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ROBUST GROUP DELAY EQUALIZATION OF DISCRETE-TIME FILTERS USING NEURAL NETWORKS

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ABSTRACT

A novel methodology for first allocating the poles and zeros of a group delay equalizer is introduced. In this paper, feed-forward neural networks are used, instead of empirical formulae. The results obtained with the networks can be applied as the initial solution of an optimization procedure that searches for the optimum group delay equalizer.

Different inputs are considered a priori for the neural networks, but after pre-processing the acquired data, some of them are discarded. By evaluating cross-correlation between the inputs and the outputs, the simplified networks are designed through an optimization procedure with mean square error back-propagation, by batch method. Armijo search method is applied in this procedure for improving convergence rate.

Simulation results proving the efficiency of the proposed method are presented. Quasi-equiripple group delay responses are obtained with the neural network initial estimate, avoiding local minima and improving the convergence rate of the optimization step of the equalizer designs.

1. INTRODUCTION

Group delay (or phase) equalization is a viable alternative, implemented with second-order allpass filters [1], to compensate for non-linear phase response of discrete-time IIR filters. The overall filter, comprised of the cascade connection of an IIR filter and an equalizer filter, has an approximately linear phase response that may satisfy frequency response requirements in many applications. Hence, phase-equalized IIR filters may find an edge in terms of computational complexity over competing linear phase FIR counterparts.

Unfortunately, the large body of knowledge and techniques developed for the design of analog equalizers cannot be directly applied to discrete-time filters, through *s*-to-*z* mapping, because of the warping effect introduced in the phase frequency response of the original equalized analog filters [2]. In view of these difficulties, various techniques have been derived in the z-domain to achieve the optimum group delay response, including genetic algorithms [3], adaptive filters [4], quasi-allpass filters [5] and allpass-based equalizers [1], [6]-[10].

Since group delay equalization involves several optimization parameters (2 parameters for each second-order allpass filter), and the convergence surface of the cost function is highly irregular, a good initial estimate is needed to avoid local minima. However, the initial choice of the parameters of the optimization procedures proposed so far in the literature does not appropriately exploit the knowledge about the distortions in the group delay to be equalized [6], [9], and in some instances, produces a number of second-order equalizing sections larger than what would be necessary [2].

In [11], an initial estimate was advanced to improve the robustness and convergence rate of the optimization algorithm. It employed a graphic, empirical procedure for the allocation of the poles radii of the equalizer.

In this paper a novel methodology is proposed, by designing neural networks that provide at their outputs the phases and radii of the equalizer pole-zero pairs, thereby avoiding the empirical formulas of [11]. Feed-forward neural networks are applied in this task. Several possible influent inputs are selected *a priori*, and after subjective and objective steps, some of them are discarded, in order to obtain simplified, but effective, networks.

In Section II, the neural networks are investigated from the basic structure to the training steps, presenting and describing the inputs applied to them, and the disposal of the non-influent inputs. Simulation results proving the effectiveness of the proposed methodology are presented in Section III. The advances achieved in this paper are summarized in Section IV.

2. NEURAL NETWORKS FOR INITIAL SOLUTION

The effectiveness of neural networks in avoiding the use of the empirical expressions developed in [11], for the initial allocation of the equalizer pole-zero pairs, is the main issue addressed in this section. While this paper considers the use of second- up to tenth-order equalizers, the technique may be readily extended to higher orders.

Feed-forward networks with back-propagation training employing the batch method [12] are considered. The network inputs and outputs are denoted as N_{in} and N_{out}, respectively. Neurons with linear and nonlinear (hyperbolic tangent) activation functions are applied. More specifically, $N_{L1}-1$ nonlinear neurons and $N_{out}+1$ linear neurons, (1 in the hidden layer and N_{out} in the output layer) are used.

Equiripple equalized group delay responses of IIR filters are used for training the networks. The outputs of the designed neural networks are the parameters of the poles of the desired equalizer, i.e., phases and radii. For the inputs, a search among several possible inputs in order to identify the most influent data was carried out. From 31 pre-selected inputs, after some subjective analyses, the 15 inputs listed in Table 1 were selected, with which cross-correlations with the desired outputs were performed. From initial simulations it was decided to apply one neural network for each equalizer order and different output. For example, for a fourth-order equalizer, two different networks with two outputs each, one for the two radii and the other one for the two phases, corresponding to the two pole-zero pairs, were designed. Therefore, up to ten different networks were designed for the desired equalizers. This is the reason why the equalizer order is not used as an input. All the inputs are investigated with respect to their correlation with the outputs, as described next.

TABLE 1: INPUTS SELECTED *A PRIORI*.

# Input	Description
e-1.	Phases of the 4 outermost pairs of poles of the IIR filter.
e-2.	
e-3.	
e-4.	
e-5.	Radii of the 4 outermost pairs of poles of the IIR filter.
e-6.	
e-7.	
e-8.	
e-9.	Difference in samples between the maximum value and the group delay response at the θ_I frequencies.
e-10.	
e-11.	
e-12.	
e-13.	
e-14.	Area below the IIR group delay.
e-15.	Area above the IIR group delay.

2.1 Description of Selected Inputs.

The inputs *e-1* up to *e-8* refer to the four outermost pairs of poles of the original, to-be-equalized IIR filter, as illustrated in Fig.1 for an eleventh-order elliptic filter having 5 complex-conjugated pairs of poles and one real pole. In this example, the four pairs of poles closest located with respect to the cutoff frequency were selected, whereas the pair closest to the real axis and the real pole were discarded. The restriction imposed to the four outermost poles to be selected as inputs of the networks stems from the fact that eighth-order elliptic filters are able to satisfy stringent frequency response specifications. Higher orders elliptic filters are not usually applied in practice, and hence would produce zero-valued inputs, in case more pairs of poles of the original IIR filter were considered. As a result, the 8 inputs selected for the networks were the 4 phases and 4 radii of the outermost pair of poles.

For performance improvement, the analytic expression introduced in [11], for the allocation of the phases of the equalizer poles, was adopted. Accordingly, for an N^{th}-order equalizer, *N/2* analytic frequencies θ_i inside the pass-band of the original IIR filter were defined. The inputs *e-9* up to *e-13* were the differences between the maximum group delay and the group delay at the frequencies θ_i, as illustrated in Fig.2 for a fourth-order equalizer, for example.

The input *e-14* is the area below the original group delay response, in the frequency range from zero up to the cutoff frequency, and is labeled as $area_1$ in Fig.3. Finally, labeled as $area_2$ in Fig. 3, the input *e-15* is the area between the original IIR group delay response and the constant line crossing the maximum original group delay value.

To verify the influence of the selected inputs to the network outputs, the acquired data was pre-processed. All the inputs and outputs were normalized with respect to their respective mean and standard deviation values, for the benefit of improved convergence. A selection of N_T = 766 input-output pairs of equiripple-equalized group delays were separated for a variety of equalizer orders, as in Table 2. For each equalizer order, 90% of the input-output equalized group delays are used for training the networks and the remaining 10% are used for checking the convergence of the network designs.

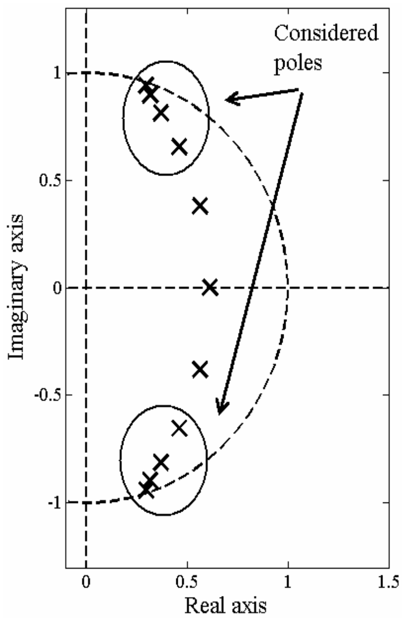

Figure 1: Four outermost pairs of poles considered as inputs.

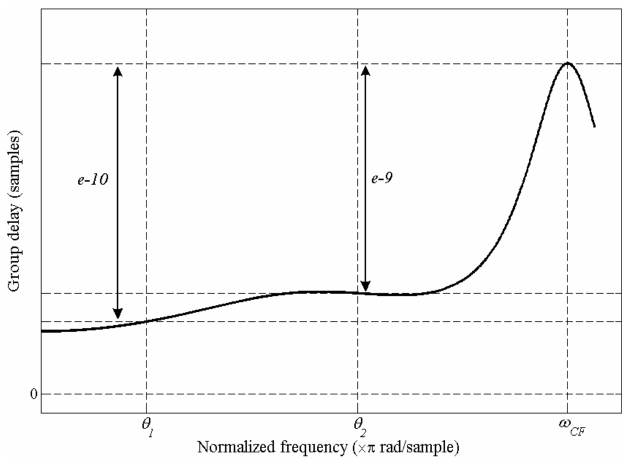

Figure 2: Visualization of inputs *e-9* to *e-13*. In this 4^{th}-order example, there are only *e-9* and *e-10* are selected.

TABLE 2: NUMBER OF INPUT-OUTPUT PAIRS ACQUIRED FOR TRAINING AND TESTING NEURAL NETWORKS.

Order (N)	2nd	4th	6th	8th	10th
# pairs	186	177	159	137	107
# training	167	159	143	123	96
# test	19	18	16	14	11

2.2 Cross-Correlation Among Inputs

An important step in neural network simplification is checking the correlation among different inputs, in order to avoid redundant information. The calculation of the cross-correlation is evaluated by

$$R_{ij}\left(x_i, x_j\right) = \frac{\sum_{k=1}^{N_T} x_i(k) \cdot x_j(k)}{\sqrt{\left(\sum_{k=1}^{N_T}\left(x_i(k)\right)^2\right)\cdot\left(\sum_{k=1}^{N_T}\left(x_j(k)\right)^2\right)}} \quad (1)$$

for $i = 1, ..., 15$, $j = 1, ..., 15$, $j \neq I$, where x_i and x_j are the inputs.

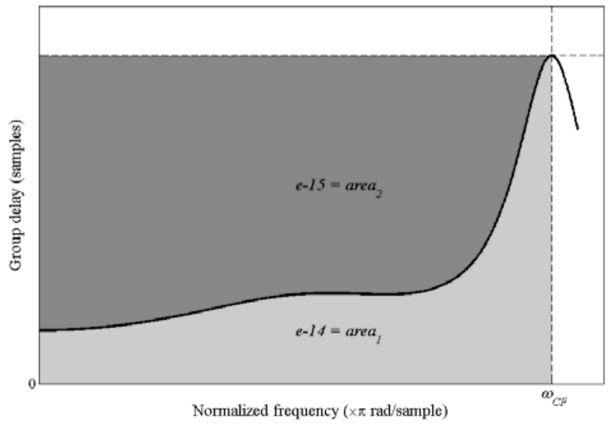

Figure 3: Parameters $area_1$ and $area_2$ as inputs *e-14* and *e-15, respectively*.

2.3 Input-Output Cross-Correlation

Even if there was no redundancy among the inputs, some of them might carry no significant information regarding the allocation of the pole parameters. Therefore, to discard unnecessary inputs, cross-correlation was evaluated for each network between the inputs and the outputs, by replacing x_j with the outputs y_j, for $j = 1,\ldots, N_{out}$. For the sake of design simplicity, all the networks assigned to the pole radii had the same set of inputs, regardless of the equalizer order. Similar procedure was adopted with the networks assigned to the phases.

Those inputs presenting correlation larger than $3/\sqrt{N_T}$, with at least one of the outputs were considered useful to the design of the networks related to the respective outputs. Hence, the inputs *e-3*, *e-4* and *e-14* were applied to the networks that determined the phases of the optimum equalizer poles, and the inputs *e-7*, *e-8*, *e-9* through *e-13* and *e-14* were applied to the networks that determined the radii of the optimum equalizer poles. It should be emphasized that inputs *e-10* to *e-13* were only considered for equalizers with orders higher than $N = 2$. I.e., for 2nd-order equalizer only inputs {*e-7, e-8, e-9, e-14*} are considered, while for 4th-order inputs {*e-7*, *e-8*, *e-9*, *e-10*, *e-14*} are considered, and so on, up to 10th-order equalizer that requires all the inputs from *e-7* to *e-14*.

Another important procedure adopted to access the influence of the inputs was their relevance. The networks were obtained through optimization, and then each of the inputs was replaced with its respective mean value. If the performance was not altered the corresponding input was discarded.

2.4 Neural Network Training

As aforementioned, the neural networks have two layers, designed with error back-propagation training with the batch method. The error function to be minimized during the network optimization was the mean square error of the difference between the desired and the obtained outputs. The update step was variable, according to the Armijo search method [13], [14], which provided fast convergence, and a low number of iterations to achieve the best solutions. These were obtained with neural networks having 4 *tanh* neurons and 1 linear neuron in the hidden layer. The output layer presents as many linear neurons as there are outputs. The design of the networks was shown very efficient, achieving errors in the order of 10% of the outputs variance, which means 90% of precision. Once the neural networks were obtained they could be applied to any equalizer design, without any need for retraining. Some simulation results are presented in the following section.

3. SIMULATION RESULTS

The above technique was applied to the group delay equalizer design for two IIR lowpass filters.

As a first design example, a fifth-order elliptic filter was considered, with a normalized cutoff frequency of 0.12, a stopband normalized frequency of 0.17, a passband ripple of 0.5 dB, and a stopband attenuation of 40 dB. Figure 4 displays, in dashed line, the group delay response of the original filter. In solid lines are the equalized responses obtained with the second- up to tenth-order equalizers produced by the neural networks. Observe that the equalized group delay responses are very close to the optimum equiripple group delays, and hence improving the profits of the optimization procedure in the search for the optimum equalizer.

The second example is a seventh-order chebyshev type-I filter, with a normalized cutoff frequency of 0.3, a stopband normalized frequency of 0.4, a passband ripple of 0.1 dB, and a stopband attenuation of 30 dB. As in the previous example, the initial responses (solid lines) obtained with the neural networks are visually close to equiripple responses, as seen in Fig.5, along with the original group delay response (dashed line).

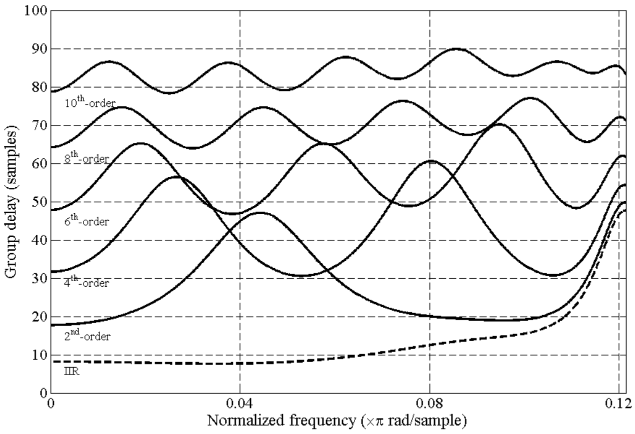

Figure 4: Original group delay (dashed line) and initially equalized group delays (solid lines) of 2nd- up to 10th-order, for a 5th-order elliptic filter.

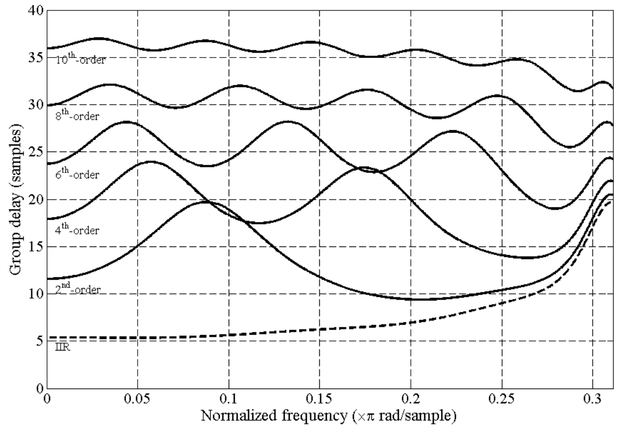

Figure 5: Original group delay (dashed line) and initially equalized group delays (solid lines) of 2nd- up to 10th-order, for 7th-order chebyshev type-I filter.

If the obtained results are applied as the initial estimates of optimization procedures, optimum equiripple group delay responses are obtained in few iterations. Therefore, the proposed design is regarded as a sufficiently good initial solution for the equalization of group delay responses of IIR filters, in general. The method described so far presents similar results to the ones presented in [11], however, there is no use of empirical formula.

Similarly good results can be obtained for higher order equalizers, but the number of training input-output pairs is reduced, as indicated in Table 2, because of the difficulty in producing relevant equalization examples in such cases.

4. CONCLUDING REMARKS

This paper proposed a new methodology for initially estimating the best allocation of poles and zeros of a group delay equalizer. The main achievement of this work was to provide a solution that avoids the drawbacks of the optimization procedures usually applied to the design of these equalizers, such as convergence to local minima, and consequently improves the design robustness and convergence rate towards the optimum solution.

The method considered feed-forward neural networks for second- up to tenth-order equalizer designs. The error back propagation of the mean square error was applied by the batch method. While higher order neural networks can be designed, it should be noted that the optimization procedure for the network training could be less accurate, because of the difficulty in producing input-output pairs of equiripple equalized group delays in such cases. Simulation results were presented, showing the efficiency of the method in providing quasi-equiripple equalization as initial solutions for the group delay equalization problem.

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