ITERATIVE TRANSCEIVER OPTIMIZATION FOR LINEAR MULTIUSER MIMO CHANNELS WITH PER-USER MMSE REQUIREMENTS

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ABSTRACT

We address the problem of jointly optimizing linear transmit and receive filters for a multi-user MIMO system, under the assumption that all users have individual Minimum-Mean-Square-Error (MMSE) requirements. Each user can perform spatial multiplexing with several data streams (layers). All users and layers are coupled by interference, so the choice of filters is intricately interwoven with the power allocation strategy. The design goal is to minimize the total power subject to MMSE constraints. This results in a non-convex problem formulation, for which we propose an iterative algorithm. The iteration consists of alternating optimization of powers, transmit filters and receive filters. We prove that the total required power obtained by the algorithm is monotonically decreasing and converges to a limit.

1. INTRODUCTION

MMSE estimation and equalization plays an important role in approaching the information-theoretic limits of linear Gaussian channels [1]. Recently, it was shown in [2] that the derivative of the mutual information (nats) with respect to the signal-to-noise ratio (SNR) is equal to half the MMSE, regardless of the input statistics. Moreover, it has been shown in [3] that the interference resulting from MMSE equalization can be considered as Gaussian distributed with zero mean for large numbers of transmit and receiver antennas. This makes it easy to derive other performance measures, such as BER, spectral efficiency, etc.

MSE optimization problems have been studied widely in the context of multipoint-to-point transmission (uplink). Point-to-multipoint transmission (downlink), is more difficult to handle since all users are coupled by the transmit filters and powers. By setting up equivalent uplink and downlink channels, it has been shown that under a total power constraint, any MSE point which can be achieved in the uplink, can be achieved in the downlink as well [4, 5]. Thus, the complicated downlink problems can be solved by focusing on the equivalent uplink problems. Furthermore, this duality makes it possible to optimize the transceivers in the uplink and the downlink in an alternating manner, so that computationally-efficient algorithms for transceiver design can be exploited [6].

In this paper, we consider a multi-user MIMO wireless communication system, where K independent receivers are each equipped with M_t^k antennas and the base station with M_r antennas. The transceiver design for sum-MSE minimization was studied in [7, 8, 9] for uplink transmission and in [5, 10] for downlink transmission. For multiuser transmissions, sum-MSE optimization is unfair to users with bad channel states. Therefore, it is also interesting to guarantee certain MSE requirements. The transceivers are optimized with respect to the following criterion.

$$\min P_{tot} \quad s.t. \ MSE_k \le \epsilon_k, \ \forall k, \tag{1}$$

where P_{tot} is the total required transmit power and ϵ_k is the MSE requirement of the kth user.

We propose an iterative algorithm for problem (1), assuming that the targets are feasible. Each iteration consists of optimizing the powers and filters in the uplink and downlink alternately. We prove that the total required power obtained by the algorithm decreases monotonically and converges. It is observed that the initial values have no effect on the total required power. Based on the dual region of the achievable MSE values, the proposed algorithm is not only suitable for the downlink transmission, but also for the uplink transmission.

2. UPLINK SUM-POWER MINIMIZATION WITH FIXED TRANSMITTERS

In this section, we assume fixed transmitters and propose an iterative algorithm which yields the power allocation that achieves given MSE targets $\epsilon_1, \dots, \epsilon_K$ with minimum total power. The joint optimization of transmitters and receivers will be studied later in Section 4.

2.1 Uplink Channel Model

Consider the uplink MIMO channel depicted in Fig 1, where

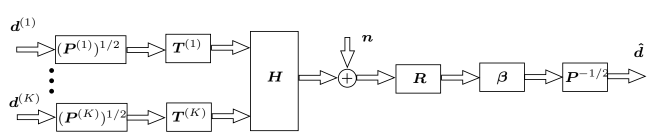

Figure 1: Uplink MIMO channel

the kth transmitter is equipped with M_t^k antennas, and the receiver has M_r antennas. The channel matrix is $\boldsymbol{H} = [\boldsymbol{H}_1, ..., \boldsymbol{H}_K]$, where $\boldsymbol{H}_k$ models the channel between the kth user and the base station. Assume that a data vector $\boldsymbol{d}^{(k)}$ of dimension $M_k \le M_t^k$ with independent, unity-power components, i.e., $\mathrm{E}\{\boldsymbol{d}^{(k)}(\boldsymbol{d}^{(k)})^H\} = \boldsymbol{I}$, is transmitted from the kth mobile to the base station. The total number of the transmit data streams is $N_d = \sum_{k=1}^K M_k$. Zero-mean white Gaussian noise $\boldsymbol{n} \sim \mathscr{N}(0, \sigma_n^2 \boldsymbol{I})$ is considered. The data and the noise are statistically independent. The transmit filters $\boldsymbol{T}^{(k)}$, $k = 1, ..., K$, have unity-norm columns. The overall power allocation over all layers (data streams) is a diagonal matrix $\boldsymbol{P} = \mathrm{diag}\{\boldsymbol{P}^{(1)},...,\boldsymbol{P}^{(K)}\}$. The total transmit power of the kth user is $p_k = \mathrm{trace}\{\boldsymbol{P}^{(k)}\}$. The powers of all users are collected in a vector $\boldsymbol{p} = [p_1,...,p_K]^T$. The receive filter $\boldsymbol{R} = [(\boldsymbol{R}^{(1)})^T,...,(\boldsymbol{R}^{(K)})^T]^T$ has unity-norm rows. We introduce $\widetilde{\boldsymbol{R}} = \boldsymbol{\beta}\boldsymbol{R}$, where $\boldsymbol{\beta} = \mathrm{diag}\{\boldsymbol{\beta}^{(1)},...,\boldsymbol{\beta}^{(K)}\}$ is a diagonal matrix. We collect all the transmit filters in a block diagonal matrix $\boldsymbol{T} = \mathrm{diag}\{\boldsymbol{T}^{(1)},...,\boldsymbol{T}^{(K)}\}$. The estimated symbol vector for the kth user is given by

$$\hat{\boldsymbol{d}}^{(k)} = (\boldsymbol{P}^{(k)})^{-1/2}\boldsymbol{\beta}^{(k)}\boldsymbol{R}^{(k)}(\sum_{l=1}^{K}\boldsymbol{H}_l\boldsymbol{T}^{(l)}(\boldsymbol{P}^{(l)})^{1/2}\boldsymbol{d}_l + \boldsymbol{n}). \quad (2)$$

It is known that with any fixed $\boldsymbol{P}$ and $\boldsymbol{T}$, for linear signal processing the MSE values of all users are minimized independently by an MMSE filters

$$\begin{aligned}\widetilde{\boldsymbol{R}}^{(k)} &= \boldsymbol{\beta}^{(k)}\boldsymbol{R}^{(k)} \\ &= \boldsymbol{P}^{(k)}(\boldsymbol{T}^{(k)})^H(\boldsymbol{H}_k)^H(\sum_{l=1}^{K}\boldsymbol{H}_l\widetilde{\boldsymbol{W}}_l\boldsymbol{H}_l^H + \sigma_n^2\boldsymbol{I})^{-1}, \forall k, \quad (3)\end{aligned}$$

where $\widetilde{\boldsymbol{W}}_k = \boldsymbol{T}^{(k)}\boldsymbol{P}^{(k)}(\boldsymbol{T}^{(k)})^H$, $\forall k$, are transmit covariances. With (2) and (3), the minimum MSE of the kth user can be expressed as

$$\begin{aligned}MSE_k &= E\{\|\boldsymbol{d}^{(k)} - \hat{\boldsymbol{d}}^{(k)}\|^2\} \\ &= M_k - \mathrm{trace}\Big\{\boldsymbol{H}_k\widetilde{\boldsymbol{W}}_k\boldsymbol{H}_k^H(\sum_{l=1}^{K}\boldsymbol{H}_l\widetilde{\boldsymbol{W}}_l\boldsymbol{H}_l^H + \sigma_n^2\boldsymbol{I})^{-1}\Big\}. \quad (4)\end{aligned}$$

We define

$$\begin{aligned}f_k(\boldsymbol{p},\sigma_n^2) &= \mathrm{trace}\Big\{\boldsymbol{H}_k\boldsymbol{W}_k\boldsymbol{H}_k^H(\sum_{l=1}^{K}p_l\boldsymbol{H}_l\boldsymbol{W}_l\boldsymbol{H}_l^H + \sigma_n^2\boldsymbol{I})^{-1}\Big\} \\ &= f_k(\mathfrak{p}), \quad (5)\end{aligned}$$

where $\boldsymbol{W}_k$, $\forall k$, are normalized transmit covariances with powers $\mathrm{trace}\{\boldsymbol{W}_k\} = 1$, and $\mathfrak{p} = [\mathfrak{p}_1,...,\mathfrak{p}_K,\mathfrak{p}_{K+1}] := [p_1,...,p_K,\sigma_n^2]$. Thus, the MSE value of the kth user is given by

$$MSE_k = M_k - p_k f_k(\boldsymbol{p},\sigma_n^2). \quad (6)$$

In the next section we show that the function $f_k(\boldsymbol{p},\sigma_n^2)$ fulfills the properties of monotonicity and scalability. This facilitates an efficient iterative algorithm to find the optimal power allocation $\boldsymbol{p}$, which achieves the MSE targets ϵ_k, $\forall k$.

2.2 Monotonicity and Scalability

Theorem 1. *The function $f_k(\mathfrak{p})$, as defined in (5), fulfills the following properties:*

P1: $f_k(\mathfrak{p})$ is non-negative on $\mathbb{R}_+^K$

P2: $f_k(\mu\mathfrak{p}) = \mu f_k(\mathfrak{p})$ for $\mu \in \mathbb{R}$

P3: $f_k(\mathfrak{p}^{(1)}) \leq f_k(\mathfrak{p}^{(2)})$, if $\mathfrak{p}^{(1)} \geq \mathfrak{p}^{(2)}$

P4: $f_k(\mathfrak{p}^{(1)}) < f_k(\mathfrak{p}^{(2)})$, if $\mathfrak{p}_{K+1}^{(1)} > \mathfrak{p}_{K+1}^{(2)}$ and $\mathfrak{p}^{(1)} \geq \mathfrak{p}^{(2)}$

Proof. P1 holds since the trace of a positive semidefinite matrix is non-negative.

P2 also follows immediately from inspection of (5).

In order to show P3, define $\boldsymbol{A} := \boldsymbol{H}_k\boldsymbol{W}_k\boldsymbol{H}_k^H$ and let $\mathfrak{p}^{(1)} \geq \mathfrak{p}^{(2)}$, thus there exists a $\hat{\mathfrak{p}} \geq 0$ such that $\mathfrak{p}^{(1)} = \mathfrak{p}^{(2)} + \hat{\mathfrak{p}}$. We define $\boldsymbol{C}(\mathfrak{p}) := \sum_{l=1}^{K}\mathfrak{p}_l\boldsymbol{H}_l\boldsymbol{W}_l\boldsymbol{H}_l^H + \mathfrak{p}_{K+1}\boldsymbol{I}$. Let $\boldsymbol{C}_1 := \boldsymbol{C}(\mathfrak{p}^{(1)})$, $\boldsymbol{C}_2 := \boldsymbol{C}(\mathfrak{p}^{(2)})$, and $\hat{\boldsymbol{C}} := \boldsymbol{C}(\hat{\mathfrak{p}})$ for short, then $\boldsymbol{C}_1 = \boldsymbol{C}_2 + \hat{\boldsymbol{C}}$. Using the *Sherman-Morrison-Woodbury* formula for matrix inversion [11], we have

$$\begin{aligned}f_k(\mathfrak{p}^{(1)}) &= \mathrm{trace}\Big\{\boldsymbol{A}(\boldsymbol{C}_2 + \hat{\boldsymbol{C}})^{-1}\Big\} \\ &= \mathrm{trace}\Big\{\boldsymbol{A}\boldsymbol{C}_2^{-1} - \boldsymbol{A}(\boldsymbol{C}_2\hat{\boldsymbol{C}}^{-1}\boldsymbol{C}_2 + \boldsymbol{C}_2)^{-1}\Big\} \\ &\leq \mathrm{trace}\Big\{\boldsymbol{A}\boldsymbol{C}_2^{-1}\Big\} = f_k(\mathfrak{p}^{(2)}) . \quad (7)\end{aligned}$$

Inequality (7) holds because the covariance matrices $\boldsymbol{A}$, $\hat{\boldsymbol{C}}$, and $\boldsymbol{C}_2$ are all Hermitian and positive semi-definite (p.s.d.). It can be shown that $\boldsymbol{C}_2\hat{\boldsymbol{C}}^{-1}\boldsymbol{C}_2$ is Hermitian and p.s.d. as well. The inverse of a sum of p.s.d. matrices is p.s.d. Since the product of two positive definite matrices has only positive eigenvalues [11, p.465], the trace is always positive.

In order to prove P4, we define $\boldsymbol{B} := \sum_l p_l\boldsymbol{H}_l\boldsymbol{W}_l\boldsymbol{H}_l^H$. Since $\boldsymbol{B} = \boldsymbol{B}^H$, there exists a decomposition $\boldsymbol{B} = \boldsymbol{U}\Sigma\boldsymbol{U}^H$, with $\Sigma = \mathrm{diag}\{\lambda_1,\ldots,\lambda_{M_r}\} \geq 0$, and $\boldsymbol{B}^{-1} = \boldsymbol{U}\Sigma^{-1}\boldsymbol{U}^H$. We have

$$\begin{aligned}f_k(\mathfrak{p}) &= \mathrm{trace}\Big\{\boldsymbol{A}\boldsymbol{U}(\Lambda + \sigma_n^2\boldsymbol{I})^{-1}\boldsymbol{U}^H\Big\} \\ &= \mathrm{trace}\Big\{\boldsymbol{U}^H\boldsymbol{A}\boldsymbol{U}(\Lambda + \sigma_n^2\boldsymbol{I})^{-1}\Big\} \\ &= \sum_{l=1}^{M_r}\frac{a_l}{\lambda_l + \sigma_n^2} , \quad (8)\end{aligned}$$

where $a_l \geq 0$ is the lth diagonal entry of $\boldsymbol{U}^H\boldsymbol{A}\boldsymbol{U}$. Property P4 follows from (8).

Notice, that P3 and P4 can also be shown by using the framework of matrix-monotone functions.[1] □

2.3 Optimal Power Allocation

The optimization goal is to achieve the targets with minimum total power, i.e.,

$$\min_{\boldsymbol{p}}\sum_l \boldsymbol{p}_l \quad \text{s.t.} \quad MSE_k \leq \epsilon_k, \forall k. \quad (9)$$

From (6), we know that MSE targets ϵ_k are fulfilled if

$$p_k f_k(\boldsymbol{p},\sigma_n^2) \geq M_k - \epsilon_k.$$

The following theorem gives an iterative algorithm which solves the problem of (9).

Theorem 2. *The following iteration converges component-wise to the unique optimizer of the power minimization problem (9)*

$$p_k^{(n+1)} = (M_k - \epsilon_k)/f_k(\boldsymbol{p}^{(n)},\sigma_n^2). \quad (10)$$

Proof. Since the function $f_k(\mathfrak{p})$ fulfills the properties P1-P4. Thus, iteration (10) can be seen as a special case of the generic fixed-point iteration proposed in [12]. See also [13]. □

3. DOWNLINK POWER ALLOCATION AND DUALITY

Consider the downlink system depicted in Fig. 2, which is obtained by switching the role of the normalized transmit and receiver filters in Section 2.1. The reason is that we want to exploit the duality between the uplink and downlink MSE regions. The filter $\boldsymbol{R}^H$ now acts as a transmitter and $(\boldsymbol{T}^{(k)})^H$, $\forall k$, as receivers. The diagonal matrix

[1] The idea of this alternative proof is due to Eduard A. Jorswieck.

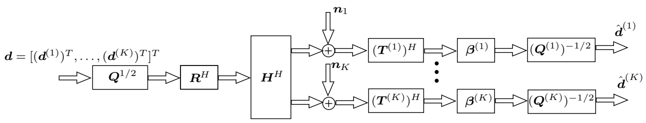

Figure 2: Downlink MIMO channel

$\boldsymbol{Q} = \text{diag}\{\boldsymbol{Q}^{(1)}, ..., \boldsymbol{Q}^{(\text{K})}\}$ contains the transmission powers. We assume that the quantities $\boldsymbol{H}$, $\boldsymbol{R}$, $\boldsymbol{T}$ and β are the same as for the uplink model. The power allocation $\boldsymbol{Q}$, however, may be different from the uplink allocation $\boldsymbol{P}$. It is assumed that both links fulfill the same sum power constraint, i.e., $\text{trace}\{\boldsymbol{Q}\} = \text{trace}\{\boldsymbol{P}\} \leq P_{tot}$.

With the equivalent downlink/uplink channels, it has been shown in [4, 5], that same MSE values $\varepsilon_i = \text{E}\{|\hat{d}_i - d_i|^2\}$, $1 \leq i \leq N_d$, can be achieved in both links.

Theorem 3. *Given $\boldsymbol{T}$, $\boldsymbol{U}$, $\boldsymbol{\beta}$ and a total power limit P_{tot}, the same MSE values $\varepsilon_1, \ldots, \varepsilon_{N_d}$ can be achieved in the downlink channel(Fig. 2) and uplink channel (Fig. 1).*

It should be noted that, although Theorem 3 shows that the duality holds for the layer-MSE values (MSE value of each layer), it implies that both links share the same user-MSE (the MSE of each user) and total MSE achievable region as well, since these quantities are the sum of the layer-MSE values.

One of the advantages of this duality is, that the downlink MSE need not be optimized directly. The equivalent uplink channel has a much more convenient structure, e.g., this allows to express the sum-MSE as a convex function with respect to the transmit covariance matrices [7, 8, 9], which is not possible in the downlink. It has been shown in [4, 5] that this duality can be exploited in order to derive optimal downlink designs.

However, for the power minimization problem (1), the $MSE_1, \ldots, MSE_K$ are all coupled by the choice of the transmit filter $\boldsymbol{R}^H$. The situation is not better for the dual uplink channel, where all users are coupled by the choice of the filter $\boldsymbol{T}$. Thus, finding algorithmic solutions for (1) is not obvious. But the transmitters and receivers can be optimized in an alternating fashion, similar to a strategy used in [6] for sum-MSE minimization. This iteration will be studied in the following section.

4. JOINT TRANSCEIVER OPTIMIZATION

In this section, we propose an iterative algorithm to minimize the total transmit power while maintaining per-user MSE requirements by jointly optimize the powers, transmitters and receivers.

First, in the uplink channel, for any given transmit covariance matrices $\boldsymbol{W}^{(k)}$, $\forall k$, the optimal power allocation $\boldsymbol{p}$ is found by using approach (10). The transmit filter $\boldsymbol{T}^{(k)}$ and power allocation $\boldsymbol{P}^{(k)}$ can be obtained by eigenvalue decomposition $\boldsymbol{T}^{(k)}\boldsymbol{P}^{(k)}(\boldsymbol{T}^{(k)})^H = p_k \boldsymbol{W}^{(k)}$. The optimal receive filter $\widetilde{\boldsymbol{R}}$ is given as MMSE filter (3), which minimizes the MSE values.

Then, consider the dual downlink channel, for fixed $\boldsymbol{R}$ and $\boldsymbol{T}$, find the power allocation $\boldsymbol{Q}$ which achieve the same MSE values as in the uplink channel. With the downlink power allocation $\boldsymbol{Q}$ and transmitter $\boldsymbol{R}^H$, update the receivers $\boldsymbol{\beta}^{(k)}(\boldsymbol{T}^{(k)})^T$ with

$$\begin{aligned}\widetilde{\boldsymbol{T}}^{(k)} &= \boldsymbol{T}^{(k)}\boldsymbol{\beta}^{(k)} \\ &= (\boldsymbol{H}_k^H \boldsymbol{R^H Q R H}_k + \sigma_n^2 \boldsymbol{I})^{-1}(\boldsymbol{R}^{(k)}\boldsymbol{H}_k)^H \boldsymbol{Q}^{(k)} \quad (11)\end{aligned}$$

where $\boldsymbol{\beta}$ is a diagonal matrix and contains the column norms of $\widetilde{\boldsymbol{T}}^{(k)}$.

The algorithm is summarized in Table 1. Superscript $(\cdot)^{(n)}$ denotes the nth iteration step.

Table 1 Algorithm for total power minimization with per-user MSE requirements

1: *initialize:* choose arbitrary *full rank* transmit covariance matrices $\boldsymbol{W}^{(1,0)}, ..., \boldsymbol{W}^{(K,0)}$ and maximal number of iterations n_{max}.
2: **repeat**
3: $n \leftarrow n + 1$
4: uplink channel:
- for given $\boldsymbol{W}^{(k,n-1)}$, $\forall k$, check the feasibility of the MSE requirements. If feasible, find the optimal uplink power allocation $\boldsymbol{p}^{(n)}$ with (10), otherwise re-adjust the MSE requirements.
- re-adjust the MSE requirements according to the number of active data streams (see remark).
- compute $\boldsymbol{T}^{(k,n)}$ and $\boldsymbol{P}^{(k,n)}$ by eigenvalue decomposition $\boldsymbol{T}^{(k,n)}\boldsymbol{P}^{(k,n)}(\boldsymbol{T}^{(k,n)})^H = p_k^{(n)}\boldsymbol{W}^{(k,n-1)}$.
- update $\boldsymbol{R}^{(k,n)}$, $\forall k$ and $\boldsymbol{\beta}^{(k,n)}$, $\forall k$. with (3)
- compute achieved MSE values of all layers.

5: downlink channel:
- compute $\boldsymbol{Q}^{(n)}$, which achieves the same layer MSE values as in step 4 with the same total power $P_{tot}(n)$ (see [6]).
- update $\boldsymbol{T}^{(n)}$ and $\boldsymbol{\beta}^{(n)}$, with (11).
- compute achieved MSE values of all layers.

6: uplink channel:
- compute $\boldsymbol{P}^{(n)}$, which achieves the same layer MSE values as in step 5 with the same total power $P_{tot}(n)$.
- update $\boldsymbol{W}^{(k,n)}$ with

$$p_k \boldsymbol{W}^{(k,n)} = \boldsymbol{T}^{(k,n)}\boldsymbol{P}^{(k,n)}(\boldsymbol{T}^{(k,n)})^H.$$

7: **until** required accuracy is reached or $n > n_{\text{max}}$

Theorem 4. *The sum power $P_{tot}(n)$ obtained by the above algorithm is monotonically decreasing. The algorithm converges to an optimum.*

Proof. Consider iteration n, in step 4, for $\boldsymbol{T}^{(k,n)}$ and $\boldsymbol{P}^{(k,n)}$, $\forall k$, the user MSE targets $\epsilon_1, ..., \epsilon_K$ are achieved with minimal total transmit power $P_{tot}(n)$(Theorem 2), i.e., $MSE_k^{(n,\text{step4})} = \epsilon_k, \forall k$.

In step 5, first, the power allocation $\boldsymbol{Q}^{(n)}$ ensures that the same MSE values are achieved with the same total transmit power $P_{tot}(n)$. Then, the MSE values are further minimized by the MMSE receivers $\widetilde{\boldsymbol{T}}^{(k,n)}$, $\forall k$, i.e., $MSE_k^{(n,\text{step5})} \leq MSE_k^{(n,\text{step4})} = \epsilon_k, \forall k$.

In the next iteration $n+1$, according to the MSE dual region between the uplink and downlink, we know that the same MSE values $MSE_k^{(n,\text{step5})}$, $\forall k$, can be achieved by $\boldsymbol{T}^{(n)}$, $\boldsymbol{R}^{(n)}$, $\boldsymbol{\beta}^{(n)}$. Thus, by optimizing the power allocation and the transmit filters, the total required power can be further minimized, i.e.,

$$P_{tot}(n+1) \leq P_{tot}(n).$$

Therefore, we can conclude that the total required power $P_{tot}(n)$ monotonically decreases with each iteration and converges to a limit. □

From a large number of simulations, we observe that regardless of the initial values (if full rank matrices are chosen), the algorithm always converges to the same total power. It is conjectured that this algorithm would return the global optimum. Another observation is that the achieved layer-MSE values depend on the initial values, i.e., with the same total transmit power, the distribution of MSE values among the layers is not unique.

Full rank initial matrices $\boldsymbol{W}^{(k,0)}$, $\forall k$, are recommended, since they have effect on the initial number of active data streams. This influences the convergence behavior of the algorithm. Unfavorable initial values can lead to a bad convergence behavior. A recommended choice of the transmitters $\boldsymbol{W}^{(k,0)}$, $\forall k$, are normalized identity matrices (trace equal to 1). From observations, the proposed algorithm does converge very fast during the first few iterations. Therefore, for practical implementations, restricting the maximal number of iterations is a good way to control the complexity of the algorithm.

Another important issue is how to check the feasibility of the given MSE requirements. This can be done in step 4. If the powers are divergent, then the MSE requirements can be determined to be infeasible. However, this method depends on the choice of the filters $\boldsymbol{W}^{(k)}$, $\forall k$, which might lead to wrong decisions of the feasibility.

Remark. Both power allocation and receiver optimization result in individual SINR for all layers. The transmitter is optimized with respect to a "virtual dual power allocation" associated with these SINRs. If one layer has a small SINR, then also its virtual power will be small. In this way, receivers and transmitters depend on each other. During the iteration the power allocated to one user is distributed among the layers in an uneven fashion. It is even possible that the power of one layer tends to zero. This will happen automatically when the initially given number of layers is not favored by the spatial structure of the propagation channel.

However, the sum-MMSE target of each user is usually chosen for a given number of layers. Switching off users means that the corresponding MSE value is decreased by the number of deactivated data streams. As an example, assume that user k has two active data streams, and the MSE target is 1.2. With (4), $MSE_k = 2 - \text{trace}\{\boldsymbol{Z}\}$. If one layer is switched off, then $MSE_k = 1 - \text{trace}\{\boldsymbol{Z}\}$. Thus, the sum-MSE is decreased and the targets are still fulfilled. But now the MSE *per layer is much better*, so it is suggested to readjust the targets in the following way: $\epsilon_k^{(n+1)}$ = (number of current active data streams) $\times\ \epsilon_k^{(n)}$ /(number of previous active data streams). This adjustment maintains the original average performance requirement per layer.

Notice, that with Theorem (3) the same algorithm can be used in order to optimize both uplink and downlink channels.

5. EXTENSION: MIN-MAX RELATIVE USER-MSE

The proposed iteration for power minimization can be extended to minimize the maximum relative MSE subject to a total power constraint, i.e.,

$$\min_{\boldsymbol{Q},\boldsymbol{T},\boldsymbol{U},\boldsymbol{\beta}} \max_{1\leq k\leq K} MSE_k/\epsilon_k \quad \text{s.t.} \quad \text{trace}\{Q\} \leq P_{max} \tag{12}$$

where P_{max} is the total transmit power limit.

With the same MSE targets, if the minimum power P_{tot}^{opt} of (1) equals the total power limit P_{max}, then problem (12) is equivalent to (1). One possible way to solve (12) is to modify the uplink power allocation in Algorithm 1 (step 4) by solving the problem of minimizing the balanced relative MSE level for fixed $\boldsymbol{W}_k$. This can be done by combining the fixed point iteration of (10) and a bi-section approach [14]. In particular, optimization (9) can be performed with the original MSE targets. If the required total power is smaller than the given power limit P_{max}, then the values of the effective MSE targets are proportionably decreased; otherwise, they are proportionably increased until the corresponding effective MSE targets ξ_k, $\forall k$, are found. That is, $\text{MSE}_k = \xi_k$, $\forall k$, are achieved with the total power P_{max}. This implies that a minimum balanced relative level $MSE_1/\xi_1 = \cdots = MSE_K/\xi_K = \xi_k/\epsilon_k$ is achieved. The remaining steps are the same as in Algorithm 1.

The bi-section strategy can only be as good as the underlying optimization strategy for power minimization. But if the proposed algorithm for sum-power minimization returns the global optimum (which remains to be shown), then also the proposed min-max balancing algorithm will converge to the global optimum.

6. SIMULATION RESULTS

The simulations are carried out under the assumption that independent unity-energy data streams are transmitted and the variance of the white Gaussian noise is 0.1. The channel realizations are randomly chosen.

We first consider a MIMO system with five antennas at the base station and three users each with two antennas. We assume that the user MSE targets are $\epsilon_1 = 0.6$, $\epsilon_2 = 0.4$ and $\epsilon_3 = 0.8$. The convergence behavior is displayed in Fig. 3. It can be observed that the required power decreases monotonically and converges to a limit. The achieved MSE values are shown in Fig. 4. We can see that the targets are fulfilled. The darkly shadowed part denotes the MSE value of the first layer and the lightly shadowed denotes the MSE value of the second layer of each user.

Fig. 5 and Fig. 6 show a case when some layers can be switched off. The original user MSE targets are set to be $\epsilon_1 = 0.6$, $\epsilon_2 = 0.4$ and $\epsilon_3 = 1.6$. From the simulation, we observe that after a few iterations, one layer of user 3 is switched off (see Fig. 6). Then, the target for user 3 is adjusted to 0.8, so that the required transmit power is further minimized (see Fig. 5).

7. CONCLUSIONS

We have studied the problem of transceiver design for minimizing the total sum-power with user-MSE requirements in multi-user MIMO systems. The proposed algorithm is based on the monotonicity and scalability of the function $f_k(\boldsymbol{p}, \sigma_n^2)$, which facilitates the iterative computation of the

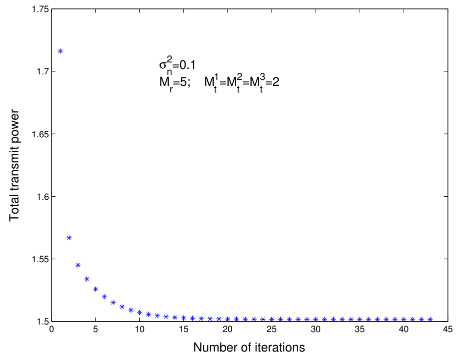

Figure 3: Iterative sum-power minimization by jointly optimizing receivers and transmitters. Case 1: all layers are active. Parameters: $M_r = 5$, $M_t^1 = M_t^2 = M_t^3 = 2$

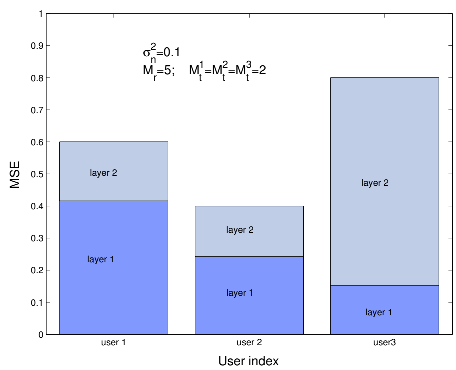

Figure 4: Achieved user MSE values by jointly optimizing receivers and transmitters. Case 1: all layers are active. Parameters: $M_r = 5$, $M_t^1 = M_t^2 = M_t^3 = 2$

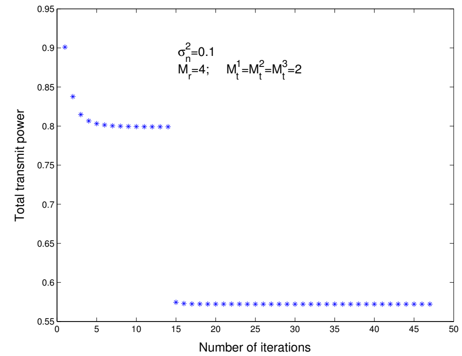

Figure 5: Iterative sum-power minimization by jointly optimizing receivers and transmitters. Case 2: some layers are switched off. Parameters: $M_r = 4$, $M_t^1 = M_t^2 = M_t^3 = 2$

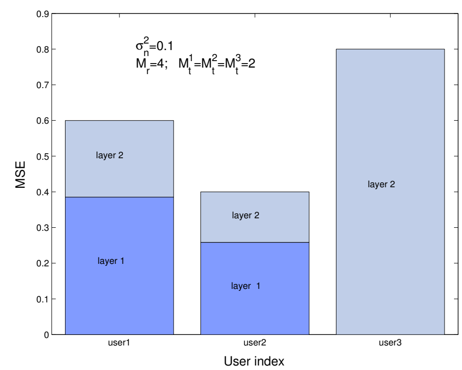

Figure 6: Achieved user MSE values by jointly optimizing receivers and transmitters. Case 2: some layers are switched off. Parameters: $M_r = 4$, $M_t^1 = M_t^2 = M_t^3 = 2$

optimal power allocation in the uplink. The duality between uplink and downlink MSE regions is used to optimize transmit filters and receive filters in an alternating way. The total required power monotonically decreases with each iteration and converges to a limit. Future work will have to show that the achieved convergence point is indeed the global optimum. This is suggested by numerical simulations, which always showed convergence to the same optimum, independent of the chosen initialization.

After finishing this paper, an alternative approach for sum-power minimization was published in [15].

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