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BLIND IDENTIFICATION OF MIMO-OSTBC CHANNELS COMBINING SECOND AND HIGHER ORDER STATISTICS

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ABSTRACT

It has been recently shown that some multiple-input multiple-output (MIMO) channels under orthogonal space-time block coding (OSTBC) transmissions can not be unambiguously identified by only exploiting the second order statistics (SOS) of the received signal. This ambiguity, which is due to properties of the OSTBC, is traduced in the fact that the largest eigenvalue of the associated eigenvalue problem has multiplicity larger than one. Fortunately, for most OSTBCs that produce ambiguity, the multiplicity is two. This means that the channel estimate lies in a rank-2 subspace, which can be easily determined applying a first principal component analysis (PCA) step. To eliminate the remaining ambiguity, we propose to apply a constant modulus algorithm (CMA). This combined PCA+CMA approach provides an effective solution for the blind identification of those OSTBCs that can not be identified using only SOS. Some simulation results are presented to show the performance of the proposed method.

1. INTRODUCTION

In recent years, orthogonal space-time block coding (OSTBC) [1,2] has emerged as one of the most promising techniques to exploit spatial diversity and to combat fading in multiple-input multiple-output (MIMO) systems. The special structure of OSTBCs implies that, assuming that the MIMO channel is known at the receiver, the optimal maximum likelihood (ML) decoder is a simple linear receiver, which can be seen as a matched filter followed by a symbol-by-symbol detector.

Training approaches are typically used to obtain an estimate of the channel at the receiver. However, the use of a training sequence implies a reduction on the bandwidth efficiency, which is avoided by other approaches like differential space-time codes [3] or blind channel estimation techniques [4–6]. Specifically, the blind channel estimation method proposed in [6] is based only on second order statistics (SOS) and it has a reduced computational complexity. However, for most of the tested OSTBCs, there exists an identifiability problem when the number of receive antennas is $n_R = 1$, which is traduced in an eigenvalue problem with a largest eigenvalue of multiplicity two. The problem of blind identifiability of OSTBC channels has been recently studied in [7–9], but unfortunately, the identifiability conditions on the underlying structure of the OSTBCs remain unclear.

In this work we consider the idea of exploiting the higher order statistics (HOS) of the signal to resolve the ambiguity problems of SOS techniques. Specifically, we propose a two-stage technique. In the first stage, the channel subspace is reduced by exploiting the SOS of the received signal, which is done by means of the principal component extraction (APEX) algorithm [10]. In the second stage, the channel is estimated by applying the constant modulus algorithm (CMA) in the previously extracted channel subspace. Although the algorithm is based on the APEX and CMA, the same idea can be applied using other algorithms, for instance, the analytical constant modulus algorithm (ACMA) [11]. Finally, the performance of the proposed technique has been analyzed by means of numerical examples in situations with SOS-ambiguity, showing a better performance than methods based on the direct application of HOS techniques.

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2. SOME BACKGROUND ON OSTBCS

Throughout this paper we will use bold-faced upper case letters to denote matrices, e.g., $\mathbf{X}$, with elements x_{ij}; bold-faced lower case letters for column vector, e.g., $\mathbf{x}$, and light-face lower case letters for scalar quantities. The superscripts $(\cdot)^T$ and $(\cdot)^H$ denote transpose and Hermitian, respectively. The real and imaginary parts will be denoted as $\Re(\cdot)$ and $\Im(\cdot)$, and superscript $\hat{(\cdot)}$ will denote estimated matrices, vectors or scalars. The trace, range (or column space) and Frobenius norm of matrix $\mathbf{A}$ will be denoted as $\mathrm{Tr}(\mathbf{A})$, $\mathrm{range}(\mathbf{A})$ and $\|\mathbf{A}\|$ respectively. Finally, the identity matrix of the required dimensions will be denoted as $\mathbf{I}$, and $E[\cdot]$ will denote the expectation operator.

A flat fading MIMO system with n_T transmit and n_R receive antennas is assumed. The $n_T \times n_R$ complex channel matrix is

$$\mathbf{H} = [\mathbf{h}_1 \cdots \mathbf{h}_{n_R}] = \begin{bmatrix} h_{11} & \cdots & h_{1n_R} \\ \vdots & \ddots & \vdots \\ h_{n_T 1} & \cdots & h_{n_T n_R} \end{bmatrix},$$

where $\mathbf{h}_j = [h_{1j}, \ldots, h_{n_T j}]^T$ contains the channel responses associated with the j-th receive antenna. The complex noise at the receive antennas is considered both spatially and temporally white with variance σ^2.

2.1 Data Model for OSTBCs

Let us consider a space-time block code (STBC) transmitting M symbols during L slots and using n_T antennas at the transmitter side. The transmission rate is defined as $R = M/L$, and the number of real symbols transmitted in each block is

$$M' = \begin{cases} M & \text{for real constellations,} \\ 2M & \text{for complex constellations.} \end{cases}$$

For a STBC, the n-th block of data can be expressed as

$$\mathbf{S}[n] = \sum_{k=1}^{M'} \mathbf{C}_k s_k[n],$$

where $\mathbf{C}_k$ are the STBC code matrices,

$$s_k[n] = \begin{cases} \Re(r_k[n]), & k \leq M, \\ \Im(r_{k-M}[n]), & k > M, \end{cases}$$

and $r_k[n]$ denotes the k-th complex symbol of the n-th STBC block.

The combined effect of the STBC code and the j-th channel can be represented by means of the vectors

$$\mathbf{w}_k(\mathbf{h}_j) = \mathbf{C}_k \mathbf{h}_j, \qquad k = 1, \ldots, M',$$

and taking into account the isomorphism between complex vectors $\mathbf{w}_k(\mathbf{h}_j)$ and real vectors $\tilde{\mathbf{w}}_k(\mathbf{h}_j) = \left[\Re(\mathbf{w}_k(\mathbf{h}_j))^T, \Im(\mathbf{w}_k(\mathbf{h}_j))^T\right]^T$ we can define the extended code matrices

$$\tilde{\mathbf{C}}_k = \begin{bmatrix} \Re(\mathbf{C}_k) & -\Im(\mathbf{C}_k) \\ \Im(\mathbf{C}_k) & \Re(\mathbf{C}_k) \end{bmatrix},$$

which imply $\tilde{\mathbf{w}}_k(\mathbf{h}_j) = \tilde{\mathbf{C}}_k \tilde{\mathbf{h}}_j$, with $\tilde{\mathbf{h}}_j = \left[\Re(\mathbf{h}_j)^T, \Im(\mathbf{h}_j)^T\right]^T$. The signal at the j-th receive antenna is

$$\mathbf{y}_j[n] = \sum_{k=1}^{M'} \mathbf{w}_k(\mathbf{h}_j) s_k[n] + \mathbf{n}_j[n],$$

where $\mathbf{n}_j[n]$ is the white complex noise with variance σ^2.

Defining now the real vectors $\tilde{\mathbf{y}}_j[n] = [\Re(\mathbf{y}_j[n])^T, \Im(\mathbf{y}_j[n])^T]^T$ and $\tilde{\mathbf{n}}_j[n] = [\Re(\mathbf{n}_j[n])^T, \Im(\mathbf{n}_j[n])^T]^T$, the above equation can be rewritten as

$$\tilde{\mathbf{y}}_j[n] = \sum_{k=1}^{M'} \tilde{\mathbf{w}}_k(\mathbf{h}_j) s_k[n] + \tilde{\mathbf{n}}_j[n] = \tilde{\mathbf{W}}(\mathbf{h}_j)\mathbf{s}[n] + \tilde{\mathbf{n}}_j[n],$$

where $\mathbf{s}[n] = [s_1[n], \ldots, s_{M'}[n]]^T$ contains the M' transmitted real symbols and $\tilde{\mathbf{W}}(\mathbf{h}_j) = \left[\tilde{\mathbf{w}}_1(\mathbf{h}_j) \cdots \tilde{\mathbf{w}}_{M'}(\mathbf{h}_j)\right]$. Finally, stacking all the received signals into $\tilde{\mathbf{y}}[n] = \left[\tilde{\mathbf{y}}_1^T[n], \ldots, \tilde{\mathbf{y}}_{n_R}^T[n]\right]^T$, we can write

$$\tilde{\mathbf{y}}[n] = \tilde{\mathbf{W}}(\mathbf{H})\mathbf{s}[n] + \tilde{\mathbf{n}}[n],$$

where $\tilde{\mathbf{W}}(\mathbf{H}) = \left[\tilde{\mathbf{W}}^T(\mathbf{h}_1) \cdots \tilde{\mathbf{W}}^T(\mathbf{h}_{n_R})\right]^T$, and $\tilde{\mathbf{n}}[n]$ is defined analogously to $\tilde{\mathbf{y}}[n]$.

In the case of orthogonal STBCs (OSTBCs), the matrix $\tilde{\mathbf{W}}(\mathbf{H})$ satisfies

$$\tilde{\mathbf{W}}^T(\mathbf{H})\tilde{\mathbf{W}}(\mathbf{H}) = \|\mathbf{H}\|^2 \mathbf{I}, \tag{1}$$

which, considering $\mathbf{H}$ known and a Gaussian distribution for the noise, reduces the complexity of the ML receiver to find the closest symbols to the estimated signal [3]

$$\hat{\mathbf{s}}_{\mathrm{ML}}[n] = \frac{\tilde{\mathbf{W}}^T(\mathbf{H})\tilde{\mathbf{y}}[n]}{\|\mathbf{H}\|^2}.$$

The necessary and sufficient conditions on the code matrices $\mathbf{C}_k \in \mathbb{C}^{L \times n_T}$, $(k, l = 1, \ldots, M')$, to satisfy (1) are [3]

$$\mathbf{C}_k^H \mathbf{C}_l = \begin{cases} \mathbf{I} & k = l, \\ -\mathbf{C}_l^H \mathbf{C}_k & k \neq l, \end{cases}$$

which also imply $\mathbf{S}^H[n]\mathbf{S}[n] = \|\mathbf{s}[n]\|^2 \mathbf{I}$ and

$$\tilde{\mathbf{C}}_k^T \tilde{\mathbf{C}}_l = \begin{cases} \mathbf{I} & k = l, \\ -\tilde{\mathbf{C}}_l^T \tilde{\mathbf{C}}_k & k \neq l. \end{cases}$$

3. PREVIOUS WORK ON BLIND CHANNEL ESTIMATION UNDER OSTBC TRANSMISSIONS

In this section, we review some previously proposed techniques for blind channel estimation under OSTBC transmissions. These methods can be divided into two groups: those solely based on second order statistics (SOS) of the received signals (or subspace methods), and those which exploit the higher order statistics (HOS) of the transmitted signals. Furthermore, the channel identifiability problems for both SOS and HOS techniques are pointed out.

3.1 Blind Channel Estimation based on SOS

Several SOS techniques have been proposed for blind channel estimation [4–6]. These subspace methods are very attractive due to their closed-form structure, and their good performance for a sufficiently large number of data samples. On the other hand, SOS methods do not exploit the particular properties of the source signals, which can even imply identifiability problems.

Among the subspace methods, the method proposed in [4] is a general technique for STBCs, which does not exploit the special structure of OSTBCs, and the OSTBC techniques proposed in [5] and [6] are based on the relaxation of the finite alphabet property of $\mathbf{s}[n]$, and the direct extraction of $\hat{\mathbf{s}}[n]$ or $\hat{\mathbf{H}}$, respectively. Specifically, it has been shown that the method proposed in [6] is able to blindly identify the channel up to a real scalar ambiguity in most of the analyzed OSTBCs when the number of receive antennas is $n_R > 1$. This method is based on the maximization of the following problem

$$\underset{\hat{\mathbf{H}}}{\operatorname{argmax}} \operatorname{Tr}\left(\tilde{\mathbf{W}}^T(\hat{\mathbf{H}})\mathbf{R}_{\tilde{\mathbf{y}}}\tilde{\mathbf{W}}(\hat{\mathbf{H}})\right), \quad \text{s. t.} \quad \tilde{\mathbf{W}}^T(\hat{\mathbf{H}})\tilde{\mathbf{W}}(\hat{\mathbf{H}}) = \mathbf{I}, \tag{2}$$

where $\mathbf{R}_{\tilde{\mathbf{y}}}$ is the correlation matrix of the observations $\tilde{\mathbf{y}}[n]$

$$\mathbf{R}_{\tilde{\mathbf{y}}} = E[\tilde{\mathbf{y}}[n]\tilde{\mathbf{y}}^T[n]] = \tilde{\mathbf{W}}(\mathbf{H})\mathbf{R}_s\tilde{\mathbf{W}}^T(\mathbf{H}) + \frac{\sigma^2}{2}\mathbf{I},$$

and $\mathbf{R}_s = E[\mathbf{s}[n]\mathbf{s}^T[n]]$ is the correlation matrix of the information symbols. Finally, it can be proved in a straightforward manner [6] that the solution of (2) is given by any estimated channel matrix $\hat{\mathbf{H}}$ with $\|\hat{\mathbf{H}}\| = 1$ satisfying

$$\operatorname{range}(\tilde{\mathbf{W}}(\hat{\mathbf{H}})) = \operatorname{range}(\tilde{\mathbf{W}}(\mathbf{H})). \tag{3}$$

3.1.1 Equivalent PCA problem

Let us start rewriting $\operatorname{Tr}\left(\tilde{\mathbf{W}}^T(\hat{\mathbf{H}})\mathbf{R}_{\tilde{\mathbf{y}}}\tilde{\mathbf{W}}(\hat{\mathbf{H}})\right)$ as

$$\sum_{k=1}^{M'} \tilde{\mathbf{w}}_k^T(\hat{\mathbf{H}})\mathbf{R}_{\tilde{\mathbf{y}}}\tilde{\mathbf{w}}_k(\hat{\mathbf{H}}),$$

where $\tilde{\mathbf{w}}_k(\hat{\mathbf{H}}) = [\tilde{\mathbf{w}}_k^T(\hat{\mathbf{h}}_1), \ldots, \tilde{\mathbf{w}}_k^T(\hat{\mathbf{h}}_{M'})]^T$ is the k-th column of $\tilde{\mathbf{W}}(\hat{\mathbf{H}})$. Assuming that the MIMO channel remains constant during N OSTBC blocks, and defining $\tilde{\mathbf{Y}} = [\tilde{\mathbf{y}}[0] \cdots \tilde{\mathbf{y}}[N-1]]$, the finite sample estimate of $\mathbf{R}_{\tilde{\mathbf{y}}}$ can be obtained as $\hat{\mathbf{R}}_{\tilde{\mathbf{y}}} = \frac{1}{N}\tilde{\mathbf{Y}}\tilde{\mathbf{Y}}^T$. Using this estimate, and taking (1) into account, the channel identification criterion (2) can be rewritten as

$$\underset{\hat{\mathbf{H}}}{\operatorname{argmax}} \sum_{k=1}^{M'} \tilde{\mathbf{w}}_k^T(\hat{\mathbf{H}})\tilde{\mathbf{Y}}\tilde{\mathbf{Y}}^T\tilde{\mathbf{w}}_k(\hat{\mathbf{H}}), \quad \text{s.t.} \quad \|\hat{\tilde{\mathbf{h}}}\| = 1,$$

where $\hat{\tilde{\mathbf{h}}} = [\hat{\tilde{\mathbf{h}}}_1^T, \ldots, \hat{\tilde{\mathbf{h}}}_{n_R}^T]^T$. Finally, considering $\tilde{\mathbf{w}}_k(\hat{\mathbf{h}}_j) = \tilde{\mathbf{C}}_k\hat{\tilde{\mathbf{h}}}_j$, the above criterion yields

$$\underset{\hat{\tilde{\mathbf{h}}}}{\operatorname{argmax}}\, \hat{\tilde{\mathbf{h}}}^T\tilde{\mathbf{Z}}^T\tilde{\mathbf{Z}}\hat{\tilde{\mathbf{h}}} \qquad \text{s.t.} \qquad \|\hat{\tilde{\mathbf{h}}}\| = 1, \tag{4}$$

where the data matrix is defined as $\tilde{\mathbf{Z}} = [\tilde{\mathbf{Z}}[0]^T \cdots \tilde{\mathbf{Z}}[N-1]^T]^T$, and

$$\tilde{\mathbf{Z}}[n] = \begin{bmatrix} \tilde{\mathbf{y}}_1^T[n]\tilde{\mathbf{C}}_1 & \cdots & \tilde{\mathbf{y}}_{n_R}^T[n]\tilde{\mathbf{C}}_1 \\ \vdots & \ddots & \vdots \\ \tilde{\mathbf{y}}_1^T[n]\tilde{\mathbf{C}}_{M'} & \cdots & \tilde{\mathbf{y}}_{n_R}^T[n]\tilde{\mathbf{C}}_{M'} \end{bmatrix},$$

i.e., the criterion (2) is equivalent to a principal component analysis (PCA) problem. Assuming a Gaussian noise distribution, the relaxed ML estimate of the signal is

$$\hat{\mathbf{s}}[n] = \tilde{\mathbf{Z}}[n]\hat{\tilde{\mathbf{h}}}.$$

Finally, we must point out that the reformulation of the estimation method as a PCA problem permits a straightforward derivation of adaptive versions of the algorithm [6], for instance, by direct application of the Oja's rule [10, 12].

3.1.2 *Identifiability problems*

The constraint $\tilde{\mathbf{W}}^T(\hat{\mathbf{H}})\tilde{\mathbf{W}}(\hat{\mathbf{H}}) = \mathbf{I}$ in (2), which implies $\|\hat{\mathbf{H}}\| = 1$, introduces a real scalar ambiguity in the estimation process. This is a common indeterminacy for all the blind estimation techniques, then in the sequel we will assume $\|\hat{\mathbf{H}}\| = \|\mathbf{H}\| = 1$. A more important indeterminacy results from (3), which can be rewritten as $\tilde{\mathbf{W}}(\hat{\mathbf{H}}) = \tilde{\mathbf{W}}(\mathbf{H})\mathbf{Q}$, where $\mathbf{Q}$ is an orthogonal matrix (i.e., real and unitary) of dimensions $M' \times M'$. Therefore, it is easy to prove [9] that the indeterminacy problems are due to the properties of the code and the channel and not to the specific criterion (2). Furthermore, by exploiting the properties of the code matrices, we have proved in [9] that if there exist an indeterminacy problem, then we can find an spurious channel $\mathbf{H}^{\perp}$ (vectorized as $\tilde{\mathbf{h}}^{\perp}$), such that $\tilde{\mathbf{h}}^T\tilde{\mathbf{h}}^{\perp} = 0$, and

$$\tilde{\mathbf{W}}(\mathbf{H}^{\perp}) = \tilde{\mathbf{W}}(\mathbf{H})\mathbf{Q}, \tag{5}$$

where $\mathbf{Q}$ is an orthogonal skew-symmetric matrix (i.e., $\mathbf{Q}^T = -\mathbf{Q}$). The effect of this indeterminacy is that the largest eigenvalue of the PCA problem (4) has a multiplicity larger than one, which is equivalent to the eigenvalue multiplicity problems pointed out in [5] and [6]. In these cases, the channel can not be unambiguously extracted without exploiting the HOS of the signals or applying linear precoding techniques [6, 9, 12].

Unfortunately, although the identifiability conditions based on SOS have been studied in [7, 9], their relationship with the underlying structure of the OSTBCs remains unclear. However, the experimental results in [6] have shown that, for most of the indeterminacy cases, the multiplicity of the associated eigenvalue problem (4) is two, i.e., there only exists a unique orthogonal skew-symmetric matrix $\mathbf{Q}$ and channel $\mathbf{H}^{\perp}$ satisfying (5).

3.2 Blind Channel Estimation based on HOS

If the belonging of the source signals to a finite set $\mathscr{S}$ is not relaxed, and under Gaussian distributed noise, the maximum likelihood decoder amounts to minimize

$$\underset{\hat{\mathbf{H}};\hat{\mathbf{s}}[n]\in\mathscr{S}^{M'}}{\operatorname{argmin}} \sum_{n=0}^{N-1} \|\tilde{\mathbf{y}}[n] - \tilde{\mathbf{W}}(\hat{\mathbf{H}})\hat{\mathbf{s}}[n]\|^2,$$

which is a extremely challenging problem. Different solutions to this problem have been presented in [5, 13]. Specifically, the suboptimal method proposed in [5], which is called the *cyclic ML*, is based on alternating minimizations over the channel and the signal estimates, whereas the techniques in [13] assume BPSK or QPSK source signals and are based on a semidefinite relaxation (SDR) approach (suboptimal) or the sphere decoder (optimal). However, the computational complexity of these algorithms remains prohibitive for adaptive processing.

In [14], the application of the well-known constant modulus algorithm (CMA) to blind channel estimation in OSTBC systems has been proposed. Firstly, the direct application of the CMA to the observations $\tilde{\mathbf{y}}[n]$ ensures the existence of M' local minima given by $\tilde{\mathbf{w}}_k(\mathbf{H})$, for $k = 1, \ldots, M'$. Secondly, taking into account the relationship between the channel $\mathbf{H}$ and the equalizers $\tilde{\mathbf{W}}(\mathbf{H})$, the authors propose a CMA problem over the channel estimate $\hat{\tilde{\mathbf{h}}}$, ensuring that the true channel $\tilde{\mathbf{h}}$ constitutes one local minimum of the CMA cost function. However, since the CMA can converge to any local minimum, there still exists an indeterminacy problem which is solved in [14] by including some pilot symbols.

3.2.1 *Identifiability problems*

Analogously to the SOS case, the identifiability of the channel exploiting the HOS of the source signals depends on the OSTBC, the channel, and the specific characteristics of the information signals. In [8] the authors study the identifiability conditions under the assumptions of real OSTBCs and BPSK signals, introducing the definition of non-rotatable and strictly non-rotatable codes. Here, we must note that, assuming that all the source signals belong to a finite alphabet $\mathscr{S}$, and considering a number of received blocks sufficiently large [15], the indeterminacy problem is reduced to the existence of a spurious channel $\mathbf{H}^{\perp}$ satisfying (5), where in this case, $\mathbf{Q}$ must be not only orthogonal and skew-symmetric, but also a permutation matrix (with only a nonzero element in each row and column), for instance

$$\mathbf{Q} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}.$$

Then, as it was expected, we can deduce that the indeterminacy conditions in the HOS case are more restrictive than that of the SOS case ($\mathbf{Q}$ must also be a permutation matrix). Then, many of the ambiguity problems of SOS methods can be avoided by exploiting the HOS. The HOS-indeterminacy problem is still present in a small set of well-known OSTBCs (see the table of non-rotatable codes in [8]), including the Alamouti code [1].

4. PROPOSED TWO-STAGE CHANNEL ESTIMATION ALGORITHM

The empirical results in [6] have shown that, for most of the tested OSTBCs, the SOS-ambiguity problem can be avoided by using $n_R > 1$ receive antennas. However, when the number of receive antennas is $n_R = 1$, most of the OSTBCs exhibit an ambiguity problem which is traduced in the largest eigenvalue of the associated problem ((4) in our case) having multiplicity two. In this section we propose a new technique which exploits the HOS of the source signals in order to avoid this indeterminacy problem. Equivalently, the dimension of the channel subspace is reduced by means of SOS, which implies a lower complexity than the direct application of the HOS algorithm. Specifically, the proposed method extracts the two main eigenvectors of the PCA problem in (4), which constitute an orthogonal basis for the channel, and finally, we apply the CMA algorithm to the principal components to obtain the correct channel estimate.

Although the algorithm has been developed considering an eigenvalue multiplicity of two (which is justified by the experimental results in [6]) and a CMA, the same idea can be applied, in a straightforward manner, to other multiplicity orders or HOS algorithms.

4.1 First Stage: PCA - APEX Algorithm

The first stage of the proposed algorithm reduces to obtain the two main eigenvectors of the PCA problem in (4). This can be done by the direct solution of the eigenvalue problem or by means of the APEX algorithm [10]. Considering, without loss of generality, a noise-free case, the two main eigenvectors $\tilde{\mathbf{g}}_1$ and $\tilde{\mathbf{g}}_2$ of the PCA problem (4) form an orthogonal basis for the correct and the spurious channels $\tilde{\mathbf{h}}$ and $\tilde{\mathbf{h}}^{\perp}$, i.e.

$$\tilde{\mathbf{G}} = \begin{bmatrix}\tilde{\mathbf{h}} & \tilde{\mathbf{h}}^{\perp}\end{bmatrix}\mathbf{V}^T,$$

where $\tilde{\mathbf{G}} = [\tilde{\mathbf{g}}_1 \quad \tilde{\mathbf{g}}_2]$ and $\mathbf{V}$ is an orthogonal matrix. The direct application of the APEX algorithm yields, for $k = 1, \ldots, M'$, the following update rules

$$\hat{u}_{k1}[n] = \tilde{\mathbf{z}}_k^T[n]\hat{\tilde{\mathbf{g}}}_1, \tag{6}$$

$$\hat{u}_{k2}[n] = \tilde{\mathbf{z}}_k^T[n]\hat{\tilde{\mathbf{g}}}_2 - a\hat{u}_{k1}[n], \tag{7}$$

$$\hat{\tilde{\mathbf{g}}}_l = \hat{\tilde{\mathbf{g}}}_l + \mu_{\text{APEX}}\left(\hat{u}_{kl}[n]\tilde{\mathbf{z}}_k[n] - \hat{u}_{kl}^2[n]\hat{\tilde{\mathbf{g}}}_l\right), \quad l = 1, 2, \tag{8}$$

$$a = a + \mu_{\text{APEX}}\left(\hat{u}_{k1}[n] - a\hat{u}_{k2}[n]\right)\hat{u}_{k2}[n], \tag{9}$$

Initialize $\hat{\tilde{\mathbf{g}}}_1 \neq \hat{\tilde{\mathbf{g}}}_2 \neq \mathbf{0}$ ($\hat{\tilde{\mathbf{g}}}_1^T \hat{\tilde{\mathbf{g}}}_2 = 0$), $\hat{\mathbf{f}} \neq \mathbf{0}$, and $\hat{\mathbf{f}}^{\perp}$ with (10).
Initialize $a = e = e^{\perp} = 0$, and select μ_{APEX}, μ_{CMA}, μ_e.
for n=1,2,... **do**
  for k=1,...,M' **do**
    APEX Algorithm
    Obtain $\hat{u}_{k1}[n]$ and $\hat{u}_{k2}[n]$ with (6) and (7).
    Update $\hat{\tilde{\mathbf{g}}}_1$, $\hat{\tilde{\mathbf{g}}}_2$ and a with (8) and (9).
    CMA
    Obtain the CMA output $\hat{s}_k[n] = [\hat{u}_{k1}[n], \hat{u}_{k2}[n]]\,\hat{\mathbf{f}}$.
    Obtain the orthogonal output $\hat{s}_k^{\perp}[n] = [\hat{u}_{k1}[n], \hat{u}_{k2}[n]]\,\hat{\mathbf{f}}^{\perp}$.
    Obtain $\hat{e}$ and $\hat{e}^{\perp}$ with (11) and (12).
    if $\hat{e} > \hat{e}^{\perp}$ **then**
      Interchange $\hat{e} \rightleftarrows \hat{e}^{\perp}$, $\hat{s}_k[n] \rightleftarrows \hat{s}_k^{\perp}[n]$ and update $\hat{\mathbf{f}} = \hat{\mathbf{f}}^{\perp}$.
    end if
    Update $\hat{\mathbf{f}}$ with (13) and obtain $\hat{\mathbf{f}}^{\perp}$ with (10).
    Obtain the channel estimate $\hat{\tilde{\mathbf{h}}} = \hat{\tilde{\mathbf{G}}}\hat{\mathbf{f}}$.
  end for
end for

Algorithm 1: Blind OSTBC channel estimation algorithm based on APEX-CMA.

where $\tilde{\mathbf{z}}_k[n] = \left[\tilde{\mathbf{y}}_1^T[n]\tilde{\mathbf{C}}_k, \ldots, \tilde{\mathbf{y}}_{n_R}^T[n]\tilde{\mathbf{C}}_k\right]^T$ is the input signal, $\hat{u}_{k1}[n]$, $\hat{u}_{k2}[n]$ are the principal components, a is the APEX deflation weight, and μ_{APEX} is the step-size.

4.2 Second Stage: HOS - CMA

Taking into account that since $\mathbf{Q}$ is skew-symmetric, its diagonal elements are zero, and considering independent source signals $s_k[n]$, it can be easily seen that the elements $s_k[n] = \tilde{\mathbf{z}}_k^T[n]\tilde{\mathbf{h}}$ and $s_k^{\perp}[n] = \tilde{\mathbf{z}}_k^T[n]\tilde{\mathbf{h}}^{\perp}$ are independent. This fact can be exploited in order to extract the information signal by means of the CMA. The CMA cost function is

$$J_{\text{CMA}}(\hat{\mathbf{f}}) = E\left[\left(\left|\tilde{\mathbf{z}}_k^T[n]\hat{\tilde{\mathbf{h}}}\right|^2 - \gamma\right)^2\right],$$

where the channel estimate is obtained as a linear combination of the two principal eigenvectors, i.e., $\hat{\tilde{\mathbf{h}}} = \tilde{\mathbf{G}}\hat{\mathbf{f}}$ and $\gamma = E[|s_k[n]|^4]/E[|s_k[n]|^2]$ is the CMA dispersion constant. Here, we must note that, since $\mathbf{V}$ is an orthogonal matrix, the local minima of the CMA cost function are given by the columns of $\mathbf{V}$ [14]. The existence of two local minima represents a new ambiguity problem, which is due to the CMA and not to the OSTBC. In order to solve this problem we propose to test the signals estimated by means of $\hat{\tilde{\mathbf{h}}} = \tilde{\mathbf{G}}\hat{\mathbf{f}}$ and its orthogonal complement $\hat{\tilde{\mathbf{h}}}^{\perp} = \tilde{\mathbf{G}}\hat{\mathbf{f}}^{\perp}$, where $\hat{\mathbf{f}}^{\perp}$ is easily obtained as

$$\hat{\mathbf{f}}^{\perp} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \hat{\mathbf{f}}. \tag{10}$$

Specifically, we form the error measures $e = E\left[\left(f\left(\hat{s}_k[n]\right) - \hat{s}_k[n]\right)^2\right]$ and $e^{\perp} = E\left[\left(f\left(\hat{s}_k^{\perp}[n]\right) - \hat{s}_k^{\perp}[n]\right)^2\right]$, which are adaptively updated as

$$\hat{e} = (1-\mu_e)\hat{e} + \mu_e\left(f\left(\hat{s}_k[n]\right) - \hat{s}_k[n]\right)^2, \tag{11}$$

$$\hat{e}^{\perp} = (1-\mu_e)\hat{e}^{\perp} + \mu_e\left(f\left(\hat{s}_k^{\perp}[n]\right) - \hat{s}_k^{\perp}[n]\right)^2, \tag{12}$$

where $\hat{s}_k[n] = \tilde{\mathbf{z}}_k^T[n]\hat{\tilde{\mathbf{h}}}$ and $\hat{s}_k^{\perp}[n] = \tilde{\mathbf{z}}_k^T[n]\hat{\tilde{\mathbf{h}}}^{\perp}$ are the estimated signals at the CMA output, $f(\cdot)$ denotes the decision function over the constellation symbols, and μ_e is a smoothing parameter. In this way, if the error measure $\hat{e}$ is higher than $\hat{e}^{\perp}$ we deduce that the CMA is converging to the spurious local minimum and $\hat{\tilde{\mathbf{h}}}$, $\hat{\mathbf{f}}$, $\hat{s}_k[n]$ and $\hat{e}$ are interchanged with $\hat{\tilde{\mathbf{h}}}^{\perp}$, $\hat{\mathbf{f}}^{\perp}$, $\hat{s}_k^{\perp}[n]$ and $\hat{e}^{\perp}$. Finally, considering a step-size μ_{CMA}, the CMA updating rule is

$$\hat{\mathbf{f}} = \hat{\mathbf{f}} + \mu_{\text{CMA}}\left(\gamma - \hat{s}_k^2[n]\right)[\hat{u}_{k1}[n], \hat{u}_{k2}[n]]^T\,\hat{s}_k[n], \tag{13}$$

and the overall algorithm is summarized in Algorithm 1.

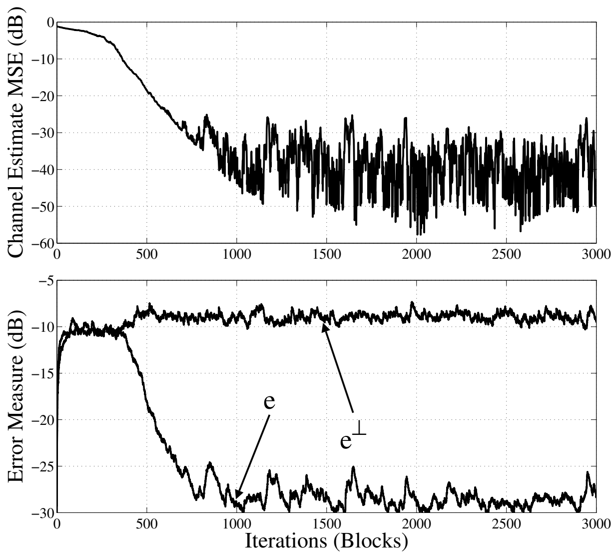

Figure 1: Channel estimate MSE and error measures. SNR=30dB.

5. SIMULATION RESULTS

In this section the performance of the proposed method is evaluated through two simulation examples. In all the simulations, the i.i.d source signal belongs to a 16-QAM constellation. The observations are affected by zero-mean, circular, complex Gaussian noise with variance σ^2. We have tested the 3/4 OSTBC code for $M = 3$ complex symbols, $L = 4$ time slots and $n_T = 4$ transmit antennas, which is presented in Eq. (7.4.10) of [3]. The number of receive antennas is $n_R = 1$, which provokes an ambiguity problem if the channel estimation is solely based on SOS [6]. Specifically, the multiplicity of the largest eigenvalue of the PCA problem in (4) is two.

In the first example we have tested the performance of the proposed algorithm in a deterministic situation. We have transmitted OSTBC blocks by a channel $\tilde{\mathbf{h}} = [-0.16, -0.62, 0.05, 0.43, 0.44, 0.44]$, with a received SNR=30dB. The learning rates are $\mu_{\text{APEX}} = 0.02$, $\mu_{\text{CMA}} = 0.02$ and $\mu_e = 0.01$. Figure 1 shows the evolution of the channel estimate MSE and the error measures $\hat{e}$ and $\hat{e}^{\perp}$, and Fig. 2 shows the evolution of the estimated channel coefficients and the real and imaginary parts of the estimated symbols. The error measure curves show that after 500 iterations the CMA converges to the correct vector $\mathbf{f}$, as we can see in Fig 2. Furthermore, the evolution of the estimated channel coefficients shows the effect of the interchanges between $\hat{\tilde{\mathbf{h}}}$ and $\hat{\tilde{\mathbf{h}}}^{\perp}$ during the first 500 iterations. The large number of transitions can be reduced by decreasing μ_e.

In the second example we have tested a realistic situation, where the elements of the flat fading MIMO channels are zero-mean, circular, complex Gaussian random variables with variance $\sigma_{\mathbf{H}}^2$, the averaged transmitted energy per antenna and time interval is $1/n_T$, and the SNR at the transmitter side is defined as $10\log_{10}(\sigma_{\mathbf{H}}^2/\sigma^2)$. We have tested the proposed algorithm and the direct application of the CMA over the observations $\tilde{\mathbf{y}}$, which is the first approximation proposed in [14]. The algorithm parameters have been selected as $\mu_{\text{APEX}} = 0.2$, $\mu_{\text{CMA}} = 0.05$ and $\mu_e = 0.001$.

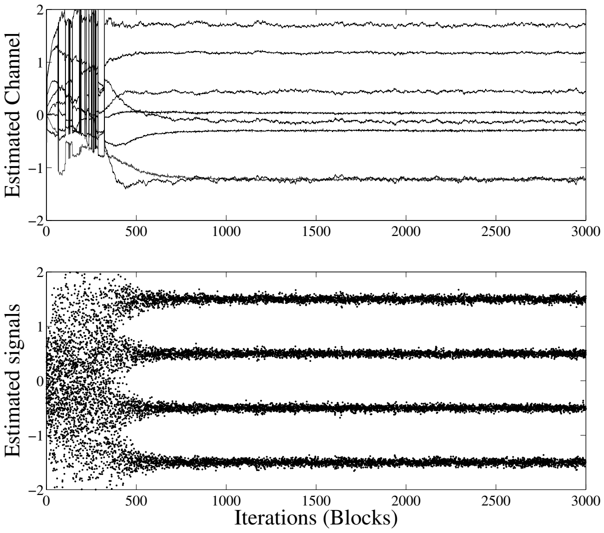

Figure 2: Evolution of the estimated channel coefficients and signal. SNR=30dB.

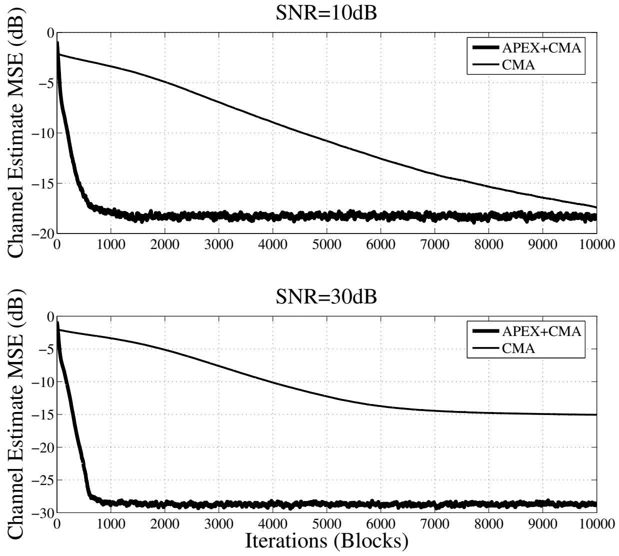

Figure 3: Performance of the proposed algorithm with a Rayleigh fading channel for two different SNRs.

In the case of the direct application of the CMA, the learning rate is $\mu = 0.005$, and in order to compare with the proposed method, the ambiguity on the estimated vector $\hat{\tilde{\mathbf{w}}}_k$ has been solved using 50 blocks of training symbols. The results of 300 independent realizations for two different SNRs have been averaged, and they are shown in Fig. 3, where we can see that the proposed method (labeled as APEX+CMA) outperforms the technique proposed in [14] (labeled as CMA).

6. CONCLUSIONS

In this paper, the problem of blind identifiability of MIMO channels under OSTBC transmissions has been analyzed. We have presented the identifiability problems when the channel estimation is based solely on second order statistics (SOS), and we have exploited the higher order statistics (HOS) of the transmitted signals to resolve the ambiguity. Specifically, we propose a two-stage algorithm which firstly reduce the subspace of possible channels by exploiting the SOS, and then apply a HOS technique. Although the proposed algorithm is based on the adaptive principal component extraction (APEX) algorithm and the constant modulus algorithm (CMA), the same idea can be exploited with other SOS and HOS techniques. Finally, we have shown by means of numerical examples that the combination of SOS and HOS provides better performance than the direct application of HOS techniques.

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