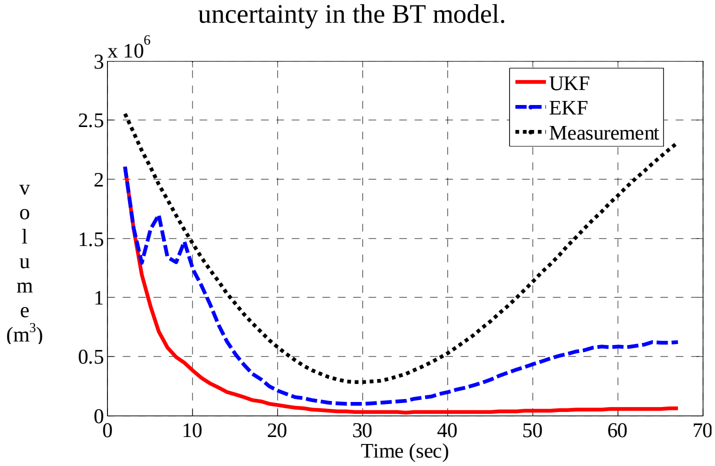

Figure 5.4. Comparison of IMM-UKF, IMM-EKF and unfiltered uncertainty volume computed from the P_{kk_calc} in presence of uncertainty in the BT model.

The performance achieved in the estimation of the BT ballistic coefficient are reported in figure 5.5 again demonstrating the advantages guaranteed by the IMM-UKF. The upper curve of figure 5.5 presents the mean value of the beta estimation error, while lower curve concerns the standard deviation of the same error.

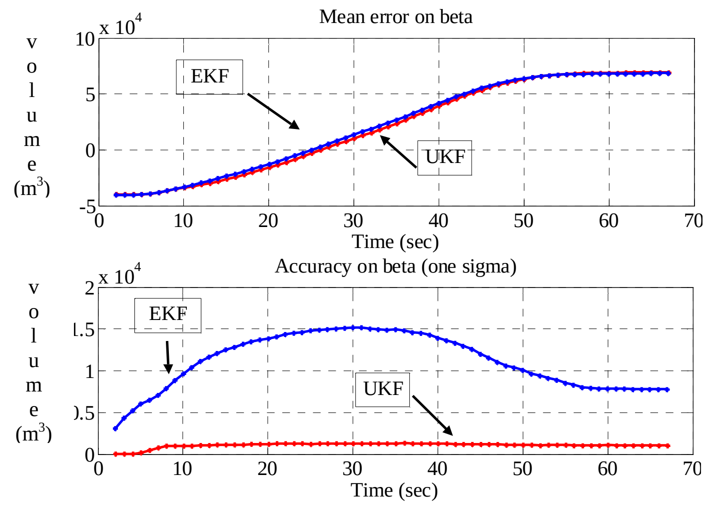

Figure 5.5. Comparison of IMM-UKF, IMM-EKF Beta estimation

The filter robustness has been also given reporting the well or badly conditions for the $\mathbf{P}_{K+1|K}$ matrices along all Monte Carlo trials. The condition number (CN) gives an indication of the accuracy of the results from matrix inversion and a well conditioned prediction covariance matrix gives good value for the Kalman gain. The CN is computed via the MATLAB® function RCOND(X) which is an estimate for the reciprocal of the CN of matrix X in the 1-norm obtained by the LAPACK condition estimator. If X is well conditioned, RCOND(X) is near 1.0. If X is badly conditioned, RCOND(X) is near 0. Next figure 5.6 presents the results achieved for the IMM-EKF (upper curves) and for the IMM-UKF (lower curves).

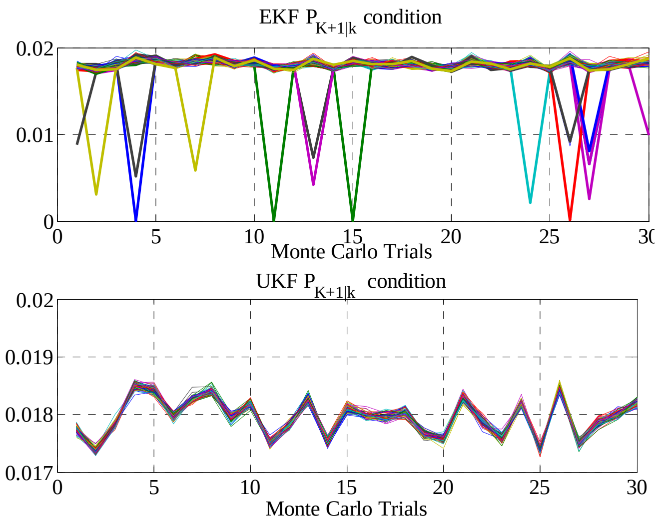

Figure 5.6. Comparison of Pk+1|k Condition Coefficient

6. Conclusions

As reported in[10], the IMM-UKF approach requires a computational load approximately three times higher than the IMM-EKF. It has been demonstrated that in presence of uncertainty in the target model, a robust and improved accuracies are obtained by the IMM-UKF with respect to the IMM-EKF. The role of the model uncertainty has been detailed in this paper and an application to the tracking of BT has been described.

References

1. D. Benvenuti, A. Farina, B. Ristic, "Estimation accuracy of a landing point of a ballistic target", Proc. of Intl. Conf. Fusion 2002, Washington DC, May 2002, pp. 2-9.

2. A. Farina, M.G. Del Gaudio , U. D'Elia , S. Immediata, L. Ortenzi, L. Timmoneri , M.R. Toma, "Detection and Tracking of Ballistic Target", 2004 IEEE Radar Conference, April 2004, Philadelphia (USA), pp. 450-456.

3. A. Farina, S. Immediata, L. Ortenzi, L. Timmoneri, "Tracking of ballistic target: comparison of an IMM architecture with the CRLB" Multinational Ballistic Missile Defense Conference, 21-24 July 2004, Berlin.

4. A. Farina, B. Ristic, D. Benvenuti," Tracking a ballistic target: comparison of several non linear filters", IEEE Trans on AES, vol. 38, issue no. 3, July 2002, pp. 854-867.

5. Y. Bar-Shalom, X. R. Li, " Estimation and Tracking: principles, techniques and software", Artech House, 1993.S.

6. Julier, S, J. Uhlmann, H. F. Durrant_Whyte, "A new method for the non linear transformation of means and covariances in filters and estimators", IEEE Trans. on Automatic Control, vol. AC-45, no. 3, March 2000, pp. 477-482.

7. S. J. Julier, J. K. Uhlmann, "Unscented Filtering and Nonlinear Estimation", Proc. of the IEEE, vol. 92, No. 3, March 2004, pp. 401-422.

8. Farina, F. A. Studer "Radar Data Processing, New York, Wiley, May 1986, Vol. II.

9. Y. Bar-Shalom, L. Campo, "The effect of the common process noise on the two sensors fused track covariance", IEEE Trans. on Aerospace and Electronic Systems, vol. 22, November 1986, pp. 803-805.

10. A. Farina, S. Immediata, B. Ristic, L. Timmoneri, "Comparison of two IMM tracking and classifier architectures based on Extended and Unscented Kalman Filters with CRLB", 2005 IEEE Workshop on Statistical Signal Processing, Bordeaux, 17-20 July 2005.

11. Capponi, A. Di Lallo, A. Farina, T. Volpi, "Accuracy of fused track for radar systems", Signal Processing 85 (2005), pp. 1189-1210.