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WIENER SOLUTION FOR OFDM PRE AND POST-FFT BEAMFORMING

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Orthogonal Frequency Division Multiplexing (OFDM) is one of the most promising techniques for high-speed transmission over severe multipath fading channel. However, once delays of secondary multipath rays are greater than the guard interval duration, intersymbol interference causes a severe degradation in the transmission performance. To solve this problem, a multiple antenna array can be used at the receiver, not only for spectral efficiency or gain enhancement, but also for interference suppression. In this paper we analyze the asymptotical behavior of two beamforming algorithms, a low complexity pre-FFT method and a more efficient post-FFT system. The optimum weight set for beamformers is derived on the basis of the minimum mean square error (MMSE) criterion and the Wiener solution is studied under different working conditions.

1. INTRODUCTION

Multipath fading is due to the presence of many reflected signals, which arrive at the receiver at different times. These echoes cause ISI and combined can produce fading. This effect is more and more severe as the distance range or the data rate of the system increase. Orthogonal Frequency Division Multiplexing (OFDM)[1] is a special form of MultiCarrier Modulations that allows reliable transmission over a channel with a relatively large maximum delay. However when the delay of the arriving signals is longer than the guard interval, ISI causes severe degradations in the system performance. To solve this problem, a multiple antenna array can be used at the receiver, not only for spectral efficiency or gain enhancement, but also for interference suppression.

In an OFDM system, the beamforming algorithm can be applied in either time domain or frequency domain. Time domain array processing has lower complexity, because only one FFT is required. In frequency domain a processing of the individual subcarriers is provided, generally with better results, but always with higher complexity. Time-domain beamforming methods are normally called pre-FFT [2][3] whereas frequency-domain algorithms are called post-FFT [4][5].

In this paper we analyze two beamforming algorithms, a low complexity pre-FFT and a more efficient post-FFT, by determining the optimum weights that satisfy the Minimum Mean Square Error (MMSE) criterion. This set is usually named Wiener solution and represents the weights to which different classes of adaptive algorithms asymptotically converge. The detailed comparison of the two methods, provided in this paper, can represent a key element in the design phase of an OFDM receiver equipped with a smart antenna, especially in the cases when it is a crucial problem to assess the best trade-off between complexity and performance. In literature only some partial results in terms of algorithm comparison are available. For example in [6] performance and computational complexity are studied, but only for the case of multipath delay within the guard interval; in [7] the analysis has been performed in different work conditions, in terms of channel model as well as applied algorithms.

An important result of this study is the proof that the two methods are equivalent when the multipath delay is greater than the cyclic prefix of the OFDM frame. For multipath delay lower than the cyclic prefix the pre-FFT tends to eliminate the multipath, while the post-FFT tends to combine the line-of-sight with its multipath rays. Therefore both methods are able to cope with the multipath problem, however the post-FFT exhibits better performance because the transmitted information is retrieved from each received ray. However we can conclude that the pre-FFT should represent the best solution in most cases of interest. In fact the cost of a smart antenna is worthwhile only in some critical applications, when multipath with long delay is expected to degrade the OFDM receiver performance, while the OFDM itself is able to directly manage shorter delays without the need of a smart antenna. On the other hand, for a very robust receiver, with a smart antenna able to perform the best operation (multipath cancellation and/or equalization) according to the delay amount, post-FFT should be preferred, even if much more computationally complex.

This paper is organized as follows: Section 2 recalls the principles of OFDM and describes the adopted channel model, introducing the basic notation used in the article; Section 3 introduces pre and post-FFT systems and Section 4 analyzes the Wiener filter solution. In Section 5 performance in term of Bit Error Rate (BER) is analyzed and finally, Section 6 concludes this paper.

2. SIGNAL AND CHANNEL MODEL

In this Section we briefly introduce the expressions of the signals we are dealing with in the considered system. The base-band channel model is also described. Let us consider a general OFDM system. The modulation parameters are:

- N, the number of data samples in the OFDM frame, with-

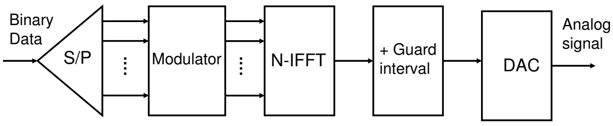

Figure 1: OFDM transmitter functional scheme

out the cyclic prefix;

- N_g, the number of samples of the cyclic prefix;
- $N_T = N + N_g$;
- N_{QAM}, the number of bits for each symbol in the QAM/PSK constellation used to modulate the OFDM frame.

At the transmitter side the input binary data are serial-to-parallel converted and mapped into the considered constellation. The obtained N samples are IFFT transformed and a block of N samples, called OFDM symbol, is generated (Figure 1). The generic kth OFDM symbol can be written as:

$$s_k[n] = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} S_{k,i} \exp\{j2\pi in/N\} \qquad 0 \le n \le N-1$$

where n denotes the sample within the symbol k. Then the cyclic prefix is attached and the symbol is transmitted. The channel is considered affected by Additive White Gaussian Noise (AWGN) and multipath. The effect of multipath can be modelled by means of a linear system, whose impulse response can be written as

$$h(t) = \delta(t) + \sum_{i=1}^{M} \alpha_i \delta(t - \tau_i)$$

where M is the number of multipaths, τ_i is the delay of arrival of the i-th path and α_i its complex attenuation with respect to the Line Of Sight (LOS).

In this work a receiver equipped with a Uniform Linear Array (ULA) is considered; this kind of multiple antenna consists in an array of L omnidirectional sensors located along a straight line and $\lambda/2$ spaced, with λ the wavelength of the signal carrier. The effect of a ULA can be modeled [8] multiplying each received ray by its correspondent steering vector that takes into account the DoA (Direction of Arrival) of each multipath. For example, if a ray arrives with an angle θ, the signal carried by this ray will be multiplied by $\exp\{\pi jh\sin\theta\}$, with $h = 0, \ldots, L-1$. The signal received by the ULA can be represented by means of a $N \times L$ matrix, the cyclic prefix being discarded by the first stages of the receiver:

$$\mathbf{X} = \mathbf{a}(\theta_d)\mathbf{s}^T + \sum_{i=1}^{M} \mathbf{a}(\theta_i)\mathbf{r}_i^T + \mathbf{M}$$

where $\mathbf{s}$ and $\mathbf{r}_i$ are the column vectors containing samples carried by the direct and the reflected rays, θ_d and θ_i are their respective angles of arrival and $\mathbf{a}(\cdot)$ is the column steering vector:

$$\mathbf{a}(\theta) = [1 \quad \exp\{j\pi\sin(\theta)\} \quad \ldots \quad \exp\{j\pi(L-1)\sin(\theta)\}]^T$$

The additive noise (AWGN) is represented by $\mathbf{M}$. Each elementary sensor receives independent noise samples and all

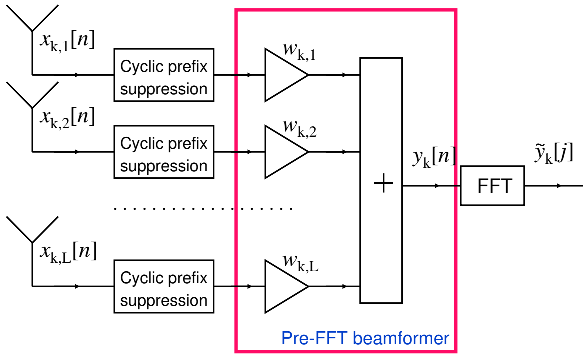

Figure 2: Pre-FFT beamforming scheme

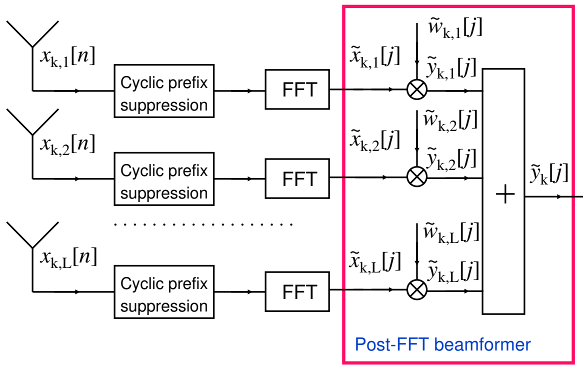

Figure 3: Post-FFT beamforming scheme

elements of $\mathbf{M}$ are independent gaussian random variables. The hth row of the matrix $\mathbf{X}$ contains the N samples received by the hth sensor, indicated as $\mathbf{x}_{k,h}$, where k is the OFDM frame.

After the down-conversion and the analog to digital stage, which is not considered in this paper, the received signals are translated into the frequency domain using at least one FFT - it depends on the smart antenna architecture - and then processed before decision. Frequency-domain signals will be indicated hereafter by a $\tilde{\cdot}$ over the correspondent time-symbol. For instance the matrix $\tilde{\mathbf{X}}$ refers to the frequency-domain translation of the matrix $\mathbf{X}$ and has as hth row $\tilde{\mathbf{x}}_{k,h} = \mathrm{FFT}\{\mathbf{x}_{k,h}\}$.

3. PRE AND POST-FFT BEAMFORMING

In Figure 2 a general pre-FFT scheme is presented: each replica of the received signal $x_{k,h}[n]$ is multiplied by the conjugate of a complex weight $w_{k,h}$ and then summed up to form the spatially filtered signal $y_k[n]$. The k index indicates the OFDM frame number, while n is the time index. It is important to notice that weights $w_{k,h}$ do not depend on n but only on k: this means that samples belonging to the OFDM frame are multiplied by the same weight $w_{k,h}$; $y_k[n]$ is a pondered mean of the different replicas of $x_{k,h}[n]$. This kind of beamforming has L freedom degrees and it does not exploit the possibility of a time filtering. In Figure 3 a general post-FFT scheme is presented: this method requires L FFT-blocks, one for each antenna, and only after the transposition of the signals in the frequency domain the beamforming is applied. In

this case each frequency sample is multiplied by a different weight $\tilde{w}_{k,h}[j]$, where j is the frequency index.
In both schemes the complex weights $\mathbf{w}$ are determined by minimizing the Mean Square Error (MSE) that has different expressions for pre and post-fft beamforming. In order to evaluate the MSE a reference signal is needed: it can be provided by a periodically transmitted preamble [5], or by pilot tones embedded into the OFDM frame. In this paper we consider the case where J pilot tones are embedded into the OFDM frame and used to form the error signal.

3.1 Pre-FFT beamforming

Let be:

- $\bar{\mathbf{s}}$, the vector of dimensions $J\times 1$, with J the number of pilot tones, containing the reference signal;
- $\mathbf{w_k}$, the $L\times 1$ vector containing the complex gains;
- $\tilde{\mathbf{X}}_k$, the $L\times J$ matrix containing in the hth row the values, received by the hth antenna, corresponding to pilot tones.

Using these notations, the MSE can be written as:

$$\mathrm{MSE} = E\left\{\left\|\bar{\mathbf{s}}^T - \mathbf{w}_k^H\tilde{\mathbf{X}}_k\right\|^2\right\} \tag{1}$$

The Wiener solution is the weight set $\mathbf{w}_k$ that minimizes Equation (1) and that nullifies the MSE gradient that is given by

$$\nabla_{\mathbf{w}_k}\mathrm{MSE} = 2E\left\{\tilde{\mathbf{X}}_k\left(\tilde{\mathbf{X}}_k^H\mathbf{w}_k - \bar{\mathbf{s}}^*\right)\right\} \tag{2}$$

Usually $\tilde{\mathbf{X}}_k$ is not available, since only one FFT is performed, and a different strategy has to be adopted. In particular it is possible to apply the Parseval equality, which, transposing the error signal $\tilde{\mathbf{X}}_k^H\mathbf{w}_k - \bar{\mathbf{s}}^*$ in the time domain, allows the utilization of $\mathbf{X}_k$ instead of $\tilde{\mathbf{X}}_k$ [9].

3.2 Post FFT beamforming

In a post-FFT scheme, the OFDM replicas are recovered by the different antennas and transposed in the frequency domain by an FFT at each sensor. Then each sample is multiplied by a complex weight $\tilde{w}_{k,h}[j]$ and combined to form $\tilde{y}_k[j]$. In the following the indexes k will be omitted to allow a more legible reading.
Let be:

- $\bar{\mathbf{s}}$: the reference signal of dimensions $J\times 1$;
- $\mathbf{b_j} = [0,0,\ldots,1,\ldots,0]^T$: a base vector with only a not-null element in position j;
- $\tilde{\mathbf{x}}_j$: the different replicas of the jth pilot tone collected by the different sensors and belonging to the kth OFDM frame;
- $\widetilde{\mathbf{w}}_j$: the complex vector containing weights corresponding to the $\tilde{\mathbf{x}}_j$ received pilot vector.

In this case the MSE is given by:

$$\mathrm{MSE} = E\left\{\left\|\bar{\mathbf{s}} - \sum_{j=1}^{J}\mathbf{b_j}\left(\widetilde{\mathbf{w}}_j^H\cdot\tilde{\mathbf{x}}_j\right)\right\|^2\right\} \tag{3}$$

Developing (3) we obtain:

$$\mathrm{MSE} = \|\bar{\mathbf{s}}\|^2 - \sum_{j=1}^{J}E\left\{\left[\bar{s}_j^*\left(\widetilde{\mathbf{w}}_j^H\cdot\tilde{\mathbf{x}}_j\right)\right]\right\} - $$
$$- \sum_{j=1}^{J}E\left\{\left[\left(\tilde{\mathbf{x}}_j^H\cdot\widetilde{\mathbf{w}}_j\right)\bar{s}_j\right]\right\} + \sum_{j=1}^{J}E\left\{|\widetilde{\mathbf{w}}_j^H\cdot\tilde{\mathbf{x}}_j|^2\right\}$$

The $\bar{s}_j$ term is the j-th element of the reference vector $\bar{\mathbf{s}}$, obtained by the scalar product $\mathbf{b_j}^H\cdot\bar{\mathbf{s}}$.
Using the generalized derivative rules we find the expression of the MSE gradient:

$$\nabla_{\widetilde{\mathbf{w}}}\mathrm{MSE} = \left[\nabla_{\widetilde{\mathbf{w}}_1}\mathrm{MSE},\ldots,\nabla_{\widetilde{\mathbf{w}}_J}\mathrm{MSE}\right]$$

with

$$\nabla_{\widetilde{\mathbf{w}}_j}\mathrm{MSE} = 2E\left\{\tilde{\mathbf{x}}_j\left(\tilde{\mathbf{x}}_j^H\widetilde{\mathbf{w}}_j - \bar{s}_j^*\right)\right\} \tag{4}$$

Observing Equation (4) it is easy to notice that:

- the expression of $\nabla_{\widetilde{\mathbf{w}}_j}\mathrm{MSE}$ involves only the replicas of the j-th pilot: the information carried by the other pilot tones is not used;
- Equation (4) is very close to the expression of the gradient in the pre-FFT case (see eq.(2)): the beamformer minimizes the MSE minimizing separately its J components. This kind of beamforming can be considered as a bank of parallel pre-FFT beamformers on J different sub-bands.

4. THE WIENER SOLUTION

In this Section we analyze the asymptotical behavior of both beamforming schemes, determining the Wiener's filter weights that are solution of

- $$E\left[2\tilde{\mathbf{X}}_k\left(\tilde{\mathbf{X}}_k^H\mathbf{w}_k - \bar{\mathbf{s}}^*\right)\right] = 0 \tag{5}$$
  in the pre-FFT case;
- $$E\left[2\tilde{\mathbf{x}}_j\left(\tilde{\mathbf{x}}_j^H\widetilde{\mathbf{w}}_j - \bar{s}_j^*\right)\right] = 0 \qquad \text{for } j = 1,\ldots,J \tag{6}$$
  in the post-FFT case.

Since Equations (5) and (6) are very similar, the two beamforming schemes have common properties: in particular it is possible to show that both $\mathbf{w}_k$ and $\widetilde{\mathbf{w}}_j$ are a linear combination of the steering vectors relative to the direct ray and to the reflected ones, such that all the noise components orthogonal to the steering vectors are eliminated. Such a condition can be expressed as

$$\mathbf{w} = \omega_d\mathbf{a}(\theta_d) + \sum_{i=1}^{M}\omega_i\mathbf{a}(\theta_i) \tag{7}$$

In the rest of the paper, the absence of index and of $\tilde{\cdot}$ indicates that the considered Equation refers to both pre and post-FFT cases. Furthermore coefficients ω_d and $\{\omega_i\}_{i=1}^M$ depend only on the correlation matrix of direct and reflected rays, on the correlation matrix of the steering vectors and on the noise variance, and in particular they are solution of the system:

$$\left[\mathbf{\Gamma S} + \sigma^2\mathbf{I}\right]\mathbf{\Omega} = \mathbf{\Gamma_1} \tag{8}$$

where $\mathbf{\Gamma}$ is the direct-reflected rays correlation matrix, $\mathbf{S}$ is the steering vectors correlation matrix or spatial signature, σ^2 the noise variance, $\mathbf{I}$ the identity matrix of size $M+1$, $\mathbf{\Omega}$ the column vector containing ω_d and $\{\omega_i\}_{i=1}^M$ and $\mathbf{\Gamma}_1$ the first column of $\mathbf{\Gamma}$. More in detail $\mathbf{\Gamma}$ is given by:

- $$\mathbf{\Gamma} = \frac{1}{J}\begin{bmatrix} E[\bar{\mathbf{s}}^T\bar{\mathbf{s}}^*] & E[\bar{\mathbf{s}}^T\bar{\mathbf{r}}_1^*] & \cdots & E[\bar{\mathbf{s}}^T\bar{\mathbf{r}}_M^*] \\ E[\bar{\mathbf{r}}_1^T\bar{\mathbf{s}}^*] & E[\bar{\mathbf{r}}_1^T\bar{\mathbf{r}}_1^*] & \cdots & E[\bar{\mathbf{r}}_1^T\bar{\mathbf{r}}_M^*] \\ \cdots & \cdots & \cdots & \cdots \\ E[\bar{\mathbf{r}}_M^T\bar{\mathbf{s}}^*] & E[\bar{\mathbf{r}}_M^T\bar{\mathbf{r}}_1^*] & \cdots & E[\bar{\mathbf{r}}_M^T\bar{\mathbf{r}}_M^*] \end{bmatrix}$$

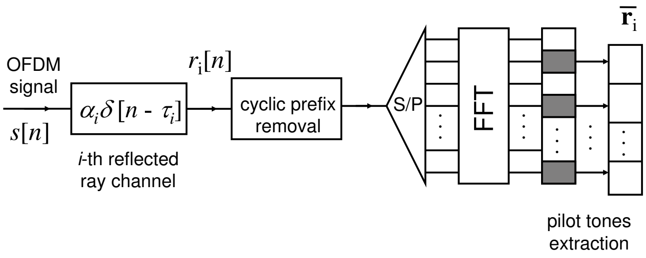

Figure 4: each reflected ray contains a distorted version of the pilot tones that are collected in the vectors $\overline{\mathbf{r}}_i$.

in the pre-FFT scheme; the $\overline{\mathbf{r}}_i$ are column vectors containing the J pilot tones extracted from the ith reflected ray and obtained as explained in Figure 4.

•

$$\mathbf{\Gamma}_j = \begin{bmatrix} E[|\bar{s}_j|^2] & E[\bar{s}_j^T\bar{r}_{1,j}^*] & \cdots & E[\bar{s}_j^T\bar{r}_{M,j}^*] \\ E[\bar{r}_{1,j}^T\bar{s}_j^*] & E[|\bar{r}_{1,j}|^2] & \cdots & E[\bar{r}_{1,j}^T\bar{r}_{M,j}^*] \\ \cdots & \cdots & \cdots & \cdots \\ E[\bar{r}_{M,j}^T\bar{s}_j^*] & E[\bar{r}_{M,j}^T\bar{r}_{1,j}^*] & \cdots & E[|\bar{r}_{M,j}|^2] \end{bmatrix}$$

in the post-FFT scheme. For a post-FFT beamforming a system like the one described in (8) must be applied, independently, for each pilot tone. For this reason a different $\mathbf{\Gamma}_j$ must be determined for each pilot tone. The variable $\bar{r}_{i,j}$ is the jth pilot tone recovered by the ith reflected ray.

The spatial signature matrix $\mathbf{S}$ is a definite positive hermitian matrix, given by

$$\mathbf{S} = \begin{bmatrix} \mathbf{a}^H(\theta_d) \\ \mathbf{a}^H(\theta_1) \\ \vdots \\ \mathbf{a}^H(\theta_M) \end{bmatrix} [\mathbf{a}(\theta_d)\mathbf{a}(\theta_1)\cdots\mathbf{a}(\theta_M)]$$

4.1 Array factor

An important parameter normally taken into account in order to evaluate the behavior of a smart antenna is the array factor, i.e. the gain offered by the antenna to a signal coming from the direction θ. The array factor is given by

$$F(\theta) = \mathbf{w}^H\mathbf{a}(\theta) \tag{9}$$

For a post-FFT beamforming an array factor is provided for each pilot tone, that means that the smart antenna offers different gains to different frequencies providing also a frequency domain filtering. Substituting expression (7) in (9) we have:

$$F(\theta) = \omega_d^*\mathbf{a}^H(\theta_d)\mathbf{a}(\theta) + \sum_{i=1}^{M}\omega_i^*\mathbf{a}^H(\theta_i)\mathbf{a}(\theta) \tag{10}$$

By using Equation (10) it is possible to evaluate the gain of the smart antenna correspondent to direct and reflected rays and it is easy to show that

$$\mathbf{F} = \begin{bmatrix} F(\theta_d) \\ F(\theta_1) \\ \vdots \\ F(\theta_M) \end{bmatrix} = \mathbf{S}^* \begin{bmatrix} \omega_d^* \\ \omega_1^* \\ \vdots \\ \omega_M^* \end{bmatrix} = \mathbf{S}^*\mathbf{\Omega}^*$$

4.2 Low noise conditions

If the noise power σ^2 is very reduced, system (8) can be approximated by

$$\mathbf{\Gamma}\mathbf{S}\mathbf{\Omega} = \mathbf{\Gamma}_1$$

that corresponds to

$$\mathbf{\Gamma}\mathbf{F}^* = \mathbf{\Gamma}_1$$

As the first column of the matrix coefficient $\mathbf{\Gamma}$ is equal to the right-hand side vector, it is easy to verify, for example using the Kramer's rule, that:

$$\mathbf{F} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{11}$$

This means that in low power noise conditions the beamformer completely eliminates the reflected rays. This condition is true as $M < L$, as supposed in the current scenario. Notice that condition (11) does not depend on the nature of $\mathbf{\Gamma}$ and it is true for both pre and post-FFT methods, so that, in low noise conditions, both methods eliminate reflected rays, the post-FFT method does not perform any frequency filtering and the $\widetilde{\mathbf{w}}_j$ are equal for all pilot tones.

4.3 Delay impact

The behavior of the post-FFT methods is strongly influenced by the delays of reflected rays. Consider a single reflected ray: when its delay is within the guard interval the samples carried by this signal belong all to the same OFDM frame, and are, thanks to the OFDM frame structure, cyclically shifted. This cyclic shift is transposed by the FFT operation in a modulation by a complex exponential, dependent on the samples position in the frequency domain. Since the post-FFT method performs separate beamforming on each pilot tone it is able to recover attenuation and modulation caused by the reflected ray channel, bettering performance. In this case, for the post-FFT methods, the matrix $\mathbf{\Gamma}_j\mathbf{S}$ is a space-frequency estimation of the channel response for the jth pilot tone and the weights $\mathbf{\Omega}_j$ are obtained by equalizing it with a MMSE criterion (the equalization is provided taking into account noise components).

When the delay is greater than the cyclic prefix, Γ_j cannot provide an estimation of the jth pilot channel because of inter-frame interference and the method perceives reflected rays as interfering that must be cancelled. As pre-FFT is not allowed to accede to frequency information, it always perceives reflected rays as interfering and it always eliminates them. For these reasons, when the delays are greater then the cyclic prefix, pre-FFT and post-FFT are likely to behave in the same manner, cancelling reflected rays: this statement has been proved by simulations.

5. BER PERFORMANCE

Figure 5 shows the bit error rate (BER) performance versus the ratio of the energy per bit received by a single sensor to the white noise power spectral density $\left(\frac{E_b}{N_0}\right)$ in different working conditions. The system parameters are the ones reported in table 1. The case of 1 reflected ray is analyzed: the LOS signal arrives from the angle $\theta = 60^o$ and the secondary ray from $\theta = 30^o$. The multipath attenuation is $\alpha_1 = 0.5$. It

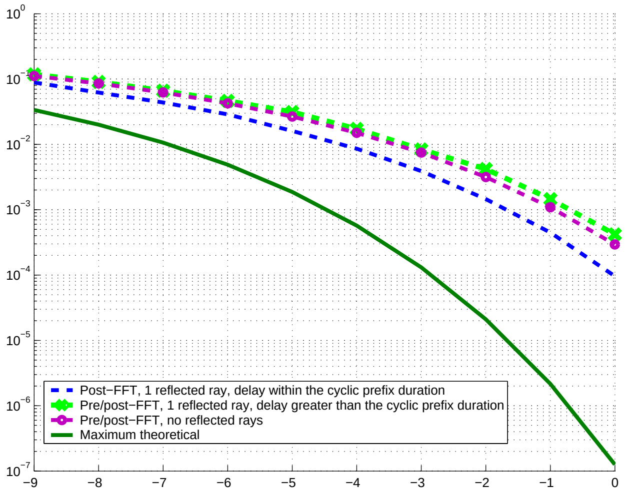

Figure 5: BER performance vs. Eb/N_0 under different working conditions, 8 antennas

Modulation scheme	4QAM/OFDM
No of data subcarriers	48
No of pilot subcarriers	4
No of zero subcarriers	12
FFT/IFFT size	64 point
Guard interval	16 samples
Antenna array	8-element linear array omni-directional antenna as elementary sensor
Element spacing	Half wavelength

Table 1: System parameters

is possible to notice that pre and post-FFT algorithms lead to the same results in absence of interfering and when the multipath delay is greater then the cyclic prefix. The pre-FFT behavior does not depend on the delay and the performance is the same even when the delay is within the guard interval; on the contrary, the post-FFT is able to retrieve useful power from the reflected ray bettering performance. In this case the BER curve results better than the one obtained in absence of reflected ray, this because the $\left(\frac{E_b}{N_0}\right)$ on the abscissa of Figure 5 only refers to the direct ray power. By the way all the curves are distant from the one labeled “Maximum theoretical”, which is obtained when two rays arrive at the same instant, from the same direction and so their powers are coherently summed.

6. CONCLUSION

In this paper a pre-FFT and a post-FFT beamformer for OFDM communications have been proposed and analyzed considering the Wiener’s solution. The analysis has clarified the behavior of both methods, showing their equivalence when the delays of reflected rays are greater than the guard interval duration. In such a case inter-frame interference corrupts both data and pilot carriers so that the post-FFT method perceives reflected rays as interfering and cancels them. When delays are within the cyclic prefix duration, the post-FFT is able to recover information carried by reflected rays, improving performances. The pre-FFT method always cancels reflected rays. For this reason in a scenario with a reduced number of reflected rays, i.e. when their number is inferior to the number of sensors, the pre-FFT method seems to be preferable for its reduced complexity, payed with a performance degradation when delays are within the cyclic prefix.

REFERENCES

[1] R. van Nee and R. Prasad *OFDM for Wireless Multimedia Communications*. Norwood, MA, USA: Artech House, Inc., 2000.

[2] C. K. Kim and K. Lee and Y. S. Cho “Adaptive beamforming algorithm for OFDM systems with antenna arrays” *IEEE Trans. Consumer Electron.*, vol. 46, pp. 1052–1058, Nov. 2000.

[3] M. Budsabathon and Y.Hara and S. Hara “Optimum Beamforming for Pre-FFT OFDM Adaptive Antenna Array” *IEEE Trans. Veh. Technol.*, vol. 53, pp. 945-955, July 2004.

[4] N. R. Sollenberger and Y. Li “Adaptive Antenna Arrays for OFDM Systems with Cochannel Interference” *IEEE Trans. Commun.*, vol. 47, pp. 217-229, Feb. 1999.

[5] A.T. Alastalo and M. Kahola “Smart-Antenna Operation for Indoor Wireless Local - Area Networks Using OFDM” *IEEE Trans. Wireless Commun.*, vol. 2, pp. 392-399, Mar. 2003.

[6] H. Matsuoka and H. Shoki, “Comparison of pre-FFT and post-FFT processing adaptive arrays for OFDM systems in presence of co-channel interference, ” in *Proc. 14th IEEE International Symposium on Personal, Indoor and Mobile Commun.*, Sept. 7-10 2003, pp. 1603-1607

[7] Z. Lei and F. P. S. Chin “Post and Pre-FFT Beamforming in an OFDM System, ” in *Proc. 59th IEEE Vehicular Technology Conference*, May 17-19 2004

[8] L.C.Godara “Application of antenna arrays to mobile communications, Part II: beam-forming and direction-of-arrival considerations”, *Proceedings of the IEEE* vol. 85, pp. 1195–1245, Aug. 1997

[9] D. Borio, L. Camoriano, L. Lo Presti and M. Mondin, “Beamforming and Synchronization Algorithms Integration for OFDM HAP-Based Communications”, *International Journal of Wireless Information Networks*, Dec. 2005.