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ITERATIVE BLIND SOURCE SEPARATION BY DECORRELATION: ALGORITHM AND PERFORMANCE ANALYSIS

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ABSTRACT

This paper presents an iterative blind source separation method using second order statistics (SOS) and natural gradient technique. The SOS of observed data is shown to be sufficient for separating mutually uncorrelated sources provided that the considered temporal coherence vectors of the sources are pairwise linearly independent. By applying the natural gradient, an iterative algorithm is derived that has a number of attractive properties including its simplicity and 'easy' generalization to adaptive or convolutive schemes. Asymptotic performance analysis of the proposed method is performed. Several numerical simulations are presented to demonstrate the effectiveness of the proposed method and to validate the theoretical expression of the asymptotic performance index.

1. INTRODUCTION

Source separation aims at recovering multiple sources from multiple observations (mixtures) received by a set of linear sensors. The problem is said to be 'blind' when the observations have been linearly mixed by the transfer medium, while having no *a priori* knowledge of the transfer medium or the sources. Blind source separation (BSS) has applications in several areas, such as communication, speech/audio processing, biomedical engineering, geophysical data processing, etc [1]. BSS of instantaneous mixtures has attracted a lot of attention due to its many potential applications [1] and its mathematical tractability that lead to several nice and simple BSS solutions [1–5].

In this paper, we present SOS based method for blind separation of temporally coherent sources. We first present SOS-based contrast functions for BSS. Then, to achieve BSS, we optimize the considered contrast function using an iterative algorithm based on the relative gradient technique. The generalization of this method to deconvolution and adaptive algorithms is then discussed. Finally, a theoretical analysis of the performance of the method has been derived and validated by simulation results.

2. PROBLEM FORMULATION

Assume that m narrow band signals impinge on an array of $n \geq m$ sensors. The measured array output is a weighted superposition of the signals, corrupted by additive noise, i.e.

$$\mathbf{x}(t) = \mathbf{y}(t) + \boldsymbol{\eta}(t) = \mathbf{A}\mathbf{s}(t) + \boldsymbol{\eta}(t) \tag{1}$$

where $\mathbf{s}(t) = [s_1(t), \cdots, s_m(t)]^T$ is the $m \times 1$ complex *source vector*, $\boldsymbol{\eta}(t) = [\eta_1(t), \cdots, \eta_n(t)]^T$ is the $n \times 1$ Gaussian complex *noise vector*, $\mathbf{A}$ is the $n \times m$ full column rank *mixing matrix* (i.e., $n \geq m$), and the superscript T denotes the transpose operator. The source signal vector $\mathbf{s}(t)$, is assumed to be a multivariate stationary complex stochastic process.

In this paper, we consider only the second order statistics and hence the signals $s_i(t)$, $1 \leq i \leq m$ are assumed to be temporally coherent and mutually uncorrelated, with zero mean and second order moments:

$$\begin{aligned}\mathbf{S}(\tau) &\stackrel{\text{def}}{=} E\left(\mathbf{s}(t+\tau)\mathbf{s}^\star(t)\right)\\ &= \operatorname{diag}[\rho_1(\tau), \cdots, \rho_m(\tau)]\end{aligned}$$

where $\rho_i(\tau) \stackrel{\text{def}}{=} E(s_i(t+\tau)s_i^*(t))$, the expectation operator is E, and the superscripts $*$ and $\star$ denote the conjugate of a complex number and the complex conjugate transpose of a vector, respectively. The additive noise $\boldsymbol{\eta}(t)$ is modeled as a stationary white (temporally but not necessarily spatially) zero-mean complex random process. In that case, the source separation is achieved by decorrelating the signals at different time lags. The purpose of blind source separation is to find a separating matrix, i.e. a $m \times n$ matrix such that $\widehat{\mathbf{s}}(t) = \mathbf{B}\mathbf{x}(t)$ is an estimate of the source signals.

Before proceeding, note that complete blind identification of separating matrix $\mathbf{B}$ (or the equivalently mixing matrix $\mathbf{A}$) is impossible. The best that can be done is to determine $\mathbf{B}$ up to a permutation and scalar multiple of its columns [3], i.e., $\mathbf{B}$ is a separating matrix iff:

$$\mathbf{B}\mathbf{y}(t) = \mathbf{P}\boldsymbol{\Lambda}\mathbf{s}(t) \tag{2}$$

where $\mathbf{P}$ is a permutation matrix and $\boldsymbol{\Lambda}$ a non-singular diagonal matrix.

3. CONTRAST FUNCTION

The following theorems serve as the basis for our method for blind separation of stationary sources.

We present here separation criteria for the stationary, temporally correlated source signals and their corresponding contrast functions. Let consider first the noiseless case. We have the following result:

Theorem 1 *Let $\tau_1 < \tau_2 < \cdots < \tau_K$ be $K \geq 1$ time lags, and define $\boldsymbol{\rho}_i = [\rho_i(\tau_1), \rho_i(\tau_2), \cdots, \rho_i(\tau_K)]$ and $\tilde{\boldsymbol{\rho}}_i = [\Re e(\boldsymbol{\rho}_i), \Im m(\boldsymbol{\rho}_i)]$ where $\Re e(x)$ and $\Im m(x)$ denote the real part and imaginary part of x, respectively. Taking advantage of the indeterminacy, we assume without loss of generality that the sources are scaled such that $\|\boldsymbol{\rho}_i\| = \|\tilde{\boldsymbol{\rho}}_i\| = 1$, for all i [1]. Then, BSS can be achieved using the output correlation matrices at time lags $\tau_1, \tau_2, \cdots, \tau_K$ if and only if for all $1 \leq i \neq j \leq m$:*

$$\tilde{\boldsymbol{\rho}}_i \text{ and } \tilde{\boldsymbol{\rho}}_j \text{ are (pairwise) linearly independent} \tag{3}$$

Interestingly, we can see from condition (3) that BSS can be achieved from only one correlation matrix $\mathbf{R}_x(\tau_k) \stackrel{\text{def}}{=} E(\mathbf{x}(t+$

[1]We implicitly assume here that $\boldsymbol{\rho}_i \neq 0$, otherwise the source signal could not be detected (and a fortiori could not be estimated) from the considered set of correlation matrices. This hypothesis will be held in the sequel.

$\tau_k)\mathbf{x}^\star(t))$ provided that the vectors $[\Re e(\rho_i(\tau_k)), \Im m(\rho_i(\tau_k)]$ and $[\Re e(\rho_j(\tau_k)), \Im m(\rho_j(\tau_k)]$ are pairwise linearly independent for all $i \neq j$.

Note also that, from (3), BSS can be achieved if at most one temporally white source signal exists. In contrast, recall that when using higher order statistics, BSS can only be achieved if at most one Gaussian source signal exists.

Under the condition of Theorem 1, the BSS can be achieved by decorrelation according to the following result:

Theorem 2 *Let $\tau_1, \tau_2, \cdots, \tau_K$ be K time lags and $\mathbf{z}(t) = [z_1(t), \cdots, z_m(t)]^T$ be an $m \times 1$ vector given by $\mathbf{z}(t) = \mathbf{B}\mathbf{x}(t)$. Define $r_{ij}(\tau_k) \stackrel{\text{def}}{=} E(z_i(t+\tau_k)z_j^*(t))$. If the identifiability condition holds, then $\mathbf{B}$ is a separating matrix if and only if*

$$r_{ij}(\tau_k) = 0 \quad \textit{and} \quad \sum_{k=1}^{K} |r_{ii}(\tau_k)| > 0 \tag{4}$$

for all $1 \leq i \neq j \leq m$ and $k = 1, 2, \cdots, K$.

Note that, if one of the time lags is zero, the result of Theorem 2 holds only under the noiseless assumption. In that case, we can replace the condition $\sum_{k=1}^{K} |r_{ii}(\tau_k)| > 0$ by $r_{ii}(0) > 0$, for $i = 1, \cdots, m$. On the other hand, if all the time lags are non-zero and if the noise is temporally white (but can be spatially colored with unknown spatial covariance matrix) then the above result holds without the noiseless assumption.

Based on Theorem 2, we can define different objective (contrast) functions for signal decorrelation. In [6], the following criterion[2] was used

$$G(\mathbf{z}) = \sum_{k=1}^{K} \log|\text{diag}(\mathbf{R}_z(\tau_k))| - \log|\mathbf{R}_z(\tau_k)| \tag{5}$$

where diag($\mathbf{A}$) is the diagonal matrix obtained by zeroing the off diagonal entries of $\mathbf{A}$. Another criterion used in [5] is

$$\begin{aligned} G(\mathbf{z}) = \sum_{k=1}^{K} \sum_{1 \leq i < j \leq m} & [|r_{ij}(\tau_k) + r_{ji}(\tau_k)|^2 + \\ |r_{ij}(\tau_k) - r_{ji}(\tau_k)|^2] & + \sum_{i=1}^{m} |\sum_{k=1}^{K} |r_{ii}(\tau_k)| - 1|^2 \end{aligned} \tag{6}$$

Equations (5) and (6) are non-negative functions which are zero if and only if $\mathbf{R}_z(\tau_k) = E(\mathbf{z}(t+\tau_k)\mathbf{z}^\star(t))$ are diagonal for $k = 1, \cdots, K$ or equivalently if (4) is met.

4. ITERATIVE ALGORITHM

The separation criteria we have presented takes the form:

$$\mathbf{B} \text{ is a separating matrix} \Longleftrightarrow G(\mathbf{z}(t)) = 0 \tag{7}$$

where $\mathbf{z}(t) = \mathbf{B}\mathbf{x}(t)$ and G is a given contrast function. The approach we choose to solving (7) is inspired from [7]. It is a block technique based on the processing of T received samples and consists of searching the zeros of the sample version of (7). Solutions are obtained iteratively in the form:

$$\mathbf{B}^{(p+1)} = (\mathbf{I} + \boldsymbol{\epsilon}^{(p)})\mathbf{B}^{(p)} \tag{8}$$

$$\mathbf{z}^{(p+1)}(t) = (\mathbf{I} + \boldsymbol{\epsilon}^{(p)})\mathbf{z}^{(p)}(t) \tag{9}$$

At iteration p, a matrix $\boldsymbol{\epsilon}^{(p)}$ is determined from a local linearization of $G(\mathbf{B}\mathbf{x}(t))$. It is an approximate Newton technique with the benefit that $\boldsymbol{\epsilon}^{(p)}$ can be very simply computed (no Hessian inversion) under the additional assumption that $\mathbf{B}^{(p)}$ is close to a separating matrix. This procedure is illustrated as follows:

We first consider the noiseless case or eventually the nonzero lag case (i.e. $\tau_i \neq 0$ for $i = 1, \ldots, K$). By using (9), we have:

$$\begin{aligned} r_{ij}^{(p+1)}(\tau_k) = r_{ij}^{(p)}(\tau_k) + \sum_{q=1}^{m} \epsilon_{jq}^{*(p)} r_{iq}^{(p)}(\tau_k) + \\ \sum_{l=1}^{m} \epsilon_{il}^{(p)} r_{lj}^{(p)}(\tau_k) + \sum_{l,q=1}^{m} \epsilon_{il}^{(p)} \epsilon_{jq}^{*(p)} r_{lq}^{(p)}(\tau_k) \end{aligned} \tag{10}$$

where

$$r_{ij}^{(p)}(\tau_k) \stackrel{\text{def}}{=} E\left(z_i^{(p)}(t+\tau_k) z_j^{*(p)}(t)\right) \tag{11}$$

$$\approx \frac{1}{T-\tau_k} \sum_{t=1}^{T-\tau_k} z_i^{(p)}(t+\tau_k) z_j^{*(p)}(t) \tag{12}$$

Under the assumption that $\mathbf{B}^{(p)}$ is close to a separating matrix, it follows that

$$|\epsilon_{ij}^{(p)}| \ll 1$$

and

$$|r_{ij}^{(p)}(\tau_k)| \ll 1 \quad \text{for} \quad i \neq j$$

and thus, a first order approximation of $r_{ij}^{(p+1)}(\tau_k)$ is given by:

$$r_{ij}^{(p+1)}(\tau_k) \approx r_{ij}^{(p)}(\tau_k) + \epsilon_{ji}^{*(p)} r_{ii}^{(p)}(\tau_k) + \epsilon_{ij}^{(p)} r_{jj}^{(p)}(\tau_k) \tag{13}$$

similarly, we have:

$$r_{ji}^{(p+1)}(\tau_k) \approx r_{ji}^{(p)}(\tau_k) + \epsilon_{ij}^{*(p)} r_{jj}^{(p)}(\tau_k) + \epsilon_{ji}^{(p)} r_{ii}^{(p)}(\tau_k) \tag{14}$$

From (13) and (14), we have:

$$\begin{aligned} r_{ij}^{(p+1)}(\tau_k) + r_{ji}^{(p+1)}(\tau_k) \;\approx\; & 2r_{jj}^{(p)}(\tau_k)\Re e(\epsilon_{ij}^{(p)}) \\ & +2r_{ii}^{(p)}(\tau_k)\Re e(\epsilon_{ji}^{(p)}) + (r_{ij}^{(p)}(\tau_k) + r_{ji}^{(p)}(\tau_k)) \end{aligned}$$

$$\begin{aligned} r_{ij}^{(p+1)}(\tau_k) - r_{ji}^{(p+1)}(\tau_k) \;\approx\; & 2\jmath r_{jj}^{(p)}(\tau_k)\Im m(\epsilon_{ij}^{(p)}) \\ & -2\jmath r_{ii}^{(p)}(\tau_k)\Im m(\epsilon_{ji}^{(p)}) + (r_{ij}^{(p)}(\tau_k) - r_{ji}^{(p)}(\tau_k)) \end{aligned}$$

with $\jmath = \sqrt{-1}$. By replacing the previous equation into (6), we obtain the following least squares (LS) minimization problem

$$\min \left\| \left[\mathbf{r}_{jj}^{(p)},\ \mathbf{r}_{ii}^{(p)}\right] \mathbf{E}_{ij}^{(p)} + \left[\frac{1}{2}(\mathbf{r}_{ij}^{(p)} + \mathbf{r}_{ji}^{(p)}),\ \frac{1}{2\jmath}(\mathbf{r}_{ij}^{(p)} - \mathbf{r}_{ji}^{(p)})\right] \right\|$$

where

$$\mathbf{E}_{ij}^{(p)} \stackrel{\text{def}}{=} \begin{bmatrix} \Re e(\epsilon_{ij}^{(p)}) & \Im m(\epsilon_{ij}^{(p)}) \\ \Re e(\epsilon_{ji}^{(p)}) & -\Im m(\epsilon_{ji}^{(p)}) \end{bmatrix} \tag{15}$$

$$\mathbf{r}_{ij}^{(p)} = [r_{ij}^{(p)}(\tau_1), \cdots, r_{ij}^{(p)}(\tau_K)]^T \tag{16}$$

A solution to the LS minimization problem is given by:

$$\mathbf{E}_{ij}^{(p)} = -\left[\mathbf{r}_{jj}^{(p)}, \mathbf{r}_{ii}^{(p)}\right]^{\#} \left[\frac{1}{2}(\mathbf{r}_{ij}^{(p)} + \mathbf{r}_{ji}^{(p)}), \frac{1}{2\jmath}(\mathbf{r}_{ij}^{(p)} - \mathbf{r}_{ji}^{(p)})\right] \tag{17}$$

where $\mathbf{A}^{\#}$ denotes the pseudo-inverse of matrix $\mathbf{A}$. Equations (15) and (17) provide the explicit expression of $\epsilon_{ij}^{(p)}$ for

[2] In that paper, only the case where $\tau_1 \neq 0$ was considered.

$i \neq j$. For $i = j$, the minimization of (6) using the first order approximation leads to:

$$\left| \sum_{k=1}^{K} r_{ii}^{(p)}(\tau_k) \left(1 + 2\Re e(\epsilon_{ii}^{(p)})\right) \right| - 1 = 0 \qquad (18)$$

Without loss of generality, we take advantage of the phase indeterminacy to assume that ϵ_{ii} are real-valued and hence $\Re e(\epsilon_{ii}) = \epsilon_{ii}$. Consequently, we obtain:

$$\epsilon_{ii}^{(p)} = \frac{1 - \sum_{k=1}^{K} |r_{ii}^{(p)}(\tau_k)|}{2 \sum_{k=1}^{K} |r_{ii}^{(p)}(\tau_k)|} \qquad (19)$$

In the case of real-valued signals, the LS minimization becomes:

$$\min \left\| \mathbf{H}_{ij}^{(p)} \mathbf{e}_{ij}^{(p)} + \boldsymbol{\psi}_{ij}^{(p)} \right\|$$

where

$$\mathbf{H}_{ij}^{(p)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \otimes \left[\mathbf{r}_{jj}^{(p)},\ \mathbf{r}_{ii}^{(p)} \right] \qquad (20)$$

$$\mathbf{e}_{ij}^{(p)} = \left[\epsilon_{ij}^{(p)}, \epsilon_{ji}^{(p)} \right]^T \qquad (21)$$

$$\boldsymbol{\psi}_{ij}^{(p)} = \begin{bmatrix} \mathbf{r}_{ij}^{(p)} \\ \mathbf{r}_{ji}^{(p)} \end{bmatrix} \qquad (22)$$

and $\otimes$ denotes the Kronecker product. A solution to the LS minimization problem is given by:

$$\mathbf{e}_{ij}^{(p)} = -\mathbf{H}_{ij}^{(p)\#} \boldsymbol{\psi}_{ij}^{(p)} \qquad (23)$$

Remark: A main advantage of the above algorithm is its flexibility and easy implementation in the adaptive case. Indeed, the adaptive version of this algorithm consists simply in replacing the iteration index p by the time index t and the correlation coefficient $r_{ij}^{(p)}(\tau_k)$ by their adaptive estimates $r_{ij}^{(t)}(\tau_k) = \beta r_{ij}^{(t-1)}(\tau_k) + z_i(t) z_j^*(t - \tau_k)$ if $k \geq 0$ or otherwise $r_{ij}^{(t)}(\tau_k) = \beta r_{ij}^{(t-1)}(\tau_k) + z_i(t + \tau_k) z_j^*(t)$ where $0 < \beta < 1$ is a forgetting factor. This algorithm can also be extended to deal with BSS of convolutive mixtures as shown next.

5. GENERALIZATION TO CONVOLUTIVE MIXTURE CASE

In the convolutive mixture case, the signal can be modeled by the following equation:

$$\mathbf{x}(t) = \mathbf{y}(t) + \boldsymbol{\eta}(t) = \sum_{\ell=0}^{L} \mathbf{A}(\ell) \mathbf{s}(t - \ell) + \boldsymbol{\eta}(t), \qquad (24)$$

where $\mathbf{A}(k)$ are $n \times m$ matrices for $\ell \in [0, L]$ representing the impulse response coefficients of the channel. The polynomial matrix $\mathbf{A}(z) = \sum_{\ell=0}^{L} \mathbf{A}(\ell) z^{-\ell}$ is assumed to be irreducible (i.e. $\mathbf{A}(z)$ is of full column rank for all z).
In this section, one will determinate the rational matrix $\mathbf{B}(z) = \sum_{\ell} \mathbf{B}(\ell) z^{-\ell}$ such that $\mathbf{B}(z)$ is a separating matrix, i.e.

$$\mathbf{w}(t) = \sum_{\ell} \mathbf{B}(\ell) \mathbf{x}(t - \ell) = [\mathrm{diag}(c_1(z) \ldots c_m(z))]\, \mathbf{s}(t) \qquad (25)$$

where $c_1(z) \ldots c_m(z)$ are m given scalar rational functions. To achieve this BSS, we consider a decorrelation criterion :

$$\widetilde{G}(\mathbf{w}) = \sum_{k=1}^{K} \sum_{1 \leq i < j \leq m} |r_{ij}(\tau_k)|^2 + \sum_{i=1}^{m} |\sum_{k=1}^{K} |r_{ii}(\tau_k)| - 1|^2 \qquad (26)$$

so that

$$\mathbf{B}(z) \ \text{ is a separating matrix} \Longleftrightarrow \widetilde{G}(\mathbf{w}(t)) = 0 \qquad (27)$$

Solutions are obtained iteratively in the form:

$$\mathbf{B}^{(p+1)}(z) = \left(\mathbf{I} + \boldsymbol{\epsilon}^{(p)}(z) \right) \mathbf{B}^{(p)}(z) \qquad (28)$$

$$\mathbf{w}^{(p+1)}(t) = \mathbf{w}^{(p)}(t) + \sum_{\ell=0}^{1} \boldsymbol{\epsilon}^{(p)}(\ell) \mathbf{w}^{(p)}(t - \ell) \qquad (29)$$

where $\boldsymbol{\epsilon}^{(p)}(z) \overset{\text{def}}{=} \boldsymbol{\epsilon}^{(p)}(0) + \boldsymbol{\epsilon}^{(p)}(1) z^{-1}$.
Similarly to the instantaneous mixture case, a first order approximation of $r_{ij}^{(p+1)}(\tau_k)$ is given by:

$$\begin{aligned} r_{ij}^{(p+1)}(\tau_k) \approx r_{ij}^{(p)}(\tau_k) + \sum_{l=0}^{1} \epsilon_{ji}^{*(p)}(\ell) r_{ii}^{(p)}(\tau_k + \ell) \\ + \sum_{l'=0}^{1} \epsilon_{ij}^{(p)}(\ell') r_{jj}^{(p)}(\tau_k - \ell') \end{aligned} \qquad (30)$$

Replacing (30) into (27) leads after straight forward derivation to:

$$\min_{\boldsymbol{\mathcal{E}}_{ij}^{(p)}} \left(\|\boldsymbol{\Phi}_{ij}^{(p)} \boldsymbol{\mathcal{E}}_{ij}^{(p)} + \mathbf{r}_{ij}^{(p)}\|^2 + \|\boldsymbol{\Phi}_{ji}^{(p)} \mathbf{M} \boldsymbol{\mathcal{E}}_{ij}^{*(p)} + \mathbf{r}_{ji}^{(p)}\|^2 \right) \qquad (31)$$

where

$$\boldsymbol{\mathcal{E}}_{ij}^{(p)} = [\epsilon_{ij}^{(p)}(0)\ \epsilon_{ij}^{(p)}(1)\ \epsilon_{ji}^{*(p)}(0)\ \epsilon_{ji}^{*(p)}(1)]^T \qquad (32)$$

$$\boldsymbol{\Phi}_{ij}^{(p)} = [\boldsymbol{\phi}_{jj}^{(p)}(0)\ \boldsymbol{\phi}_{jj}^{(p)}(-1)\ \boldsymbol{\phi}_{ii}^{(p)}(0)\ \boldsymbol{\phi}_{ii}^{(p)}(1)] \qquad (33)$$

$$\boldsymbol{\phi}_{ii}^{(p)}(k) = [r_{ii}^{(p)}(\tau_1 + k), \ldots, r_{ii}^{(p)}(\tau_K + k)]^T, \qquad k \in \mathbb{Z} \qquad (34)$$

and $\mathbf{M}$ is the matrix verifying $\boldsymbol{\mathcal{E}}_{ji}^{(p)} = \mathbf{M} \boldsymbol{\mathcal{E}}_{ij}^{*(p)}$, i.e.

$$\mathbf{M} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad (35)$$

Finally, one obtains the solution:

$$\begin{bmatrix} \Re e \left(\boldsymbol{\mathcal{E}}_{ij}^{(p)} \right) \\ \Im m \left(\boldsymbol{\mathcal{E}}_{ij}^{(p)} \right) \end{bmatrix} = - \begin{bmatrix} \boldsymbol{\Theta}_{ij}^{(p)} \\ \boldsymbol{\Theta}_{ji}^{(p)} \end{bmatrix}^{\#} \boldsymbol{\nu}_{ij}^{(p)} \qquad (36)$$

where

$$\boldsymbol{\Theta}_{ij}^{(p)} = \begin{bmatrix} \Re e \left(\boldsymbol{\Phi}_{ij}^{(p)} \right) & -\Im m \left(\boldsymbol{\Phi}_{ij}^{(p)} \right) \\ \Im m \left(\boldsymbol{\Phi}_{ij}^{(p)} \right) & \Re e \left(\boldsymbol{\Phi}_{ij}^{(p)} \right) \end{bmatrix} \qquad (37)$$

$$\boldsymbol{\Theta}_{ji}^{(p)} = \begin{bmatrix} \Re e \left(\boldsymbol{\Phi}_{ji}^{(p)} \mathbf{M} \right) & \Im m \left(\boldsymbol{\Phi}_{ji}^{(p)} \mathbf{M} \right) \\ \Im m \left(\boldsymbol{\Phi}_{ji}^{(p)} \mathbf{M} \right) & -\Re e \left(\boldsymbol{\Phi}_{ji}^{(p)} \mathbf{M} \right) \end{bmatrix} \qquad (38)$$

and

$$\boldsymbol{\nu}_{ij}^{(p)} = \left[\Re e(\mathbf{r}_{ij}^{(p)})^T \Im m(\mathbf{r}_{ij}^{(p)})^T \Re e(\mathbf{r}_{ji}^{(p)})^T \Im m(\mathbf{r}_{ji}^{(p)})^T \right]^T \qquad (39)$$

For $i = j$, we take again advantage of the problem indeterminacy to assume without loss of generality that the diagonal entries of $\epsilon_{ii}^{(p)}(0)$ are real-valued and those of $\epsilon_{ii}^{(p)}(1)$ are zero. This assumption leads to the $\epsilon_{ii}^{(p)}(0)$ given by equation (19).

6. ASYMPTOTIC PERFORMANCE

In this section, asymptotic (i.e. for large sample sizes) performance analysis results of the proposed method in instantaneous real case is given. We consider the case of instantaneous mixture with i.i.d real-valued sources satisfying, in addition to the identifiability condition $\sum_{k\in\mathbb{Z}} |\rho_i(k)| < +\infty$ for $i = 1, \ldots, m$. The noise is assumed Gaussian with variance $\sigma^2\mathbf{I}$. Assuming that the permutation indeterminacy is $\mathbf{P} = \mathbf{I}$, one can write:

$$\mathbf{BA} = \mathbf{I} + \boldsymbol{\delta} \tag{40}$$

and hence, the separation quality is measured using the mean rejection level criterion [3] defined as:

$$\begin{aligned} \mathcal{I}perf &\stackrel{\text{def}}{=} \sum_{p\neq q} \frac{E\left(|(\mathbf{BA})_{pq}|^2\right)\rho_q(0)}{E\left(|(\mathbf{BA})_{pp}|^2\right)\rho_p(0)} && (41)\\ &= \sum_{p\neq q} E\left(|\delta_{pq}|^2\right)\frac{\rho_q(0)}{\rho_p(0)} && (42)\end{aligned}$$

Our performance analysis consists in deriving the closed-form expression of the asymptotical variance errors:

$$\lim_{T\to+\infty} T\, E\left(|\delta_{pq}|^2\right) \tag{43}$$

By using a similar approach to that in [7] based on the central-limit and continuity theorems, one obtains the following result (the proof is omitted due to space limitation).

Theorem 3 *Vector* $\boldsymbol{\delta}_{ij} \stackrel{\text{def}}{=} [\delta_{ij}\ \ \delta_{ji}]^T$ *is asymptotically Gaussian distributed with asymptotic covariance matrix*

$$\begin{aligned} \boldsymbol{\Delta}_{ij} &\stackrel{\text{def}}{=} \lim_{T\to+\infty} T\, E\left(\boldsymbol{\delta}_{ij}\boldsymbol{\delta}_{ij}^T\right) && (44)\\ &= \lim_{T\to+\infty} T \begin{bmatrix} E\left(\delta_{ij}^2\right) & E\left(\delta_{ij}\delta_{ji}\right) \\ E\left(\delta_{ji}\delta_{ij}\right) & E\left(\delta_{ji}^2\right)\end{bmatrix} && (45)\\ &= \boldsymbol{\mathcal{H}}_{ij}^{\#}\boldsymbol{\Psi}_{ij}\boldsymbol{\mathcal{H}}_{ij}^{\#T} && (46)\end{aligned}$$

where

$$\boldsymbol{\mathcal{H}}_{ij} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \otimes [\boldsymbol{\rho}_i\ \ \boldsymbol{\rho}_j] \tag{47}$$

$$\boldsymbol{\Psi}_{ij} = \begin{bmatrix} \boldsymbol{\Gamma}_{11}^{(ij)} & \boldsymbol{\Gamma}_{12}^{(ij)} \\ \boldsymbol{\Gamma}_{21}^{(ij)} & \boldsymbol{\Gamma}_{22}^{(ij)} \end{bmatrix} \tag{48}$$

with

$$\begin{aligned} \boldsymbol{\Gamma}_{11}^{(ij)}(k,k') &= \sum_{\tau\in\mathbb{Z}} r_{ii}(k+\tau)r_{jj}(k'+\tau) && (49)\\ \boldsymbol{\Gamma}_{22}^{(ij)}(k,k') &= \sum_{\tau\in\mathbb{Z}} r_{ii}(k'+\tau)r_{jj}(k+\tau) && (50)\\ \boldsymbol{\Gamma}_{12}^{(ij)}(k,k') &= \sum_{\tau\in\mathbb{Z}} r_{ii}(k+\tau)r_{jj}(k'-\tau) && (51)\\ r_{ii}(k) &= \rho_i(k) + \delta(k)\sigma^2\mathbf{b}_i\mathbf{b}_i^T && (52)\end{aligned}$$

and $\boldsymbol{\Gamma}_{21}^{(ij)} = \boldsymbol{\Gamma}_{12}^{(ij)T}$ *and* $\mathbf{b}_i$ *represents the* i^{th} *row of* $\mathbf{B} = \mathbf{A}^{\#}$.

7. SIMULATION RESULTS

We present here some numerical simulations to evaluate the performance of our algorithm. We consider in our simulation an array of $n = 4$ sensors receiving two signals in the presence of stationary real temporally white noise. The two source signals are generated by filtering real white Gaussian processes by an AR model of order 1 with coefficient $a_1 = 0.95$ and $a_2 = 0.50$ (except for Figure 4). The sources arrive from the directions $\theta_1 = 30$ and $\theta_2 = 45$ degree. The number of time lags is $K = 5$ (except for Figure 5). The signal to noise ratio is defined as $\text{SNR} = -10\log_{10}\sigma^2$, where σ^2 is the noise variance. The mean rejection level is estimated over 1000 Monte-Carlo runs.

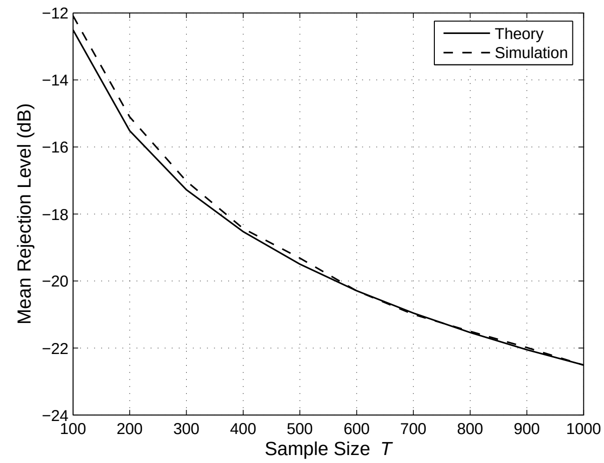

Figure 1: Mean Rejection Level in dB versus the sample size T for 2 autoregressive sources, 4 sensors and SNR=40dB.

In Figure 1, The mean rejection level $\mathcal{I}perf$ is plotted in dB against the sample size. The figure is for SNR = 40dB. This figure shows that the asymptotic closed form expressions of the rejection level are pertinent from snapshot length of about 100 samples. In the plots $E\left(\delta_{ij}^2\right)$ and $E\left(\delta_{ji}^2\right)$ are replaced by $\boldsymbol{\Delta}_{ij}(1,1)/T$ and $\boldsymbol{\Delta}_{ij}(2,2)/T$ respectively. This means that asymptotic conditions are reached for short data block size.

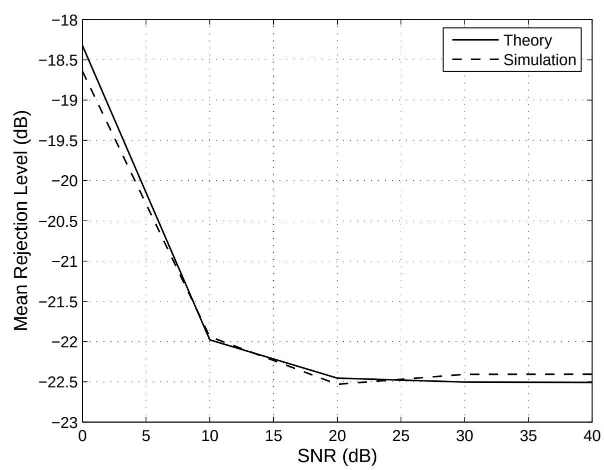

Figure 2: Mean Rejection Level in dB versus the SNR for 2 autoregressive sources, 4 sensors and T=1000.

Figure 2 shows the mean rejection level against the signal to noise ratio SNR. We compare the empirical performance with theoretical performance for $T = 1000$ sample size.
Figure 3 shows the mean rejection level versus the number of sensors using the theoretical formulation for $T = 1000$ sam-

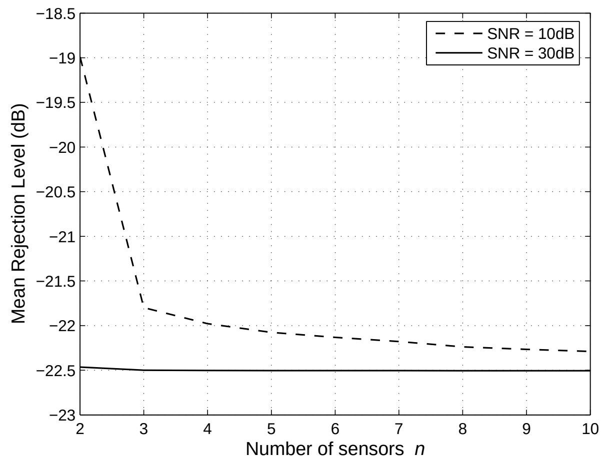

Figure 3: Mean Rejection Level in dB versus the number of sensors n for 2 autoregressive sources and T=1000.

ple size. We observe that, the greater the number of sensors, the lower the rejection level is in the low SNR case. For high SNRs the number of sensors has negligible effect on the separation performance.

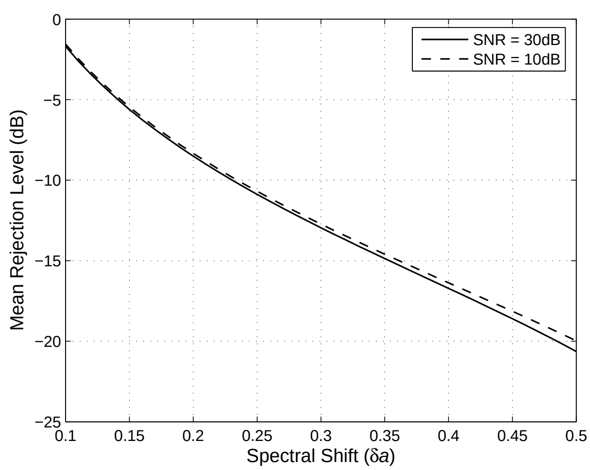

Figure 4: Mean Rejection Level in dB versus the spectral shift δa for 2 autoregressive sources, 4 sensors and T=1000.

Figure 4 shows $\mathcal{I}perf$ versus the spectral shift δa. the spectral shift δa represents the spectral overlap of the two sources. In this figure, the noise is assumed to be spatially white and its level is kept constant at 10dB and 30dB. We let $a_1 = 0.4$ and $a_2 = a_1 + \delta a$. The plot evidences a significant increase in rejection performance by increasing δa.
The plots in Figure 5 illustrate the effect of the number of time lags K for different SNRs. In this simulation the sources arrive from the directions $\theta_1 = 10$ and $\theta_2 = 13$ degree.

8. CONCLUSION

This paper presents a blind source separation method for temporally correlated stationary sources. An SOS-based contrast function is introduced and iterative algorithm based on relative gradient technique is proposed to minimize it and perform BSS. Generalization to adaptive and convolutive

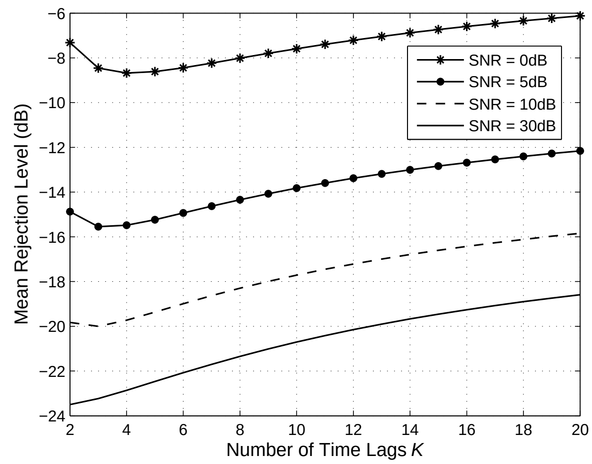

Figure 5: Mean Rejection Level in dB versus the number of time lags K for 2 autoregressive sources, 4 sensors and T=1000.

cases are discussed. A theoretical analysis of the asymptotic performance of the method has been derived. Numerical simulations have been performed to evidence the usefulness of the method and to support our theoretical performance study.

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