RANDOMIZED DISTRIBUTED MULTI-ANTENNA SYSTEMS IN MULTI-PATH CHANNELS

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ABSTRACT

A great deal of research on MIMO systems is now trying to focus on distributed designs to bring the advantages of co-located antenna systems to nodes with a single RF front end, by leveraging on the other nodes resources. Yet, most schemes that are considered assume that the nodes encode their signals in a fashion that requires at least the knowledge of the number of nodes involved and in many cases the specific encoding rule to use. Hence, while the hardware resources are distributed, the protocols that are proposed are not. Recently we have proposed schemes that are totally decentralized and using random matrix theory we have studied the diversity attainable through these schemes in flat fading channels. The goal of this paper is to show that our general randomized designs are suitable to work in frequency selective channels and can easily be adapted to block space-time precoding schemes that are known to harvest diversity not only from the multiple antennas but also from the multi-path.

1. INTRODUCTION AND SYSTEM MODEL

Consider a network with N radios. Each radio has a baseband equivalent, discrete-time transmit signal $X_i[k]$, with average power constraint $\sum_{k=0}^{K-1} |X_i[k]|^2 \leq KP_i$, where K is the duration of the signal, and the receive signal is $Y_i[k]$, $i = 1, 2, ..., N$. Including the 'half-duplex' constraint the discrete-time received signal at radio i and time sample k is, if radio i receives a time k:

$$Y_i[k] = \sum_{j=1, j\neq i}^{N} H_{i,j}[k] * X_j[k] + W_i[k] \tag{1}$$

if radio i transmits a time k:

$$Y_i[k] = 0 \tag{2}$$

where $H_{i,j}[k]$ captures the combined effects of frequency-selective, quasi-static multi-path fading, shadowing, path-loss between radios i and j and, last but not least, symbol asynchronism among the cooperative nodes; $W_i[k]$ is a sequence of mutually independent, white circularly-symmetric, complex Gaussian random variables with common variance N_0 modelling thermal noise and other interference received at radio i. The complex-valued random impulse response representing the channel $H_{i,j}[k]$ is assumed to be fixed during the block length and estimated exactly at the destination i. Nodes that cooperate share a common message, which was transmitted previously by one or more nodes and received by the group of cooperating nodes.

When the nodes do not have channel state information at the transmitter and the receivers do not cooperate, the encoding strategies that are appropriate are distributed forms of space time coding [11] designed for multi-input single output (MISO) channels. More specifically, we can represent a portion or the entire common message to transmit as a vector of length M denoted by $\mathbf{s} = (S[0], \ldots, S[M-1])^T$; each one of the T cooperating relay nodes transmits a specific column $\mathbf{x}_t = (X_t[0], ...X_t[K-1])^T$ of a $K \times T$ matrix code be $\mathbf{X} = \mathbf{G}_{K\times T}(\mathbf{s})$. M/K is the spectral efficiency of the code and the number of columns T is the number of cooperative nodes. Different cooperative schemes, starting from the early examples in [5] and [3], and including the simple amplify and forward or decode and forward strategies, can all be cast into different instantiations of the mapping $\mathbf{s} \to \mathbf{G}(\mathbf{s})$. For frequency selective channels there are various forms of cooperative *precoded* transmission schemes that can be employed but for the MISO setup, without channel feedback, using OFDM in combination with ST and error correction coding [2] is an efficient option. The of the cooperative frequency selective channels can be cast in exactly the same terms of the non cooperative case (c.f. [6]). Scaling laws for the relay channel with inter-symbol interference (ISI) are, for example, in [4]. Interestingly, schemes such as [10], require only to know the number of nodes that cooperate, but the code assignment is not required. With the exception of an handful of papers [7], [8], [12] most other the approaches considered in the literature assume that the nodes are informed about the code to use or of the number of cooperating nodes. The shortcoming of all these schemes is that they would require a tight

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management of the relay activities that would not result in a scalable design for large networks.

Suppose that the node that transmits the message $\mathbf{s}$ has a unknown number of nodes in the vicinity that are qualified to relay its data. If neither the number of nodes cooperating nor the codes to be used need to be specified, the nodes in the vicinity can synchronize to the source message using its preamble sequence and can cooperate in the following time slot without need of *any control message exchange* and *without contending for the medium*. In [9] we showed how such fully decentralized policies could be designed in general and analyze the effective amount of diversity that can be harvested in this fashion. These architectures include [7], [8], [12] as special cases. Specifically, in our model each node is unaware of the effective code being employed by the other cooperating nodes and their number and a local, independent random assignment of the code is performed at each node. The randomized coding rule targets a *fixed maximum order of diversity L, which is independent of the actual number of nodes cooperating in each set*. We review briefly the main results in [9] in Section 2. The novel contribution in this paper compared to [9] is that of considering a frequency selective model. This generalization is important in practice because of the real difficulty of attaining perfect symbol synchronism among the cooperating nodes. For this reason we develop a randomized strategy that generalizes not only [9], but also better captures other schemes that have been proposed in the past which use cooperative multi-path to generate diversity, such as [7], [8], [12] and [1].

2. RANDOMIZED COOPERATION IN FLAT FADING: A BRIEF REVIEW

In [9] the cooperative nodes generate their codes by forming a matrix code $\mathbf{X} = \mathbf{G}_{K\times L}(\mathbf{s})$. Instead of using a pre-assigned column $\mathbf{x}_l \in \mathbf{X}$ each node projects the rows of the matrix over a random, independently generated, $L \times 1$ vector $\mathbf{r}_t$, $t = 1,\ldots,T$. For a flat fading channel, $H_{i,j}[k] \equiv H_{i,j}\delta[k]$ and, thus, the i-th receiver data are (1):

$$\begin{aligned}\mathbf{y}_i &= \sum_{t=1}^{T} H_{i,t}\mathbf{G}(\mathbf{s})\mathbf{r}_t + \mathbf{w}_i & (3)\\ &= \mathbf{G}(\mathbf{s})\mathbf{R}\mathbf{h}_i + \mathbf{w} = \mathbf{G}(\mathbf{s})\tilde{\mathbf{h}}_i + \mathbf{w}_i. & (4)\end{aligned}$$

From the last equation it can be noticed that the whole system behaves as an equivalent set of L co-located transmit antennas with fading coefficients relative to the ith receiver that are the entries of the vector $\tilde{\mathbf{h}}_i$. Hence, to decode the receiver one can simply estimate directly $\tilde{\mathbf{h}}_i$, ignoring both $\mathbf{R}$ and the true number of nodes that are transmitting, leading to a great simplification of the receiver side as well.

Under Rayleigh i.i.d. fading $\mathbf{h}_i \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Phi_h})$ the pair-wise error probability is [11]:

$$P(\mathbf{s}_i \to \mathbf{s}_j) = E_{\mathbf{R}}\{det\,(\mathbf{I} + SNR/4\mathbf{A}_{i,j})\} \qquad (5)$$

where:

$$\mathbf{A}_{i,j} \triangleq \mathbf{R}^{\star}(\mathbf{G}(\mathbf{s}_i) - \mathbf{G}(\mathbf{s}_j))^{\star}\mathbf{\Phi_h}(\mathbf{G}(\mathbf{s}_i) - \mathbf{G}(\mathbf{s_j}))\mathbf{R} \quad (6)$$

The diversity that a code can bring is defined as:

$$d \triangleq \lim_{SNR\to\infty} \frac{-\log P_e(SNR)}{\log SNR}. \qquad (7)$$

Since:

$$d = \min_{k,i}\left(rank\left(\mathbf{R}^*(\mathbf{G}(\mathbf{s}_k) - \mathbf{G}(\mathbf{s}_i))^*\mathbf{\Phi_h}(\mathbf{G}(\mathbf{s}_k) - \mathbf{G}(\mathbf{s}_i))\mathbf{R}\right)\right)$$

using the union bound and the theory of random determinants in [9] we have shown that, provided that one uses codes $\mathbf{G}(\mathbf{s})$ that achieve full diversity, i.e. such that $\forall\, k \neq i$:

$$rank\left((\mathbf{G}(\mathbf{s}_k) - \mathbf{G}(\mathbf{s}_i))^*\mathbf{\Phi_h}(\mathbf{G}(\mathbf{s}_k) - \mathbf{G}(\mathbf{s}_i))\right) = L$$

there are several options for the randomization matrix $\mathbf{R}$ *that enable achieving the full diversity L of the code* $\mathbf{G}(\mathbf{s})$*when the number of nodes exceeds the number of virtual antennae L even by only one extra node, i.e. if* $T = L+1$. *If* $T \leq L$ *the same random selection rules give a diversity that is* $O(T)$. Specifically, under a wide variety of distributions for $\mathbf{R}$:

$$d = \begin{cases} T \text{ if } L \geq T+1 \\ L \text{ if } L \leq T-1 \end{cases}. \qquad (8)$$

In [9] one can find the sufficient conditions on the distributions for $\mathbf{R}$ that make (8) hold true. These distributions have, obviously, all independent columns by design and the entries of the column can be drawn from a wide variety of standard distributions; they can for example be i.i.d. complex Gaussian random variables, complex exponential with a uniformly distributed phase or the entire vector can be selected as a random vector uniformly distributed in an L-dimensional hyper-sphere.

3. RANDOMIZED COOPERATION IN ISI CHANNELS

We extended the methodology to the frequency selective case as follows. To model the most general case, we can consider for each i,j pair a doubly selective channel with baseband complex equivalent time varying impulse response $h_{ij}(t,\tau)$[1]. Indicating by $p(t)$ the pulse shaping filter and by B the Nyquist sampling rate, the block discrete-time model

[1] Some of the considerations made in this section assume a channel that is stationary. The model is still valid, since all that changes is that in (9) $H_{ij}[l,k] = H_{ij}[k]$ independent of l.

will have each input vector mapped onto an output vector through the linear transformation matrix:

$$[\mathbf{H}_{ij}]_{l,k} = H_{ij}[l, l-k] \quad (9)$$

where:

$$H_{ij}[l,k] = \iint h_{ij}(l/B - \nu, \tau) p(k/B - \nu - \tau) p(-v) d\tau dv.$$

Assuming that the channel has finite memory D the transmission can be structured in blocks which are interleaved with appropriate guard-periods. For simplicity we assume that the guard periods are formed with at least D zeros, although the use of a cyclic prefix is also possible. Hence the block of size $K > 2D$ is mapped into a vector of size $K+D$ (i.e. $\mathbf{H}_{ij}$ in (9) is a tall $(K+D)\times K$ matrix):

$$\begin{aligned} \mathbf{y}_i &= \sum_{j=1}^{T} \mathbf{H}_{i,j} \underbrace{\mathbf{G}(\mathbf{s})}_{\mathbf{X}_{K\times L}} \mathbf{r}_j + \mathbf{w}_i \quad (10) \\ &= \sum_{l=1}^{L} \left(\sum_{j=1}^{T} \mathbf{H}_{i,j} r_{j,l} \right) \mathbf{x}_l + \mathbf{w}_i = \sum_{l=1}^{L} \tilde{\mathbf{H}}_{i,l} \mathbf{x}_l + \mathbf{w}_i \end{aligned}$$

The diversity that can be earned through this scheme depends on the statistics of the resulting equivalent channels $\tilde{\mathbf{H}}_{il}, l = 1, \ldots, L$ and on the particular selection of the code $\mathbf{G}(\mathbf{s})$. Clearly, also in this case the receiver i does not need to know $\mathbf{R}$ it just needs to acquire the parameters of the equivalent L frequency selective channels, as it would in any block precoded space time scheme.

Cooperative multi-path [7], [8] and [12] can also be cast into the randomized coding framework with a $\mathbf{G}(\mathbf{s})$ with Toeplitz structure and $\mathbf{r}_i$ which is a randomly chosen canonical vector, i.e.:

$$\mathbf{G}_{ij}(\mathbf{s}) = S[i-j], \quad \mathbf{r}_i = \mathbf{e}_q \quad \rightarrow q\text{-th canonical vector.} \quad (11)$$

Rather than forming $\mathbf{G}(\mathbf{s})$ directly using the message $S[k]$ or could construct it from precoded data. The method, in fact, can be combined with spread spectrum techniques or to Orthogonal Frequency Division Multiplexing (OFDM). The latter, combined with error correction coding, achieves diversity in a spectrally efficient fashion compared to spread spectrum methods. Correspondingly, the code matrix is constructed in the two following ways:

$$\begin{gathered} \mathbf{u} = \mathbf{c}S[i],\ \mathbf{c}: \text{ spreading code} \\ \mathbf{u} = \mathbf{F}\mathbf{s},\ \mathbf{F}: \text{ IFFT matrix+prefix} \\ \rightarrow \mathbf{G}_{ij}(\mathbf{s}) = \mathcal{T}(\mathbf{u}). \end{gathered} \quad (12)$$

where $\mathcal{T}(\mathbf{u})$ is a Toeplitz convolution matrix of size $(M+L)\times L$ with first column equal to $[\mathbf{u}^T, 0, .., 0]^T$ and first row equal to $(U[0], 0, \ldots, 0)$, $U[0] \equiv \mathbf{u}_1$ and the number of zeros appended equal to $L-1$. Note that in the spread-spectrum alternative the message has length $M = 1$ while in the OFDM scheme there are multiple symbols that are multiplexed in the same block. To have diversity order L one needs to have an equivalent channel of length L and, therefore, there will be a bandwidth expansion due to the need of adding a cyclic prefix (or a zero padding sequence, like in our model) of length $L-1$. In either case, the scheme can also include space time coding, within the OFDM block or across blocks. *Cooperative multi-path, has numerous advantages over cooperative space time coding which are seldom acknowledged: 1) it enables receiver architectures that have reduced complexity, that are fairly standard, and that provide simple options for multiplexing multiple sources using different spreading codes and/or sub-carriers; 2) cooperative multi-path in combination with OFDM precoding can provide diversity gains in the order of the equivalent channel memory D and, yet, have very high spectral efficiency. In fact, if the channel coherence time is large, one can utilize a large number of sub-carriers and reduce the inefficiency caused by the need of adding cyclic prefixes (or zero padding). Hence, this simple strategy leads to spectrally-efficient high dimensional space-time codes as well.* Even unintentional effects of time-asynchronism and carrier frequency offset (CFO) can be incorporated in the model by appropriately defining the code matrix and the randomization vector. The effect of asynchronism can be captured by including the time and frequency offset in the channel response $h_{ij}(t,\tau)$; the variability in the equivalent channels $\tilde{\mathbf{H}}_{il}, l = 1, \ldots, L$ that is induced by the CFO effectively increases the Doppler spread of the equivalent channel. Overall, what can be noticed is that dealing with a cooperative channel is no different than dealing with multi-antenna transmission over dispersive time-varying channels, which is challenging but within our reach.

Note that there are benefits and drawbacks in targeting large or small degrees L of diversity; in fact, the greater is L the greater is the number of dimensions of the code and correspondingly not only the complexity increases but also either the bandwidth used or the duration of the codes increases. Having large degrees of diversity allows harvesting the greatest gains if the nodes cooperating are $T > L$ but when their number is much smaller, because the maximum diversity attainable is in the order of T, coding for large L requires an investment in complexity, increased bandwidth or latency that are strong disincentives towards choosing large values of L.

3.1. Diversity analysis

To assess what are the potential performance gains that can be attained by randomized cooperation in multi-path channels with asynchronous cooperative relays, in this section we extend the diversity analysis done in [9] for a stationary channel. The key step is to rewrite the model in such a way that it can be mapped one to one in a special instance of (3). If the channels are all linear time-invariant, each equivalent channel each matrix $\tilde{\mathbf{H}}_{il}$ has a Sylvester structure and therefore the product $\tilde{\mathbf{H}}_{il}\mathbf{x}_l = \mathcal{T}(\mathbf{x}_l)\tilde{\mathbf{h}}_{il}$, where $\mathcal{T}(\mathbf{x}_l)$ has an Toeplitz structure analogous to the one described in (12), except that the size depends now on the equivalent channel order D and not only on the design parameter L. Hence, with simple manipulations (10) can be rewritten as follows:

$$\mathbf{y}_i = \sum_{l=1}^{L} \mathcal{T}(\mathbf{x}_l)\tilde{\mathbf{h}}_{il} + \mathbf{w}_i = \mathcal{X}\tilde{\eta}_i + \mathbf{w}_i \quad (13)$$

$$= \mathcal{X}(\mathbf{I}_{D\times D} \otimes \mathbf{R})\eta_i + \mathbf{w}_i \quad (14)$$

where $\mathcal{X} \triangleq (T(\mathbf{x}_1), ..., T(\mathbf{x}_L))$; $\tilde{\eta}_i \triangleq (\tilde{\mathbf{h}}_{i1}^T, ..., \tilde{\mathbf{h}}_{iL}^T)^T$; and $\eta_i \triangleq (\mathbf{h}_{i1}^T, ..., \mathbf{h}_{iT}^T)^T$. Comparing equation (13) with (3), we can see that the only difference is that both the equivalent code matrix and random mapping have a very peculiar structure. If, within the constraints for the structure of $\mathcal{X}$, it is possible to find codes that attain maximum diversity without randomization, there are results in [9] that can be extended to work in the ISI model. Hence, the diversity can potentially be as large as the channel order times the number of cooperating users.

4. NUMERICAL RESULTS AND CONCLUSIONS

We analyzed by simulation the performance of a randomized applied to OFDM transmission in a block fading frequency selective channel whose discrete-time impulse response is of length 3. The number of sub-carriers used is $M = 32$ and even and odd subcarriers are used to transmit an Alamouti code. The results are not surprising: as soon as we have at least three nodes cooperating the full diversity of order 2 of the Alamouti code is obtained. However, since there is no additional coding, there is no diversity gain earned from multi-path (which in this simulation is of order $D = 3$). While the slope of the SER curve remains the same, there is obviously a power gain in recruiting more users. Hence one can either decrease the power per user or enjoy better performance over the same communication range.

The conclusion that can be reached from our studies is that cooperative diversity schemes can be made extremely practical through the idea of randomization and that the design of the receiver at the physical layer is well within our reach. Hence, cooperating radios should be seriously considered in future wireless networks standards.

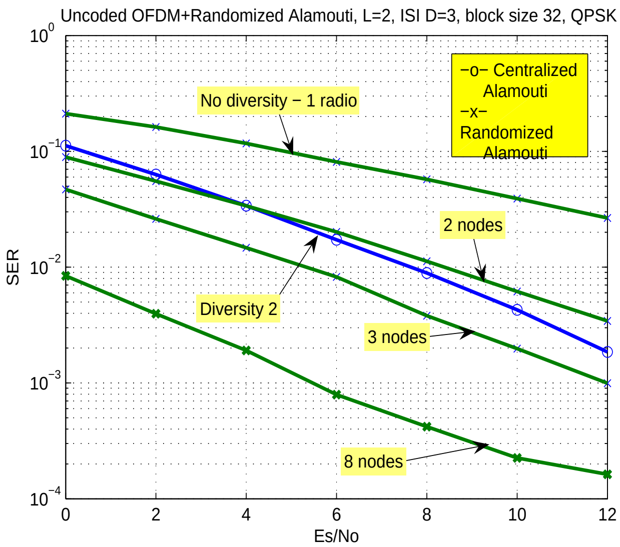

Fig. 1. SER of centralized (blue) and randomized (green) STC-OFDM; $L = 2$ Alamouti code over frequency bins.

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