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ADAPTIVE EQUALIZER BASED ON A POWER-OF-TWO-QUANTIZED-LMF ALGORITHM

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ABSTRACT

High speed and reliable data transmission over a variety of communication channels, including wireless and mobile radio channels, has been rendered possible through the use of adaptive equalization. In practice, adaptive equalizers rely heavily on the use of the least-mean square (LMS) algorithm which performs sub-optimally in the real world that is largely dominated by non-Gaussian interference signals. This paper proposes a new adaptive equalizer which relies on the judicious combination of the least-mean fourth (LMF) algorithm, which ensures a better performance in a non-Gaussian environment, and the power-of-two quantizer (PTQ) which reduces the high computational load brought about by the LMF and hence renders the proposed low-complexity equalizer capable of tracking fast-changing channels. This paper also presents a performance analysis of the proposed adaptive equalizer, based on a new linear approximation of the PTQ. Finally, the extensive simulation carried out here using the quantized LMF corroborates very well the theoretical predictions provided by the analysis of the linearized proposed algorithm.

1. INTRODUCTION

Ever since its introduction in digital communication by Lucky [1], adaptive equalization continues to enjoy a plethora of practical applications and to offer researchers in this area a rich source of deep theoretical challenges. The vibrancy of this area of research is clearly evidenced by its many footprints of success and the steady flow of interesting and practical research results. Central to the wide success and applicability of adaptive equalization is the ubiquitous adaptive least-mean square (LMS) algorithm [2] which is well-known to be optimal for Gaussian interference signals. Unfortunately, the real world is largely dominated by non-Gaussian interference signals, thus rendering the performance of any LMS-based adaptive equalizer sub-optimal. Moreover, a LMS-based adaptive equalizer suffers from a further loss in performance when applied to wireless and mobile radio channels that are fast-changing, both time and frequency-dispersive and where long bursts may get unacceptably corrupted if a fast-tracking operation is not in place.

To address this difficulty, several approaches, all aiming at simplifying the structure of the underlying adaptive scheme, were proposed [3, 4, 5]with varying degrees of success. Most notable of these contributions are the approaches used in [3] and [4] which achieve the required structural simplicity through the use of the power-of-two quantizer (PTQ) instead of the conventional analog-to-digital converter. Whereas [3] relies on the use of nonlinear correlation multipliers, [4] hinges on the use of the popular LMS and attributes the improvement gained in the overall performance of the adaptive equalizer to the combined use of the LMS and the PTQ.

In this paper, we propose a new approach which aims to effectively address the 2 main difficulties (sub-optimality in a non-Gaussian environment and lack of fast tracking rapidly-changing channels) plaguing the use of the LMS-based adaptive equalization. Our new approach is inspired from the work of [6] which showed that the LMF, which essentially relies on a non-mean square cost function, yields a better performance than the LMS in some non-Gaussian environments, e.g., uniform, sine, and square, but at the cost of a higher (than in the LMS case) computational load and from the work of [4] which demonstrated that the use of the PTQ leads to a structural simplicity of the adaptive equalization scheme and hence to an important reduction of the normally high computational load of the LMF, thus endowing an LMF-based equalizer with a fast tracking capability.

In addition to the beneficial and judicious combination of the LMF and the PTQ, this paper also presents a derivation of a new and very useful linear approximation of the PTQ's input/output characteristics. This linear approximation greatly simplifies the performance analysis of the proposed LMF-PTQ equalizer. The extensive simulation work carried out here in various practical scenarios substantiates very well the theory behind the proposed equalizer.

2. THE LMF ALGORITHM FOR ADAPTIVE EQUALIZATION

Consider the model of a linear channel with N-tap equalizer shown in Figure 1.

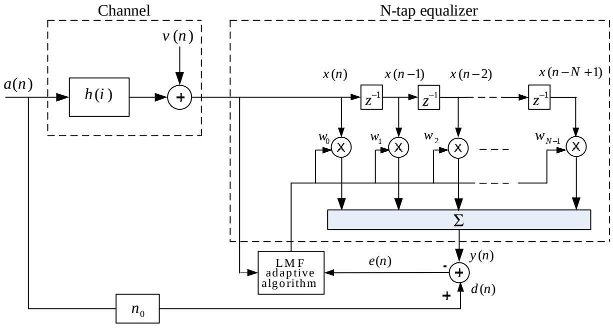

Fig. 1. Adaptive channel equalizer.

The equalizer input samples can be written as

$$x(n) = \sum_{i=0}^{N-1} h(i)a(n-i) + \nu(n), \tag{1}$$

where $h(i)$, $i = 0, 1, \ldots, N-1$, is the channel impulse response, $a(n)$ denotes the nth data sample, $\nu(n)$ is the additive noise added to the channel and N represent the length of the equalizer. The estimated output, $y(n)$, is defined as:

$$y(n) = \mathbf{w}^T(n)\mathbf{x}(n), \tag{2}$$

where $\mathbf{w}(n) = [w(0), w(1), \cdots, w(N-1)]^T$ is the current value of the adaptive weights, superscript T denotes transpose operation, and $\mathbf{x}(n) = [x(n), x(n-1), \cdots, x(n-N+1)]^T$ represents the input vector.

The weight vector, $\mathbf{w}(n)$, is updated by the LMF algorithm [6] according to:

$$\mathbf{w}(n+1) = \mathbf{w}(n) + 2\mu e^3(n)\mathbf{x}(n), \tag{3}$$

where μ is the step-size constant which controls stability and rate of convergence and $e(n)$ is the system's output error sample at the nth moment and found by:

$$e(n) \quad = \quad d(n) - \mathbf{w}^T(n)\mathbf{x}(n), \tag{4}$$

where $d(n)$ is the desired signal.

3. THE SIMPLIFIED ALGORITHM–LMF BASED POWER-OF-TWO QUANTIZER

A power-of-two quantizer is defined by Duttweiler [3] as:

$$q(u) = 2^{\lfloor \ln |u| \rfloor}\mathrm{sgn}(u), \tag{5}$$

where $\lfloor u \rfloor$ is the largest integer less than u and sgn(u) is the sign of u defined as:

$$\mathrm{sgn}(u) = \begin{cases} 1 & u \geq 0 \\ -1 & u < 0 \end{cases}$$

The quantizer defined by (5) is an infinite bit quantizer. However, in a real application, a finite bit quantizer is often used. The analysis of a finite bit power-of-two quantizer incorporated with LMS algorithm is given by Xue and Liu in [4], where they have indicated that a B-bit power-of-two quantizer converts an input u to a "one-bit" word according to:

$$q(u) = \begin{cases} \mathrm{sgn}(u), & |u| \geq 1; \\ 2^{\lfloor \ln |u| \rfloor}\mathrm{sgn}(u), & 2^{-B+1} \leq |u| < 1; \\ 0, & |u| < 2^{-B+1}. \end{cases} \tag{6}$$

In this work, we have adopted the simplification of equation (6) and applied it to LMF algorithm resulting in LMF based power-of-two quantizer (LMF-PTQ). Instead of the updating algorithm (3), the equalizer coefficient update is carried out according to:

$$\mathbf{w}(n+1) = \mathbf{w}(n) + 2\mu q[e^3(n)]\mathrm{sgn}[\mathbf{x}(n)], \tag{7}$$

where $q[e^3(n)]$ is the modified power-of-two quantizer for LMF algorithm and is defined by:

$$q[e^3(n)] = \begin{cases} \mathrm{sgn}[e(n)], & |e(n)| \geq 1; \\ 2^{\lfloor 3\ln |e(n)| \rfloor}\mathrm{sgn}[e(n)], & 2^{\frac{-B+1}{3}} \leq |e(n)| < 1; \\ 0, & |e(n)| < 2^{\frac{-B+1}{3}}. \end{cases} \tag{8}$$

Note here that (8) has been straightforwardly obtained from (6) by replacing the quantizer input u by $e^3(n)$.

Finally, Figure 2 illustrates the transfer characteristics of such quantizer with $B = 4$ bits .

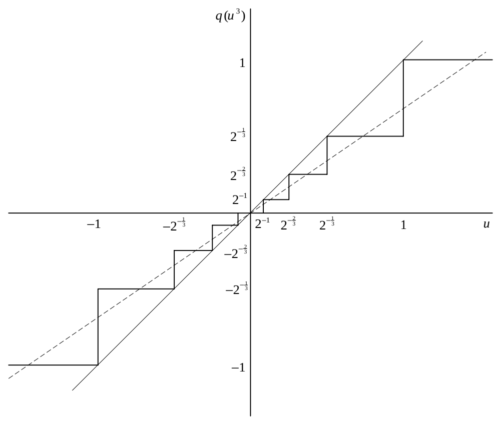

Fig. 2. Input-Output characteristics of a 4-bit power-of-two quantizer.

4. CONVERGENCE ANALYSIS OF LMF-PTQ ALGORITHM

In the following, a linearized analysis is presented. It is obvious that the analysis of Equation (8) will be complex because of the presence of the error cube update. We therefore resort to a linearized approach through approximation of the quantizer function $q[e^3(n)]$ to a linear function. This is done by drawing a straight line passing through the center of each step of the quantizer transfer characteristic. Such line is shown dotted in Figure 2. Although, the approach may give less accurate results, but it will surely render the analysis more tractable.

A geometrical analysis of Figure 2 leads to the following approximation, as shown in Appendix A:

$$q[e^3(n)] \approx \frac{7}{8}e(n). \qquad (9)$$

4.1. Convergence in the Mean

On using approximation (9), equation (7) becomes

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \frac{7}{4}\mu e(n)\text{sgn}[\mathbf{x}(n)]. \qquad (10)$$

Now, let us define the coefficient error vector $\mathbf{v}(n) = \mathbf{w}(n) - \mathbf{w}_{opt}$, where $\mathbf{w}_{opt}$ denotes the optimal coefficient vector. Subtracting $\mathbf{w}_{opt}$ from both sides of (10) and taking the expected value of both sides of it, using the independence assumption [2] and applying Price theorem [7], the mean behaviour for the coefficient misalignment vector of the LMF-PTQ is shown to be governed by the following recursion:

$$E\{\mathbf{v}(n+1)\} = \left[\mathbf{I} - \frac{7}{4}\sqrt{\frac{2}{\pi}}\,\mu\frac{\mathbf{R}}{\sigma_x}\right]E\{\mathbf{v}(n)\}, \qquad (11)$$

where σ_x and $\mathbf{R}$ are, respectively, the standard deviation of the input signal and the input autocorrelation matrix.

Therefore, sufficient condition for the convergence in the mean of the LMF-PTQ algorithm is governed by:

$$0 < \mu < \frac{4}{7N\sigma_x}\sqrt{2\pi}. \qquad (12)$$

4.2. Convergence in the Mean-Square

We begin by subtracting the optimal coefficient vector, $\mathbf{w}_{opt}$ from both sides of (7). Accordingly, we have

$$\mathbf{v}(n+1) = \mathbf{v}(n) + 2\mu q[e^3(n)]\text{sgn}[\mathbf{x}(n)]. \qquad (13)$$

Let us define

$$E[\|\mathbf{v}(n)\|^2] = \theta(n). \qquad (14)$$

It follows from (13) that with white inputs, Equation (14) looks like the following:

$$\theta(n+1) = \theta(n) + 4N\mu^2E\{q^2[e^3(n)]\} + 4\mu \times E\{q[e^3(n)]\mathbf{v}^T(n)\text{sgn}[\mathbf{x}(n)\}. \qquad (15)$$

Misadjustment	$\dfrac{N\mu\sigma_x}{\frac{16}{7}\sqrt{\frac{1}{2\pi}} - N\mu\sigma_x}$
Time constant	$-\dfrac{1}{\ln\left(1 - \frac{7}{2}\sqrt{\frac{2}{\pi}}\mu\sigma_x + \frac{49}{16}N\mu^2\sigma_x^2\right)}$

Table 1. A summary of the convergence analysis results for the LMF-PTQ algorithm.

Again using approximation (9), Equation (15) becomes

$$\theta(n+1) = \theta(n) + \frac{49}{16}N\mu^2E\{e^2(n)\} + \frac{7}{2}\mu \times E\{e(n)\mathbf{v}^T(n)\text{sgn}[\mathbf{x}(n)]\}. \qquad (16)$$

With the assumption that the sequence $x(n)$ is an i.i.d. with zero mean and variance σ_x^2, it can be shown that the LMF-PTQ algorithm will converge in the mean-square sense if the step size μ is governed by:

$$0 < \mu < \frac{8}{7N\sigma_x}\sqrt{\frac{2}{\pi}}. \qquad (17)$$

Finally, Table 1 gives the expressions of the misadjustment factor and the time constant of the proposed algorithm. were not derived here (due to space limitations) but included for completeness of the study.

5. SIMULATION RESULTS

An 11-tap transversal equalizer with varying quantization resolution ranging from 3 to 9 bits has been simulated. The input symbol sequence is a random bipolar signals, i.e., $(\{a(n)\} = \pm 1)$. As an example of a non-Gaussian environment, a uniformly distributed noise with SNR=20 dB was added to the output of the channel. The dispersive channel considered here has an impulse response of the raise-cosine type [2] given by:

$$h(n) = \begin{cases} \frac{1}{2}\left[1 + \cos\left(\frac{2\pi(n-1)}{W}\right)\right], & n = 1,2,3 \\ 0, & \text{otherwise} \end{cases} \qquad (18)$$

where W is a parameter that determines the eigenvalue spread of the signal input. W was set equal to 3.1 and 3.3 to provide an input autocorrelation matrix eigenvalue spread of 11 and 21, respectively. A sample of our simulation results is shown here in Figures 3 and 4, where the learning curves are shown for the cases: B=3, 5, 7 and 9. As can be seen from these figures, a good agreement between simulation and theory has been achieved.

The proposed algorithm was also successfully tested in other practical scenarios involving time varying channels (e.g., mobile channels) and different noise environments.

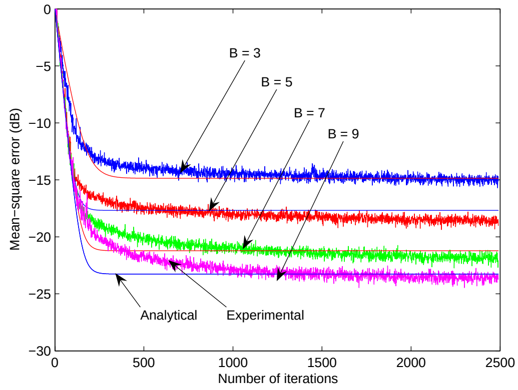

Fig. 3. Analytical and experimental learning curves for adaptive equalization using LMF-PTQ algorithm. Eigen-value spread of 11.

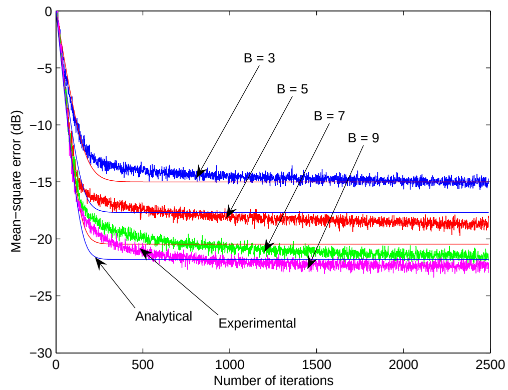

Fig. 4. Analytical and experimental learning curves for adaptive equalization using LMF-PTQ algorithm. Eigen-value spread of 21.

Due to space limitation, these results cannot be reported on here. Moreover, it should be noted that there are other low complexity algorithms (e.g., block adaptive, frequency-domain, partial update, just to name a few); however, a comparison between these algorithms and our proposed low complexity algorithm warrants by itself a separate study.

6. CONCLUSION

A new equalizer based on a judicious combination of the LMF algorithm and the PTQ quantizer was presented. As such, this equalizer enjoys better performance in a non-Gaussian environment and a fast tracking capability needed for a successful handling of rapidly-changing channels. The proposed equalizer's performance was also analyzed and all the theoretical predictions were very well supported by our extensive simulation work. Finally, the success of this work provides ample encouragement to investigate the performance of a whole class of new equalizers that are driven by more general adaptive algorithms governed by non-mean square cost functions of the general form : $J = E[e^{2p}(n)]$, with $p > 2$.

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A. APPENDIX: DERIVATION OF THE LINEAR APPROXIMATION IN (9)

From the first quadrant of Figure 2, the mid points of the steps are:
$\frac{2^{\frac{-B+2}{3}}+2^{\frac{-B+1}{3}}}{2}$, $\frac{2^{\frac{-B+3}{3}}+2^{\frac{-B+2}{3}}}{2}$, $\frac{2^{\frac{-B+4}{3}}+2^{\frac{-B+3}{3}}}{2}$, and so on. The slope of the line joining these mid points can be expressed as:

$$\frac{\Delta q(u^3)}{\Delta u} = \frac{2^{\frac{-B+3}{3}} - 2^{\frac{-B+2}{3}}}{\frac{2^{\frac{-B+4}{3}}+2^{\frac{-B+3}{3}}}{2} - \frac{2^{\frac{-B+3}{3}}+2^{\frac{-B+2}{3}}}{2}} \quad (19)$$

Putting $B = 4$, we obtain $\frac{\Delta q(u^3)}{\Delta u} = 0.885 \approx \frac{7}{8}$.

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