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EIGENSPACE ADAPTIVE FILTERING FOR EFFICIENT PRE-EQUALIZATION OF ACOUSTIC MIMO SYSTEMS

S. Spors [1], *H. Buchner* [2] *and R. Rabenstein* [2]

[1] Deutsche Telekom Laboratories
Ernst-Reuter Platz 7
10587 Berlin, Germany
Sascha.Spors@telekom.de

[2] Telecommunications Laboratory
University Erlangen-Nuremberg
Cauerstrasse 7, 91058 Erlangen, Germany
{rabe,buchner}@LNT.de

ABSTRACT

Pre-equalization of MIMO systems is required for a wide variety of applications, e. g. in channel equalization and spatial sound reproduction. However, traditional adaptive algorithms fail for channel numbers of some ten or more. It is shown that the problem becomes tractable by decoupling of the MIMO adaptation problem, e. g. by a generalized singular value decomposition. This method is called eigenspace adaptive filtering. The required singular vectors depend on the unknown system response to be equalized. An reasonable approximation by data-independent transformations is derived for the example of listening room compensation yielding the approach of wave-domain adaptive filtering.

1. INTRODUCTION

The pre-equalization of MIMO systems by adaptive filtering is a well-known topic, e.g. in communications for channel equalization in multi-antenna scenarios. The same problem arises as well in acoustics for massive multichannel reproduction systems such as wave field synthesis (WFS) or higher order Ambisonics. The theoretical background of these reproduction methods assumes free-field propagation of acoustic waves. However, reproduction systems in theaters, studios, and homes are always subject to acoustic reflections at the walls of the listening room. Passive measures like damping material will diminish these reflections to some extent, but they are not very effective for low frequencies.

An active measure is the pre-equalization of the loudspeaker signals such that their emitted waves together with the unavoidable reflections produce the desired wave field. This process is also called active room compensation and constitutes an inverse filtering problem. Unfortunately, the response of the listening room to acoustic excitations is non-stationary and depends on several imponderabilities, like opening of doors, motion of persons, and changes in the room temperature. Consequently, only adaptive algorithms are suitable for active room compensation. It will be shown in this paper that conventional adaptive algorithms are ineffective for acoustic pre-equalization in the context of MIMO systems with a high number of channels.

An escape to this situation has been shown in a previous contribution [1]. By using a set of spatio-temporal transformations the MIMO adaptation problem was approximately decoupled. This resulted in the highly efficient wave-domain adaptive filtering (WDAF) algorithm. This contribution looks at the MIMO pre-equalization problem from a more theoretical perspective and provides the foundations to the WDAF approach from [1].

The paper is organized as follows: Sec. 2 prepares the ground by discussing a generic MIMO pre-equalization approach. Sec. 3 attacks the decoupling problem by a generalized singular value decomposition (GSVD) which leads to a solution called eigenspace adaptive filtering (Sec. 4). However, for perfect decoupling, the singular vectors have to be determined according to the unknown acoustical characteristics of the systems. Sec. 5 presents an approximate solution at the example of active listening room compensation based on a physical interpretation of the singular vectors. For listening rooms with moderate reverberation they may be represented by circular harmonics which are independent of the particular listening room. The presented results reveal that a thorough mathematical method (GSVD) in combination with physical insight (circ. harmonics) gives the theoretical foundation of the WDAF method proposed in [1].

2. ADAPTIVE PRE-EQUALIZATION OF MIMO SYSTEMS

The following section briefly reviews adaptive pre-equalization of MIMO systems and outlines the fundamental problems of traditional adaptation algorithms within the context of massive MIMO systems.

2.1 Description of Scenario

The generic MIMO pre-equalization scenario illustrated by the discrete time and space block diagram shown in Fig. 1 will be considered in the following. The matrices of impulse responses $\mathbf{R}(k)$, $\mathbf{F}(k)$ and $\mathbf{C}(k)$ describe discrete linear multiple-input/multiple-output (MIMO) FIR systems. The driving signals are denoted by the vector $\mathbf{d}^{(\mathrm{N})}(k) = [d_1(k), d_2(k), \cdots, d_N(k)]^T$, where $d_n(k)$ denotes the signal of the n-th channel. The filtered driving signals are denoted by the vector $\mathbf{w}^{(\mathrm{N})}(k)$, the output signals by $\mathbf{l}^{(\mathrm{M})}(k)$, the desired signal by $\mathbf{a}^{(\mathrm{M})}(k)$ and the error between the analyzed and the desired signal by $\mathbf{e}^{(\mathrm{M})}(k)$. The elements of these vectors are given according to $\mathbf{d}^{(\mathrm{N})}(k)$.

The unknown system $\mathbf{R}(k)$ with N-input and M-output channels and impulse responses $r_{m,n}(k)$ is pre-equalized by the equalization filter $\mathbf{C}(k)$ with coefficients $c_{n,n'}(k)$. The fundamental problem of adaptive pre-equalization is to compute a pre-equalization filter such that the overall response of $\mathbf{C}(k)$ and $\mathbf{R}(k)$ matches the desired system response $\mathbf{F}(k)$ as closely as possible. For few input and output channels numerous solutions to this problem have been developed in the past, e. g. [2]. However, algorithms for MIMO systems with a high number of channels (massive MIMO systems) still remain a challenge as will be shown in the following.

2.2 Non-adaptive Computation of Pre-equalization Filters

In order to gain more insight into the solution of the pre-equalization problem the non-adaptive case will be discussed first. For this purpose a frequency-domain description of the pre-equalization problem depicted in Fig. 1 is used.

Performing a discrete-time Fourier transformation (DTFT) [3] of the respective signals and systems yields the signal at the M analysis points in the frequency domain as

$$\underline{\mathbf{l}}^{(\mathrm{M})}(\omega) = \underline{\mathbf{R}}(\omega)\, \underline{\mathbf{w}}^{(\mathrm{N})}(\omega)\,, \tag{1}$$

where $\underline{\mathbf{R}}(\omega)$ denotes the DTFT transformed matrix of impulse responses from each synthesis to each analysis position, $\underline{\mathbf{l}}^{(\mathrm{M})}(\omega)$ and $\underline{\mathbf{w}}^{(\mathrm{N})}(\omega)$ the DTFT transformed signals at the analysis positions and filtered driving signals respectively.

The error $\underline{\mathbf{e}}^{(\mathrm{M})}(\omega)$ between the desired $\underline{\mathbf{a}}^{(\mathrm{M})}(\omega)$ and the actual

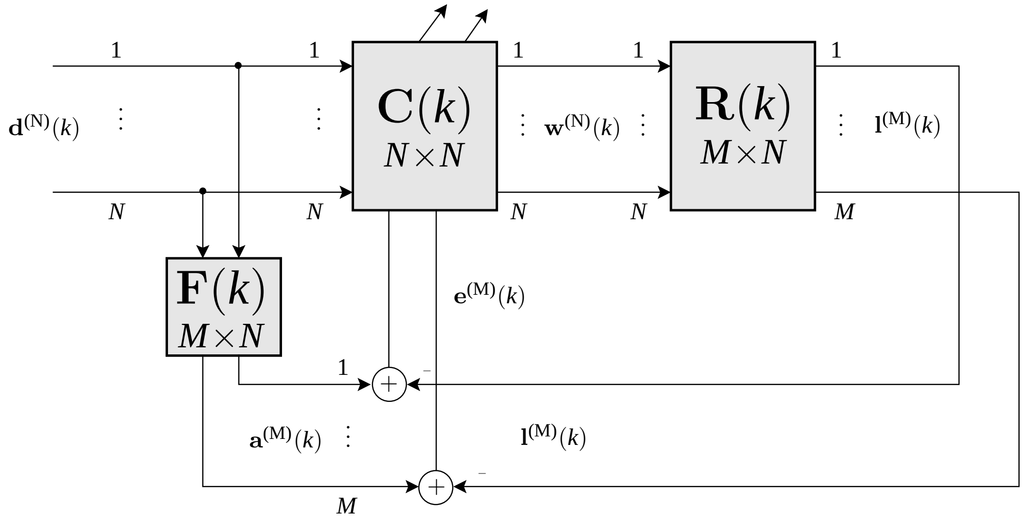

Figure 1: Block diagram illustrating the generic MIMO pre-equalization approach.

$\underline{\mathbf{l}}^{(\mathrm{M})}(\omega)$ signal at the M analysis positions can be derived as

$$\begin{aligned}\underline{\mathbf{e}}^{(\mathrm{M})}(\omega) &= \underline{\mathbf{a}}^{(\mathrm{M})}(\omega) - \underline{\mathbf{l}}^{(\mathrm{M})}(\omega) = \\ &= \underline{\mathbf{F}}(\omega)\,\underline{\mathbf{d}}^{(\mathrm{N})}(\omega) - \underline{\mathbf{R}}(\omega)\,\underline{\mathbf{C}}(\omega)\,\underline{\mathbf{d}}^{(\mathrm{N})}(\omega)\,.\end{aligned} \tag{2}$$

Optimal pre-equalization is obtained by minimizing this error: $\underline{\mathbf{e}}^{(\mathrm{M})}(\omega) \rightarrow \mathbf{0}$. The least-squares solution to Eq. (2) in this sense, with respect to the compensation filter, is given as [4]

$$\underline{\mathbf{C}}(\omega) = \underline{\mathbf{R}}^{+}(\omega)\,\underline{\mathbf{F}}(\omega)\,, \tag{3}$$

where $\underline{\mathbf{R}}^{+}(\omega)$ denotes the pseudoinverse of $\underline{\mathbf{R}}(\omega)$.
Calculating the pre-equalization filters by evaluation of Eq. (3) has two major drawbacks: (1) the filters cannot cope with time-variant system characteristics and (2) the exact solution may require filters with a high number of coefficients [5]. In order to overcome these drawbacks an adaptive computation of the pre-equalization filters based on a least-squares error (LSE) criterion is derived in the following section.

2.3 Least-Squares Error Adaptation of the Pre-Equalization Filter

The following section briefly reviews the derivation of the normal equation for the adaptive pre-equalization problem introduced in Section 2.1. A detailed discussion for acoustic MIMO systems can be found e. g. in [2, 6]. The normal equation is the basis for the derivation of the filtered-x recursive least squares algorithm (X-RLS).
The cost function of the filtered-x RLS algorithm is given as

$$\xi(\hat{\mathbf{c}},k) = \sum_{\kappa=0}^{k} \lambda^{k-\kappa} \sum_{m=1}^{M} |e_m(\kappa)|^2\,, \tag{4}$$

where $0 < \lambda \leq 1$ denotes an exponential weighting factor. The optimal filter coefficients in the mean-squared error (MSE) sense are found by setting the gradient with respect to the estimated filter coefficients $\hat{\mathbf{c}}$ of the cost function to zero. The normal equation is then derived by expressing the error $\mathbf{e}^{(\mathrm{M})}(k)$ in terms of the filter coefficients, introducing the result into the cost function (4) and calculating its gradient. The resulting normal equation is given as

$$\hat{\mathbf{\Phi}}_{dd}(k)\;\hat{\mathbf{c}}(k) = \hat{\mathbf{\Phi}}_{da}(k)\,, \tag{5}$$

where $\hat{\mathbf{c}}(k)$ denotes the vector of all filter coefficients at the time instant k

$$\hat{\mathbf{c}}_n^{(\mathrm{N})}(k) = [\;\hat{\mathbf{c}}_{n,1}^T(k)\quad \hat{\mathbf{c}}_{n,2}^T(k)\quad \cdots\quad \hat{\mathbf{c}}_{n,N}^T(k)\;]^T\,, \tag{6a}$$

$$\hat{\mathbf{c}}(k) = [\;\hat{\mathbf{c}}_1^{(\mathrm{N})}(k)^T\quad \hat{\mathbf{c}}_2^{(\mathrm{N})}(k)^T\quad \cdots\quad \hat{\mathbf{c}}_N^{(\mathrm{N})}(k)^T\;]^T\,. \tag{6b}$$

The vector $\hat{\mathbf{c}}_{n,n'}(k)$ of estimated filter coefficients at time-instant k is given as $\hat{\mathbf{c}}_{n,n'}(k) = [\hat{c}_{n,n'}(0), \hat{c}_{n,n'}(1), \cdots, \hat{c}_{n,n'}(N_c-1)]^T$ where N_c denotes the number of filter coefficients. The $N^2N_c \times N_cN^2$ matrix $\hat{\mathbf{\Phi}}_{dd}$ denotes the time and analysis position-averaged auto-correlation matrix of the filtered driving signals

$$\hat{\mathbf{\Phi}}_{dd}(k) = \sum_{\kappa=0}^{k} \lambda^{k-\kappa}\;\mathbf{D}_R(\kappa)\,\mathbf{D}_R^T(\kappa)\,, \tag{7}$$

where $\mathbf{D}_R(\kappa)$ denotes the matrix of filtered driving signals. The matrix of filtered driving signals is given as follows

$$\mathbf{d}_{m,n}^{(\mathrm{N})}(k) = [\;\mathbf{d}_{m,n,1}^T(k)\quad \mathbf{d}_{m,n,2}^T(k)\quad \cdots\quad \mathbf{d}_{m,n,N}^T(k)\;]^T\,, \tag{8a}$$

$$\mathbf{d}_{m}^{(\mathrm{N,N})}(k) = [\;\mathbf{d}_{m,1}^{(\mathrm{N})}(k)^T\quad \mathbf{d}_{m,2}^{(\mathrm{N})}(k)^T\quad \cdots\quad \mathbf{d}_{m,N}^{(\mathrm{N})}(k)^T\;]^T\,, \tag{8b}$$

$$\mathbf{D}_R(k) = [\;\mathbf{d}_1^{(\mathrm{N,N})}(k)\quad \mathbf{d}_2^{(\mathrm{N,N})}(k)\quad \cdots\quad \mathbf{d}_M^{(\mathrm{N,N})}(k)\;]\,, \tag{8c}$$

where $\mathbf{d}_{m,n,n'}(k)$ denotes a $N_c \times 1$ vector composed from the result of the convolution $d_{n'}(k) * r_{m,n}(k)$ of the driving signals with the system response. The $N^2N_c \times 1$ vector $\hat{\mathbf{\Phi}}_{da}$ can be interpreted as the time and analysis position-averaged cross-correlation vector between the filtered driving signals and the desired signals which is defined as

$$\hat{\mathbf{\Phi}}_{da}(k) = \sum_{\kappa=0}^{k} \lambda^{k-\kappa}\;\mathbf{D}_R(\kappa)\,\mathbf{a}^{(\mathrm{M})}(\kappa)\,. \tag{9}$$

The optimal pre-equalization filter with respect to the cost function (4) is given by solving the normal equation (5).
The filtered-x RLS algorithm can be derived from the normal equation (5) by computing the sums (7) and (9) in a recursive fashion and by applying the matrix inversion lemma. The filtered-x RLS algorithm deviates from the standard RLS algorithm by using a filtered version of the driving signal for adaptation.

2.4 Fundamental Problems of Adaptive Inverse Filtering

Four fundamental problems of adaptive pre-equalization can be concluded from the normal equation (5). These are:

1. non-uniqueness of the solution,
2. ill-conditioning of the auto-correlation matrix $\hat{\Phi}_{dd}(k)$,
3. computational complexity for massive MIMO systems, and
4. required a-priori knowledge of the room transfer function.

The first problem is related to the minimization of the cost function $\xi(\hat{\mathbf{c}},k)$. The optimal pre-equalization filter is given by calculating the inverse filter to the system response (see Eq. (3)). However, minimization of the cost function $\xi(\hat{\mathbf{c}},k)$ may not provide the optimal solution in these terms. Depending on the driving signals $\mathbf{d}^{(\mathrm{N})}(k)$ there may be multiple possible solutions for $\hat{\mathbf{c}}$ that minimize $\xi(\hat{\mathbf{c}},k)$ [7]. This problem is termed as the *non-uniqueness problem* in the following.
The second and third fundamental problem is related to the solution of the normal equation (5). The normal equation has to be solved with respect to the coefficients of the room compensation filter. However, due to the dimensionality and potential ill-conditioning of the auto-correlation matrix $\hat{\Phi}_{dd}(k)$ this may become an infeasible task for a large number of input and output channels. Additionally an exact solution may not always exist [2].
The calculation of the filtered driving signals requires knowledge of the system response $\mathbf{R}(k)$. Hence, the system response has to be identified additionally. There are various on-line identification methods for this task. An overview on possible methods can be found in [2, 8]. However, most of these algorithms are not capable of handling the massive multichannel case [7] for the same reasons as mentioned above.
The following section will derive a generic framework for pre-equalization which explicitly solves the third problem by utilizing signal and system transformations. It will be shown additionally that the other problems are highly alleviated by the proposed approach. The basic idea is to perform a decoupling of the MIMO systems $\underline{\mathbf{R}}(\omega)$ and $\underline{\mathbf{F}}(\omega)$. This decoupling yields a decoupling of the MIMO adaptation problem and the auto-correlation matrix $\hat{\Phi}_{dd}(n)$ as will be shown in the remainder of this paper.

3. DECOUPLING OF THE MIMO SYSTEMS

This section shows how the desired decoupling of the transfer matrices $\underline{\mathbf{R}}(\omega)$ and $\underline{\mathbf{F}}(\omega)$ can be obtained using the concept of the (generalized) singular value decomposition (SVD).

3.1 Generalized Singular Value Decomposition

It will be assumed in the following that $\underline{\mathbf{R}}(\omega)$ and $\underline{\mathbf{F}}(\omega)$ have the dimensions $M \times N$ with $N \geq M$. However, the derived results can be generalized straightforwardly to arbitrary M and N.
The singular value decomposition (SVD) states that any matrix can be decomposed into two unitary matrices and a diagonal matrix [4, 9]. The concept of the SVD can be generalized to the diagonalization of a pair of matrices. This decomposition is known as generalized singular value decomposition (GSVD) [9]. The GSVD for the matrices $\underline{\mathbf{R}}(\omega)$ and $\underline{\mathbf{F}}(\omega)$ is given as follows:

$$\underline{\mathbf{R}}(\omega) = \underline{\mathbf{X}}(\omega)\,\underline{\tilde{\mathbf{R}}}(\omega)\,\underline{\mathbf{V}}^H(\omega)\,, \tag{10a}$$
$$\underline{\mathbf{F}}(\omega) = \underline{\mathbf{X}}(\omega)\,\underline{\tilde{\mathbf{F}}}(\omega)\,\underline{\mathbf{U}}^H(\omega)\,. \tag{10b}$$

The matrices $\underline{\mathbf{X}}(\omega)$, $\underline{\mathbf{V}}(\omega)$ and $\underline{\mathbf{U}}(\omega)$ are unitary matrices with the dimensions $M \times M$, $N \times M$ and $N \times M$, respectively. The matrix $\underline{\mathbf{X}}(\omega)$ is the generalized singular matrix of $\underline{\mathbf{R}}(\omega)$ and $\underline{\mathbf{F}}(\omega)$, the matrices $\underline{\mathbf{V}}(\omega)$ and $\underline{\mathbf{U}}(\omega)$ the respective right singular matrices of $\underline{\mathbf{R}}(\omega)$ and $\underline{\mathbf{F}}(\omega)$. The matrices $\underline{\tilde{\mathbf{R}}}(\omega)$ and $\underline{\tilde{\mathbf{F}}}(\omega)$ are diagonal matrices constructed from the singular values of $\underline{\mathbf{R}}(\omega)$ and $\underline{\mathbf{F}}(\omega)$. The diagonal matrix $\underline{\tilde{\mathbf{R}}}(\omega)$ is defined as

$$\underline{\tilde{\mathbf{R}}}(\omega) = \mathrm{diag}\{\,[\sigma_{\mathrm{R},1}, \sigma_{\mathrm{R},2}, \cdots, \sigma_{\mathrm{R},M}]\,\}\,, \tag{11}$$

where $\sigma_{\mathrm{R},1} \geq \sigma_{\mathrm{R},2} \geq \cdots \geq \sigma_{\mathrm{R},B} > 0$ denote the B nonzero singular values $\sigma_{\mathrm{R},b}$ of $\underline{\mathbf{R}}(\omega)$. Their total number B is given by the rank of the matrix $\underline{\mathbf{R}}(\omega)$ with $1 \leq B \leq M$. For $B < M$ the remaining singular values $\sigma_{\mathrm{R},B+1}, \sigma_{\mathrm{R},B+2}, \cdots, \sigma_{\mathrm{R},M}$ are zero. Similar definitions as given above for $\underline{\tilde{\mathbf{R}}}(\omega)$ apply to the matrix $\underline{\tilde{\mathbf{F}}}(\omega)$.
The relation given by Eq. (10a) can be inverted by exploiting the unitary property of the joint and right singular matrices. This results in

$$\underline{\tilde{\mathbf{R}}}(\omega) = \underline{\mathbf{X}}^H(\omega)\,\underline{\mathbf{R}}(\omega)\,\underline{\mathbf{V}}(\omega)\,. \tag{12}$$

Hence each matrix $\underline{\mathbf{R}}(\omega)$ can be transformed into a diagonal matrix $\underline{\tilde{\mathbf{R}}}(\omega)$ using the joint and right singular matrix $\underline{\mathbf{X}}(\omega)$ and $\underline{\mathbf{V}}(\omega)$. A similar relation as given by Eq. (12) can be derived straightforwardly for $\underline{\tilde{\mathbf{F}}}(\omega)$. The GSVD transforms the matrices $\mathbf{R}(\omega)$ and $\mathbf{F}(\omega)$ into their joint eigenspace using the singular matrices $\underline{\mathbf{X}}(\omega)$, $\underline{\mathbf{V}}(\omega)$ and $\underline{\mathbf{U}}(\omega)$. In general, these singular matrices depend on the matrices $\underline{\mathbf{R}}(\omega)$ and $\underline{\mathbf{F}}(\omega)$. The GSVD is a *data-dependent transformation*.
The SVD can be used to define the pseudoinverse $\underline{\mathbf{R}}^+(\omega)$ of the matrix $\underline{\mathbf{R}}(\omega)$ [4]

$$\underline{\mathbf{R}}^+(\omega) = \underline{\mathbf{V}}(\omega)\,\underline{\tilde{\mathbf{R}}}^{-1}(\omega)\,\underline{\mathbf{X}}^H(\omega)\,. \tag{13}$$

Equation (10b) and Eq. (13) can be combined to derive the following result

$$\underline{\mathbf{R}}^+(\omega)\,\underline{\mathbf{F}}(\omega) = \underline{\mathbf{V}}(\omega)\;\underline{\tilde{\mathbf{R}}}^{-1}(\omega)\,\underline{\tilde{\mathbf{F}}}(\omega)\;\underline{\mathbf{U}}^H(\omega)\,, \tag{14}$$

where it is assumed that $\underline{\mathbf{R}}(\omega)$ and $\underline{\mathbf{F}}(\omega)$ have both full rank. Equation (14) will be used to derive the desired decoupling of the MIMO adaptation problem.

3.2 Decoupling of the MIMO System $\underline{\mathbf{R}}(\omega)$

The SVD, as introduced in the previous section, can be used to transform the MIMO system into a decoupled representation. Equation (12) together with the unitary property of the joint and right singular matrices can be used to reformulate Eq. (1) as follows

$$\underbrace{\underline{\mathbf{X}}^H(\omega)\,\underline{\mathbf{l}}^{(\mathrm{M})}(\omega)}_{\underline{\tilde{\mathbf{l}}}^{(\mathrm{M})}(\omega)} = \underline{\tilde{\mathbf{R}}}(\omega)\;\underbrace{\underline{\mathbf{V}}^H(\omega)\,\underline{\mathbf{w}}^{(\mathrm{N})}(\omega)}_{\underline{\tilde{\mathbf{w}}}^{(\mathrm{M})}(\omega)}\,, \tag{15}$$

where the $M \times 1$ vectors $\underline{\tilde{\mathbf{l}}}^{(\mathrm{M})}(\omega)$ and $\underline{\tilde{\mathbf{w}}}^{(\mathrm{M})}(\omega)$ denote the transformed signals at the analysis positions and the transformed driving signals respectively. Hence, in the context of signals and systems the SVD can be understood as a transformation. The joint and right singular matrices $\underline{\mathbf{X}}(\omega)$ and $\underline{\mathbf{V}}(\omega)$ constitute the kernels of this transformation. The transformation of the MIMO system $\underline{\mathbf{R}}(\omega)$ can be performed by pre- and post-filtering the system with $\underline{\mathbf{V}}(\omega)$ and $\underline{\mathbf{X}}^H(\omega)$. The pre- and post-filters constitute MIMO systems themselves. Thus, Eq. (1) can be expressed entirely in the transformed domain. The benefit of using this transform domain description of the system lies in the simplified structure of $\underline{\tilde{\mathbf{R}}}(\omega)$. As stated in the previous section $\underline{\tilde{\mathbf{R}}}(\omega)$ denotes the diagonal matrix composed of the singular values of $\underline{\mathbf{R}}(\omega)$. Due to its diagonal structure, the transformed signals $\underline{\tilde{\mathbf{l}}}^{(\mathrm{M})}(\omega)$ at the analysis positions can be computed by scalar multiplication of the main diagonal elements $\tilde{R}_m(\omega)$ of $\underline{\tilde{\mathbf{R}}}(\omega)$ with the transformed driving signals $\underline{\tilde{\mathbf{w}}}^{(\mathrm{M})}(\omega)$

$$\tilde{L}_m(\omega) = \tilde{R}_m(\omega)\,\tilde{W}_m(\omega)\,, \tag{16}$$

where $\tilde{L}_m(\omega)$ and $\tilde{W}_m(\omega)$ denote the m-th component of the vector $\underline{\tilde{\mathbf{l}}}^{(\mathrm{M})}(\omega)$ and $\underline{\tilde{\mathbf{w}}}^{(\mathrm{M})}(\omega)$ respectively. Hence, the transformation of the signals and systems using the SVD decomposes the MIMO system given by $\underline{\mathbf{R}}(\omega)$ into M single-input/single-output (SISO) systems.

4. EIGENSPACE ADAPTIVE FILTERING

The previous section derived a decomposition of the MIMO system $\underline{\mathbf{R}}(\omega)$ into a series of SISO systems by using an SVD based transformation. This section derives a decoupling of the entire adaptive

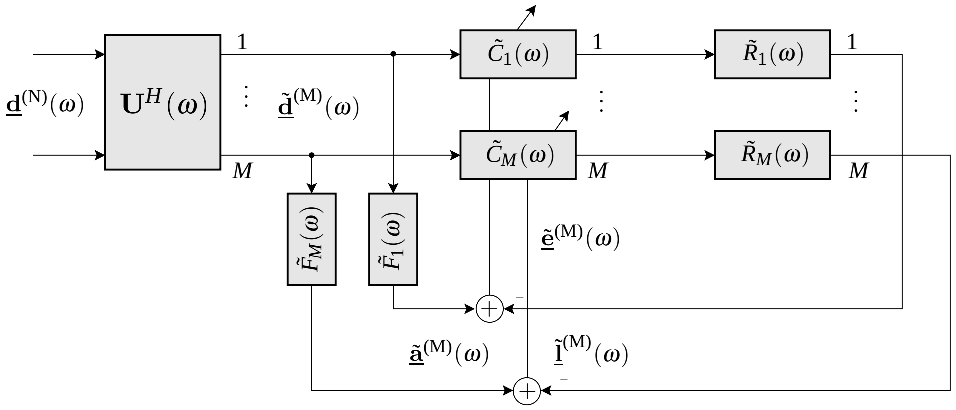

Figure 2: Block diagram illustrating the eigenspace adaptive inverse filtering approach to room compensation.

system depicted by Fig. 1 using the GSVD. For this purpose a decoupling of Eq. (2) will be derived, resulting in a decoupling of the MIMO adaptive inverse filtering problem. The basic idea is to diagonalize the system transfer matrix $\underline{\mathbf{R}}(\omega)$ and the desired system response $\underline{\mathbf{F}}(\omega)$ using the GSVD. It will be assumed first that both transfer matrices are known and have full rank. The results can be generalized straightforwardly to the case that $\underline{\mathbf{F}}(\omega)$ and/or $\underline{\mathbf{R}}(\omega)$ do not have full rank.

4.1 Decoupling of the Adaptive System

The decompositions of the transfer matrices $\underline{\mathbf{F}}(\omega)$ and $\underline{\mathbf{R}}(\omega)$ are given by Eq. (10). It remains to choose a suitable decomposition of the compensation filter $\underline{\mathbf{C}}(\omega)$. The non-adaptive solution for the pre-equalization filter is given by Eq. (3). Hence, an eigenspace expansion of $\underline{\mathbf{C}}(\omega)$ is given by Eq. (14). However, the system transfer matrix $\underline{\mathbf{R}}(\omega)$ is not known in general and has to be identified additionally. An expansion of the pre-equalization filter can be given by using Eq. (14) but with unknown expansion coefficients $\tilde{\underline{\mathbf{C}}}(\omega)$

$$\underline{\mathbf{C}}(\omega) = \underline{\mathbf{V}}(\omega)\,\tilde{\underline{\mathbf{C}}}(\omega)\,\underline{\mathbf{U}}^H(\omega)\;, \tag{17}$$

where $\tilde{\underline{\mathbf{C}}}(\omega)$ denotes a diagonal matrix, where some diagonal elements may be zero. Using Eq. (17) together with Eq. (10a) yields the transformed signal $\tilde{\underline{\mathbf{l}}}^{(\mathrm{M})}(\omega)$ at the analysis points

$$\tilde{\underline{\mathbf{l}}}^{(\mathrm{M})}(\omega) = \tilde{\underline{\mathbf{R}}}(\omega)\,\tilde{\underline{\mathbf{C}}}(\omega)\,\tilde{\underline{\mathbf{d}}}^{(\mathrm{M})}(\omega)\;, \tag{18}$$

where $\tilde{\underline{\mathbf{l}}}^{(\mathrm{M})}(\omega) = \underline{\mathbf{X}}^H(\omega)\,\underline{\mathbf{l}}^{(\mathrm{M})}(\omega)$ and $\tilde{\underline{\mathbf{d}}}^{(\mathrm{M})}(\omega) = \underline{\mathbf{U}}^H(\omega)\,\underline{\mathbf{d}}^{(\mathrm{N})}(\omega)$. Decomposition of the desired system response according to Eq. (10b) yields the desired signal in the transformed domain as

$$\tilde{\underline{\mathbf{a}}}^{(\mathrm{M})}(\omega) = \tilde{\underline{\mathbf{F}}}(\omega)\,\tilde{\underline{\mathbf{d}}}^{(\mathrm{M})}(\omega)\;, \tag{19}$$

where $\tilde{\underline{\mathbf{a}}}^{(\mathrm{M})}(\omega) = \underline{\mathbf{X}}^H(\omega)\,\underline{\mathbf{a}}^{(\mathrm{M})}(\omega)$. Equation (18) together with Eq. (19) allows to decouple Eq. (2) in the transformed domain

$$\tilde{\underline{\mathbf{e}}}^{(\mathrm{M})}(\omega) = \tilde{\underline{\mathbf{F}}}(\omega)\,\tilde{\underline{\mathbf{d}}}^{(\mathrm{M})}(\omega) - \tilde{\underline{\mathbf{R}}}(\omega)\,\tilde{\underline{\mathbf{C}}}(\omega)\,\tilde{\underline{\mathbf{d}}}^{(\mathrm{M})}(\omega)\;, \tag{20}$$

where $\tilde{\underline{\mathbf{e}}}^{(\mathrm{M})}(\omega)$ denotes the error signal for all M components in the transformed domain. Since $\tilde{\underline{\mathbf{R}}}(\omega)$, $\tilde{\underline{\mathbf{C}}}(\omega)$ and $\tilde{\underline{\mathbf{F}}}(\omega)$ are diagonal matrices, the m-th component of the error signal $\tilde{E}_m(\omega)$ in the transformed domain is given by

$$\tilde{E}_m(\omega) = \tilde{F}_m(\omega)\,\tilde{D}_m(\omega) - \tilde{R}_m(\omega)\,\tilde{C}_m(\omega)\,\tilde{D}_m(\omega)\;, \tag{21}$$

where $\tilde{R}_m(\omega)$, $\tilde{C}_m(\omega)$ and $\tilde{F}_m(\omega)$ denote the m-th component of the main diagonal of $\tilde{\underline{\mathbf{R}}}(\omega)$, $\tilde{\underline{\mathbf{C}}}(\omega)$ and $\tilde{\underline{\mathbf{F}}}(\omega)$ respectively. The error $\tilde{E}_m(\omega)$ is only dependent on the m-th component of the respective signals and systems. Thus, Eq. (21) states that the MIMO adaptive inverse filtering problem can be decomposed into M SISO adaptive inverse filtering problems using the GSVD. The computation of the pre-equalization filters can be performed independently for each of the M transformed components. The transformation of the systems and signals is performed by transforming them into the joint eigenspace of $\underline{\mathbf{R}}(\omega)$ and $\underline{\mathbf{F}}(\omega)$ using the GSVD. Therefore this approach will be termed as *eigenspace inverse adaptive filtering*. Please note that the transformation is not dependent on the driving signals. Figure 2 illustrates the eigenspace inverse adaptive filtering approach.

4.2 Adaptation of the Decoupled Pre-Equalization Filter

In the following the normal equation of the multichannel adaptive pre-equalization problem presented in Section 2.3 will be specialized to the decoupled MIMO system. Due to the decoupling, the cost function $\xi(\hat{\mathbf{c}}, k)$ given by Eq. (4) can be minimized independently for each component $m = 1 \ldots M$. The normal equation in the transformed domain is then given as

$$\hat{\tilde{\Phi}}_{dd,m}(k)\;\hat{\tilde{\mathbf{c}}}_m(k) = \hat{\tilde{\Phi}}_{da,m}(k)\;, \tag{22}$$

where $\hat{\tilde{\Phi}}_{dd,m}(n)$ denotes the time-averaged auto-correlation matrix of the m-th component of the transformed filtered loudspeaker driving signal, $\hat{\tilde{\Phi}}_{da,m}(n)$ the corresponding cross-correlation matrix between the filtered loudspeaker driving signal and the desired signal and $\hat{\tilde{\mathbf{c}}}_m(k)$ the filter coefficients. The auto-correlation matrix $\hat{\tilde{\Phi}}_{dd,m}(n)$ has the dimensions $N_c \times N_c$. Due to this reduction in dimensionality, the solution of the M equations given by Eq. (22) is much more efficient than for the adaptation using the original (not transformed) signals. Equation (22) corresponds to the well known single channel normal equation [4]. The cross-channel correlations present in $\hat{\Phi}_{dd}(k)$ have been removed in the transformed domain by the spatial decoupling of the MIMO systems. Thus, the non-uniqueness and ill-conditioning problem discussed in Section 2.4 are highly alleviated. There may still be time-domain correlations present in the filtered input signals which cause problems when solving the normal equation (22). However, there are numerous approaches known in the literature on single-channel adaptive filtering to overcome these problems [4].

5. APPLICATION TO ACTIVE LISTENING ROOM COMPENSATION

Sound reproduction aims at recreating an (virtual) acoustic scene at a remote place or at a later time. When realized properly a perfect auditory illusion of the original scene is created. However, the perfect acoustic illusion has not been realized by the currently available

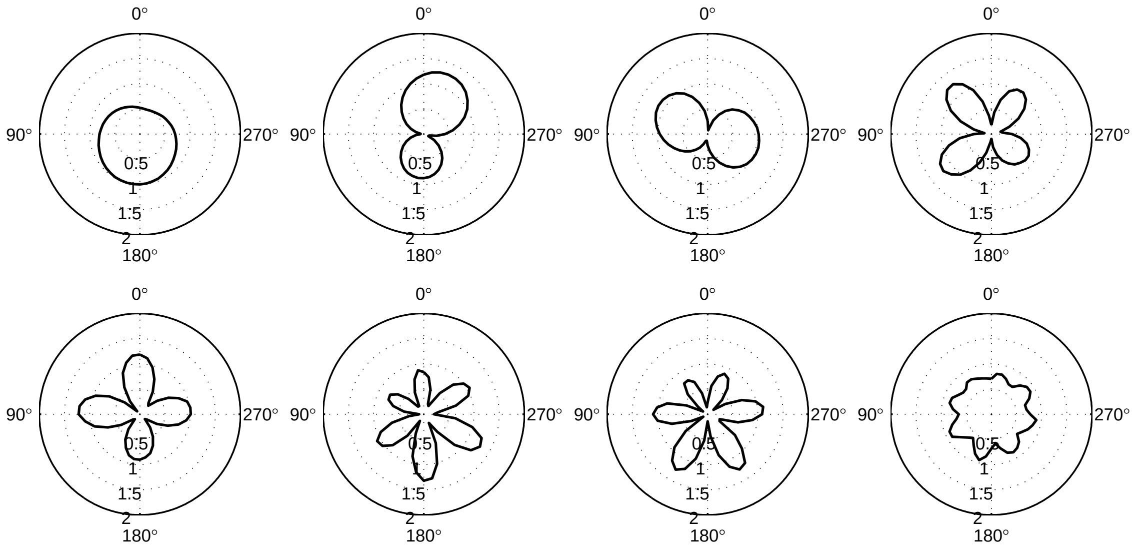

Figure 3: Absolute value of the first eight right singular vectors ($f = 80$ Hz) of a circular WFS system sorted by descending singular values.

reproduction systems. One source of impairments is the acoustics of the room were the reproduction system is placed (listening room). Most reproduction systems assume an anechoic listening room, an assumption which is typically not met.

Spatial sound reproduction systems with a large number of loudspeakers are increasingly being used. These advanced reproduction systems, like WFS, provide a reasonable amount of control over the reproduced wave field. This control can be used to perform active compensation of the listening room acoustics by pre-equalization of the loudspeaker driving signals. Hence, in the context of multichannel reproduction systems listening room compensation is subject to the same fundamental problems as discussed in Section 2.4. The concept of eigenspace adaptive filtering provides a solution to most of these problems. In general, the computation of the GSVD will be too complex to benefit from the complexity reduction given by this decomposition of the MIMO adaptation problem. However, presuming an efficient transformation of $\underline{\mathbf{R}}(\omega)$ and $\underline{\mathbf{F}}(\omega)$ with equivalent properties as the GSVD based transformation of the systems and signals may result in a highly reduced complexity.

Recently active listening room compensation using wave-domain adaptive filtering (WDAF) has been proposed [1]. Here a transformation based on circular harmonics has been proposed for the decoupling of the MIMO adaptation problem. The results presented in this paper provide a theoretical background to WDAF as will be shown in the following. For this purpose the right singular vectors of a particular measured room transfer matrix are computed. The considered WFS-based reproduction system consists of a circular loudspeaker array with diameter $D_{LS} = 3$ m with 48 equidistantly positioned loudspeakers. The loudspeaker array is placed in the center of the listening room with the size 5.9 m $\times$ 5.8 m $\times$ 3.1 m at a height of 1.80 m. The room has a reverberation time of $T_{60} \approx 400$ ms. A circular microphone array with 48 microphone positions and a diameter of $D_{Mic} = 1.50$ m is placed concentric inside the loudspeaker array. Figure 3 shows the absolute value of the right singular values for this particular scenario. The presented results resemble strong similarities with the basis functions of circular harmonics. The basis functions of the circular harmonics are given by the free-field solutions of the two-dimensional wave equation in polar coordinates [6]. Hence, a transformation based on the circular harmonics provides an optimal decomposition of the free-field transfer matrix $\underline{\mathbf{F}}(\omega)$ only. However, for rectangular listening rooms with not too much reverberation they will also provide a reasonable basis for the representation of $\underline{\mathbf{R}}(\omega)$ as illustrated by Fig. 3. This has also been proven by simulations of various other rooms and loudspeaker setups [6].

The combination of prior physical knowledge and eigenspace adaptive filtering thus yields an efficient practical solution.

6. CONCLUSIONS

A novel framework for efficient pre-equalization of massive MIMO systems has been presented. It is based on a decomposition of the MIMO adaptation problem into a series of single channel adaptation problems by decomposing the MIMO system into the joint eigenspace of the desired system response $\underline{\mathbf{F}}(\omega)$ and the system response $\underline{\mathbf{R}}(\omega)$. The presented concept of eigenspace adaptive filtering provides the framework for wave-domain adaptive filtering. It was further shown that the investigation of the singular vectors for a particular problem may lead to efficient algorithms for active listening room compensation. The same procedure can also be applied to other massive multichannel adaptation problems.

REFERENCES

[1] S. Spors, H. Buchner, and R. Rabenstein, “Adaptive listening room compensation for spatial audio systems,” in *European Signal Processing Conference (EUSIPCO)*, 2004.

[2] J. Garas, *Adaptive 3D Sound Systems*, Kluwer Academic Publishers, 2000.

[3] A.V. Oppenheim and R.W. Schafer, *Discrete-time signal processing*, Prentice-Hall, 1999.

[4] S. Haykin, *Adaptive Filter Theory*, Prentice-Hall, 2002.

[5] M. Miyoshi and Y. Kaneda, “Inverse filtering of room acoustics,” *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 36, no. 2, pp. 145–152, February 1988.

[6] S. Spors, *Active Listening Room Compensation for Spatial Sound Reproduction Systems*, Ph.D. thesis, University of Erlangen-Nuremberg, 2006.

[7] J. Benesty, D.R. Morgan, and M.M. Sondhi, “A better understanding and an improved solution to the specific problems of stereophonic acoustic echo cancellation,” *IEEE Transactions on Speech and Audio Processing*, vol. 6, no. 2, pp. 156–165, March 1998.

[8] C.H. Hansen and S.D. Snyder, *Active Control of Noise and Vibration*, E&FN Spon, 1997.

[9] G.H. Golub and C.F. Van Loan, *Matrix Computations*, The Johns Hopkins University Press, 1989.