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LINEAR TRANSMITTER DESIGN FOR THE MISO COMPOUND CHANNEL WITH INTERFERENCE

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ABSTRACT

We consider the problem of linear transmitter design in multiple input single output (MISO) compound channels with interference. The motivation for this channel model is communication in MISO broadcast channels with partial channel state information (CSI). Since the compound capacity is unknown for this model, we consider optimal linear transmit methods for maximizing the data rate. We provide efficient numerical solutions with and without perfect CSI. We then discuss the optimality of beamforming and the existence of a saddle point in the compound channel.

1. INTRODUCTION

One of the emerging topics in modern wireless communication is multiuser systems in which there are multiple transmit antennas. It is well known that the use of multiple antennas can improve the capacity and reliability of wireless links. In order to exploit these gains the system must efficiently use the antennas and optimize its transmission method. One of the main obstacles in designing such systems in the lack of perfect channel state information (CSI) at the transmitter side. The problem becomes even more interesting and difficult in the multiuser broadcast setting, due to interference directed at the other users.

The optimal transmit strategies for achieving capacity in single user multiple input single output (MISO) channels are well known. If perfect CSI is available at the transmitter, then linear beamforming along the MISO channel is optimal. There are also many results using different partial CSI models, e.g., [1]. We consider the compound model in which the channel is a deterministic variable within a known set of possible values. The compound capacity is defined as the achievable rate when the transmitter does not know the exact channel realization and is designed for the worst case realization within the set [2]. Due to its importance, the compound capacity recently gained considerable attention [3, 4, 5]. In [5] we addressed the optimization of the single user multiple input multiple output (MIMO) compound capacity. We provided a simple solution for finding the optimal transmit scheme, and proved that beamforming is optimal for maximizing the capacity in this case. Like many results in the context of worst case optimization, our proof is based on the existence of a saddle point. Using a standard minimax theorem we showed that the compound capacity (maxmin) is equal to the capacity of the worst case channel within the uncertainty set (minmax). For a known channel the capacity achieving strategy is beamforming, and therefore all we had to do is find the worst case channel and beamform in its direction. Thus, the solution is simple, its beamforming implementation is practical, and there is no advantage in additional feedback since we can achieve the minimax capacity using the maximin compound capacity. Finally, most of these results can be generalized to the multiuser multiple access channel.

On the other hand, many of the questions regarding the multiuser broadcast channel are still open. The main problem is that each user must cope with the interference which is directed to the other users. When perfect CSI is available, this interference can be eliminated using dirty paper coding (DPC) [6, 7]. Unfortunately, there are not many results in the more practical case of partial CSI. It is not clear whether one can still use a robust version of DPC, and whether it is optimal in some way [8, 9, 10]. In fact, even if a robust DPC scheme was available and the interference to some of the users was eliminated, there are not many results on the linear transmitter design to the other users. Moreover, in some applications, such as co-located transmitters [11], there is no access to the interference and linear transmission methods should be used. These scenarios motivate us to consider the problem of linear transmitter design in the compound channel. Interestingly, even in this simplified channel model, many questions are still open. For example, it is unknown whether a saddle point always exists and whether beamforming is still optimal. In this paper, we try to answer some of these questions.

As explained, we are interested in compound channel with interference. We address the maximization of the worst case rate (using linear processing) in this compound channel with and without perfect CSI. Mathematically, we consider the maxmin rate problem and its complementary minmax rate problem. We reformulate both problems using semidefinite programs (SDP) which can be efficiently solved. We discuss the numerical results and give special emphasis to the optimality of beamforming and to the existence of a saddle point. We show that, due to the interference, both of these properties are not necessarily true anymore.

The paper is organized as follows. We begin in Section 2 by introducing the problem. Then, we provide the main results in Section 3. The mathematical derivations and proofs are developed in Section 4. A numerical example using computer simulations is described in Section 5.

The following notation is used: The operators $(\cdot)^T$, $\mathrm{Tr}\{\cdot\}$, $\mathrm{rank}(\cdot)$ and $\|\cdot\|$ denote the transpose, the trace, the rank and the Euclidean norm, respectively. $\lambda_{\max}(\mathbf{X})$ is the maximal eigenvalue of $\mathbf{X}$. Finally, $\mathbf{X} \succeq 0$ denotes that a matrix $\mathbf{X}$ is Hermitian positive semidefinite.

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2. PROBLEM FORMULATION

Consider a MISO compound channel with interference. The signal at the output of the receiver can be expressed as

$$y = \mathbf{g}^T\left[\mathbf{x}+\mathbf{i}\right]+w \tag{1}$$

where y is the received sample, $\mathbf{g}$ is the length-K channel vector, $\mathbf{x}$ is the length-K transmitted vector, $\mathbf{i}$ is a length-K zero mean, normal interference vector with covariance $\mathbf{Z}\succeq\mathbf{0}$, and w is a zero-mean, unit-variance normal noise sample. We assume that $\mathbf{i}$ and w are statistically independent. The compound channel $\mathbf{g}$ is a deterministic vector within the following set:

$$\mathscr{G} = \left\{\mathbf{h}+\mathbf{d} : \mathbf{d}^T\mathbf{d}\leq\varepsilon\right\} \tag{2}$$

where $\mathbf{h}$ is the center of the ellipsoid and the parameter ε controls its volume. For simplicity, we assume that $\varepsilon<\mathbf{h}^T\mathbf{h}$. We emphasize that there is no prior distribution on $\mathbf{g}$.

The capacity of the compound channel in (1) is the maximal achievable transmission rate in the channel. This rate must be achievable for any $\mathbf{g}\in\mathscr{G}$. To our knowledge, there is no solution to this compound capacity, and it is not clear what is the optimal transmission strategy for achieving it. Therefore, although it is clearly suboptimal, we restrict ourselves to the class of linear transmission schemes and assume that $\mathbf{x}$ is a zero mean, normal vector with covariance $\mathbf{Q}\succeq\mathbf{0}$. When the rank of $\mathbf{Q}$ is one, we call the scheme beamforming along the principal eigenvector. As usual, we assume there is a standard constraint on the average transmitted power

$$\mathrm{Tr}\left\{\mathbf{Q}\right\}\leq P \tag{3}$$

where P is the available power.

Under this setting, we define two achievable rates depending on the available CSI. In both cases we assume that the receiver has perfect CSI and knows $\mathbf{g}$. In the first case, the transmitter does not know the exact realization of $\mathbf{g}$, and must design $\mathbf{Q}$ to deal with the worst case $\mathbf{g}$. The maximal achievable rate is then

$$R_1 = \max_{\substack{\mathbf{Q}\succeq\mathbf{0}\\ \mathrm{Tr}\{\mathbf{Q}\}\leq P}}\ \min_{\mathbf{g}\in\mathscr{G}} \log\left(1+\frac{\mathbf{g}^T\mathbf{Q}\mathbf{g}}{\mathbf{g}^T\mathbf{Z}\mathbf{g}+1}\right). \tag{4}$$

On the other hand, when the transmitter knows $\mathbf{g}$ (due to feedback from the receiver), the transmitter may design $\mathbf{Q}$ as a function of the specific $\mathbf{g}$. Nonetheless, our compound channel model assumes that the set $\mathscr{G}$ does not change due to feedback, i.e., feedback does not cause the set to shrink and does not decrease ε. Thus, even in the case of perfect CSI, the system's code must be designed to satisfy the SINR (rate) in all channel realizations, and in particular in the worst case channel. The resulting rate is then

$$R_2 = \min_{\mathbf{g}\in\mathscr{G}}\ \max_{\substack{\mathbf{Q}\succeq\mathbf{0}\\ \mathrm{Tr}\{\mathbf{Q}\}\leq P}} \log\left(1+\frac{\mathbf{g}^T\mathbf{Q}\mathbf{g}}{\mathbf{g}^T\mathbf{Z}\mathbf{g}+1}\right). \tag{5}$$

The above rates $R_i = \log(1+\gamma_i)$ for $i=1,2$ are monotonic increasing in their SINRs γ_i. This property allows us to avoid the $\log(1+x)$ function and optimize the SINRs instead of the rates.

In the sequel, we derive efficient numerical solutions for finding γ_1 and γ_2 (respectively, R_1 and R_2). In addition, our goal is to provide insight regarding two important issues. First, we compare γ_1 with γ_2 and check whether $\gamma_1=\gamma_2$, i.e., whether there is any advantage in additional feedback. Second, we focus on the rank of the optimal covariance matrices and on the optimality of beamforming.

3. MAIN RESULTS

We now provide the main results. We begin with two special cases which have been published before and then address the general setting.

3.1 Perfect CSI

The first case is when the transmitter has perfect CSI, i.e., $\varepsilon=0$ and $\mathbf{g}=\mathbf{h}$. It is easy to see that the two problems are equivalent and $\gamma_1=\gamma_2$. The optimal solution is beamforming along the channel $\mathbf{g}$, which results in

$$\mathbf{Q} = P\frac{\mathbf{g}\mathbf{g}^T}{\mathbf{g}^T\mathbf{g}}. \tag{6}$$

3.2 No interference

The second scenario is when there is no interference, i.e., $\mathbf{Z}=\mathbf{0}$. In this case, we have recently shown that the optimal solution of both problems is a saddle point and $\gamma_1=\gamma_2$. The resulting strategy is beamforming along the worst case channel denoted by $\mathbf{g}_w$:

$$\mathbf{Q} = P\frac{\mathbf{g}_w\mathbf{g}_w^T}{\mathbf{g}_w^T\mathbf{g}_w}. \tag{7}$$

The worst case channel $\mathbf{g}_w$ can be found using a simple one dimensional search as explained in [5]. Alternatively, the optimal $\mathbf{Q}$ can be obtained by solving the following SDP

$$\gamma_1=\gamma_2=\left\{\begin{array}{ll}\max_{\mathbf{Q},\alpha,\gamma} & \gamma\\ \text{s.t.} & \begin{bmatrix}\mathbf{Q}+\alpha\mathbf{I} & -\alpha\mathbf{h}\\ -\alpha\mathbf{h}^T & \alpha\left(\mathbf{h}^T\mathbf{h}-\varepsilon\right)-\gamma\end{bmatrix}\succeq\mathbf{0}\\ & \mathrm{Tr}\left\{\mathbf{Q}\right\}\leq P\\ & \mathbf{Q}\succeq\mathbf{0}\\ & \alpha\geq 0.\end{array}\right. \tag{8}$$

3.3 The general case

We now consider the general case. In the next section, we show that problems (4) and (5) can be solved using the following standard SDPs:

$$\gamma_1=\left\{\begin{array}{ll}\max_{\mathbf{Q},\alpha,\gamma} & \gamma\\ \text{s.t.} & \begin{bmatrix}\mathbf{Q}-\gamma\mathbf{Z}+\alpha\mathbf{I} & -\alpha\mathbf{h}\\ -\alpha\mathbf{h}^T & \alpha\left(\mathbf{h}^T\mathbf{h}-\varepsilon\right)-\gamma\end{bmatrix}\succeq\mathbf{0}\\ & \mathrm{Tr}\left\{\mathbf{Q}\right\}\leq P\\ & \mathbf{Q}\succeq\mathbf{0}\\ & \alpha\geq 0\end{array}\right. \tag{9}$$

and

$$\gamma_2=\left\{\begin{array}{ll}\max_{\alpha,\gamma} & \gamma\\ \text{s.t.} & \begin{bmatrix}P\mathbf{I}-\gamma\mathbf{Z}+\alpha\mathbf{I} & -\alpha\mathbf{h}\\ -\alpha\mathbf{h}^T & \alpha\left(\mathbf{h}^T\mathbf{h}-\varepsilon\right)-\gamma\end{bmatrix}\succeq\mathbf{0}\\ & \alpha\geq 0.\end{array}\right. \tag{10}$$

Unlike the previous cases, problems (9) and (10) are not equivalent in general. This can be seen by examining the optimality of beamforming. When perfect CSI is available, the optimal $\mathbf{Q}$ is always of rank one and aligned with the worst case channel[1]. But this is not necessarily true for the optimal $\mathbf{Q}$ in (9). Although not too frequent, it is not difficult to find a numerical example where (9) results in $\mathrm{rank}(\mathbf{Q}) > 1$.

Moreover, even if we allow a high rank solution, the resulting SINRs γ_1 and γ_2 are not necessarily equal. Due to the standard minimax inequality, we always have

$$\gamma_1 \leq \gamma_2. \tag{11}$$

Interestingly, the following theorem provides a condition for equality in (11) without the need for solving (10):

Theorem 1. *Let $\mathbf{Q}$, α and γ be the optimal solution of (9). If $\mathbf{Q}-\gamma\mathbf{Z}+\alpha\mathbf{I} \succ \mathbf{0}$, then $\gamma_1=\gamma_2$.*

4. DERIVATIONS AND PROOF

The formulation of (4) and (5) as SDPs is based on optimizing the SINRs instead of the rates, and rewriting each of these problems as a minimization of a ratio of quadratic forms subject to a quadratic constraint (RQ). Then, we use a recent result that transforms such problems into SDPs [12].

We begin with (4). It is easy to see that after omitting the $\log(1+x)$ function, the inner minimization

$$\min_{\|\mathbf{g}-\mathbf{h}\|^2\leq\varepsilon} \frac{\mathbf{g}^T\mathbf{Q}\mathbf{g}}{\mathbf{g}^T\mathbf{Z}\mathbf{g}+1} \tag{12}$$

is an RQ. We define

$$\mathbf{y} = s\begin{bmatrix} \mathbf{g} \\ 1 \end{bmatrix} \tag{13}$$

where $s \neq 0$, and rewrite (12) as[2]

$$\begin{array}{lll} \min & \mathbf{y}^T\begin{bmatrix} \mathbf{Q} & \mathbf{0} \\ \mathbf{0}^T & 0 \end{bmatrix}\mathbf{y} \\ \text{s.t.} & \mathbf{y}^T\begin{bmatrix} \mathbf{Z} & \mathbf{0} \\ \mathbf{0}^T & 1 \end{bmatrix}\mathbf{y} = 1 \\ & \mathbf{y}^T\begin{bmatrix} \mathbf{I} & -\mathbf{h} \\ -\mathbf{h} & \mathbf{h}^T\mathbf{h}-\varepsilon \end{bmatrix}\mathbf{y} \leq 0 \end{array} \tag{14}$$

Due to a special case of strong duality, (14) can be solved using its Lagrange dual (see [12] for more details)

$$\max_{\gamma,\alpha\geq 0}\min_{\mathbf{y}}\left\{\mathbf{y}^T\begin{bmatrix} \mathbf{Q} & \mathbf{0} \\ \mathbf{0}^T & 0 \end{bmatrix}\mathbf{y} + \gamma\left(1-\mathbf{y}^T\begin{bmatrix} \mathbf{Z} & \mathbf{0} \\ \mathbf{0}^T & 1 \end{bmatrix}\mathbf{y}\right) + \alpha\mathbf{y}^T\begin{bmatrix} \mathbf{I} & -\mathbf{h} \\ -\mathbf{h} & \mathbf{h}^T\mathbf{h}-\varepsilon \end{bmatrix}\mathbf{y}\right\}. \tag{15}$$

Quadratic forms are bounded from below only if they are positive semidefinite. Therefore, the dual is

$$\begin{array}{ll} \max_{\alpha\geq 0,\gamma} & \gamma \\ \text{s.t.} & \begin{bmatrix} \mathbf{Q}-\gamma\mathbf{Z}+\alpha\mathbf{I} & -\alpha\mathbf{h} \\ -\alpha\mathbf{h}^T & \alpha\left(\mathbf{h}^T\mathbf{h}-\varepsilon\right)-\gamma \end{bmatrix} \succeq \mathbf{0} \end{array} \tag{16}$$

[1] See (18) in the proof below.

[2] Actually, there is an additional implicit constraint $[\mathbf{y}]_{K+1} \neq 0$ which ensures that $s \neq 0$, but it is easy to see that this constraint is implied by the other constraints since $\mathbf{y}=\mathbf{0}$ is infeasible.

Omitting the $\log(1+x)$ function and replacing the inner minimization in (4) with (16) yields (9).

We now turn to (5). Again, we omit the $\log(1+x)$ function and obtain the following simple solution for the inner maximization

$$P\frac{\mathbf{g}^T\mathbf{g}}{\mathbf{g}^T\mathbf{Z}\mathbf{g}+1} = \max_{\substack{\mathbf{Q}\succeq\mathbf{0} \\ \mathrm{Tr}\{\mathbf{Q}\}\leq P}} \frac{\mathbf{g}^T\mathbf{Q}\mathbf{g}}{\mathbf{g}^T\mathbf{Z}\mathbf{g}+1} \tag{17}$$

along with the optimal argument

$$\mathbf{Q} = P\frac{\mathbf{g}\mathbf{g}^T}{\mathbf{g}^T\mathbf{g}}. \tag{18}$$

From a communication theory point of view, this is the well known result that when the channel is known, the optimal linear transmit method is beamforming along it. Plugging this result into the outer minimization yields

$$\min_{\|\mathbf{g}-\mathbf{h}\|^2\leq\varepsilon} \frac{P\mathbf{g}^T\mathbf{g}}{\mathbf{g}^T\mathbf{Z}\mathbf{g}+1} \tag{19}$$

which is again an RQ. As before, we define $\mathbf{y}$ as in (13). This yields

$$\begin{array}{lll} \min & \mathbf{y}^T\begin{bmatrix} P\mathbf{I} & \mathbf{0} \\ \mathbf{0}^T & 0 \end{bmatrix}\mathbf{y} \\ \text{s.t.} & \mathbf{y}^T\begin{bmatrix} \mathbf{Z} & \mathbf{0} \\ \mathbf{0}^T & 1 \end{bmatrix}\mathbf{y} = 1 \\ & \mathbf{y}^T\begin{bmatrix} \mathbf{I} & -\mathbf{h} \\ -\mathbf{h} & \mathbf{h}^T\mathbf{h}-\varepsilon \end{bmatrix}\mathbf{y} \leq 0. \end{array} \tag{20}$$

and its dual

$$\max_{\gamma,\alpha\geq 0}\min_{\mathbf{y}}\left\{\mathbf{y}^T\begin{bmatrix} P\mathbf{I} & \mathbf{0} \\ \mathbf{0}^T & 0 \end{bmatrix}\mathbf{y} + \gamma\left(1-\mathbf{y}^T\begin{bmatrix} \mathbf{Z} & \mathbf{0} \\ \mathbf{0}^T & 1 \end{bmatrix}\mathbf{y}\right) + \alpha\mathbf{y}^T\begin{bmatrix} \mathbf{I} & -\mathbf{h} \\ -\mathbf{h} & \mathbf{h}^T\mathbf{h}-\varepsilon \end{bmatrix}\mathbf{y}\right\} \tag{21}$$

which is given by (10).

We now prove the theorem by showing equivalence between the Lagrange duals of (9) and (10). The dual of (9) is

$$\gamma_1 = \left\{\begin{array}{ll} \min_{\mathbf{W}\succeq\mathbf{0},\lambda\geq 0} & \lambda P \\ \text{s.t.} & \lambda\mathbf{I} - \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix}\mathbf{W}\begin{bmatrix} \mathbf{I} \\ \mathbf{0}^T \end{bmatrix} \succeq \mathbf{0} \\ & \mathrm{Tr}\left\{\mathbf{W}\begin{bmatrix} \mathbf{Z} & \mathbf{0} \\ \mathbf{0}^T & 1 \end{bmatrix}\right\} = 1 \\ & \mathrm{Tr}\left\{\mathbf{W}\begin{bmatrix} \mathbf{I} & \mathbf{h} \\ \mathbf{h}^T & \mathbf{h}^T\mathbf{h}-\varepsilon \end{bmatrix}\right\} \leq 0 \end{array}\right. \tag{22}$$

which can be written as

$$\gamma_1 = \left\{\begin{array}{ll} \min_{\mathbf{W}\succeq\mathbf{0},\lambda\geq 0} & P\lambda_{\max}\left(\begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix}\mathbf{W}\begin{bmatrix} \mathbf{I} \\ \mathbf{0}^T \end{bmatrix}\right) \\ \text{s.t.} & \mathrm{Tr}\left\{\mathbf{W}\begin{bmatrix} \mathbf{Z} & \mathbf{0} \\ \mathbf{0}^T & 1 \end{bmatrix}\right\} = 1 \\ & \mathrm{Tr}\left\{\mathbf{W}\begin{bmatrix} \mathbf{I} & \mathbf{h} \\ \mathbf{h}^T & \mathbf{h}^T\mathbf{h}-\varepsilon \end{bmatrix}\right\} \leq 0. \end{array}\right. \tag{23}$$

Similarly, the dual of (10) (which is also the bidual and the tight SDP relaxation of (20)) is

$$\gamma_2 = \begin{cases} \min_{\mathbf{W}\succeq \mathbf{0},\lambda\geq 0} & P\mathrm{Tr}\left\{\left[\begin{array}{cc}\mathbf{I} & \mathbf{0}\end{array}\right]\mathbf{W}\left[\begin{array}{c}\mathbf{I}\\ \mathbf{0}^T\end{array}\right]\right\} \\ \text{s.t.} & \mathrm{Tr}\left\{\mathbf{W}\left[\begin{array}{cc}\mathbf{Z} & \mathbf{0}\\ \mathbf{0}^T & 1\end{array}\right]\right\} = 1 \\ & \mathrm{Tr}\left\{\mathbf{W}\left[\begin{array}{cc}\mathbf{I} & \mathbf{h}\\ \mathbf{h}^T & \mathbf{h}^T\mathbf{h}-\varepsilon\end{array}\right]\right\} \leq 0. \end{cases} \quad (24)$$

It remains to show that if $\mathbf{Q}-\gamma\mathbf{Z}+\alpha\mathbf{I}\succ\mathbf{0}$, then (23) and (24) are equivalent. The positive definiteness implies that

$$\mathrm{rank}\left(\mathbf{Q}-\gamma\mathbf{Z}+\alpha\mathbf{I}\right) = n \quad (25)$$

and that

$$\mathrm{rank}\left(\left[\begin{array}{cc}\mathbf{Q}-\gamma\mathbf{Z}+\alpha\mathbf{I} & \alpha\mathbf{h}\\ \alpha\mathbf{h}^T & \alpha\left(\mathbf{h}^T\mathbf{h}-\varepsilon\right)-\gamma\end{array}\right]\right) \geq n. \quad (26)$$

Due to the complementary slackness condition associated with the first constraint in (9) we have

$$\mathbf{W}\left[\begin{array}{cc}\mathbf{Q}-\gamma\mathbf{Z}+\alpha\mathbf{I} & \alpha\mathbf{h}\\ \alpha\mathbf{h}^T & \alpha\left(\mathbf{h}^T\mathbf{h}-\varepsilon\right)-\gamma\end{array}\right] = \mathbf{0}, \quad (27)$$

where $\mathbf{W}\succeq\mathbf{0}$ is the Lagrange dual matrix in (23). Combining (26) and (27) yields

$$\mathrm{rank}\left(\mathbf{W}\right) \leq 1. \quad (28)$$

Finally, if the rank of a matrix is less than or equal to one, then $\lambda_{\max}\{\cdot\} = \mathrm{Tr}\{\cdot\}$. Thus, the objective functions of (23) and (24) are equal, i.e., $\gamma_1 = \gamma_2$.

5. NUMERICAL EXAMPLE

In this section, we provide more insight using a numerical example. The SDP formulations in Section 3 allow us to numerically solve the maxmin and the minmax problems and compare their solution. The parameters in this example are as follows. The number of transmit antennas is $K = 4$. The interference is modeled as $\mathbf{Z} = \mathbf{Y}\mathbf{Y}^T$ where $\mathbf{Y}$ is a $K\times K$ matrix with independent, zero mean and unit variance normal random variables. Similarly, the channel $\mathbf{h}$ is a length K vector with independent, zero mean and unit variance normal random variables. The volume of the ellipsoid is given by $\varepsilon = 0.3\mathbf{h}^T\mathbf{h}$. The available transmitted power is $P = 10$. Given these parameters, we estimated the probability density function (PDF) of R_1 and R_2 using Monte Carlo simulations. For comparison, we also simulated the resulting rate R_0 using the naive approach where

$$\mathbf{Q} = P\frac{\mathbf{h}\mathbf{h}^T}{\mathbf{h}^T\mathbf{h}} \quad (29)$$

i.e., when the transmitter designs its transmission assuming that $\mathbf{g} = \mathbf{h}$. The results are presented in Figure 1.

It is easy to see that the naive approach performs worse than both the maximin and the minimax approaches. Comparing the PDF of R_1 and the PDF of R_2 shows that there is a slight advantage for perfect CSI. However this advantage is very small and it seems that in most applications the maximin approach in (9) performs sufficiently well. It is not shown in the figure, but the resulting $\mathbf{Q}$s in all of the simulations were always of rank one, i.e., in this specific example beamforming was always found optimal. However, it is not difficult to find a case where this is not true.

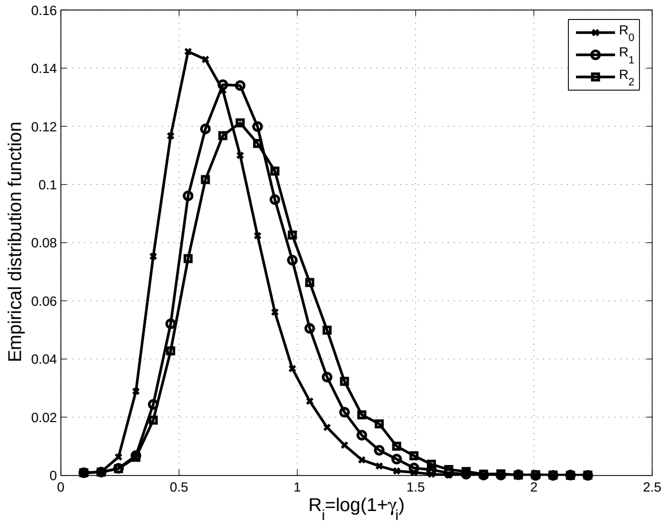

Figure 1: Estimated probability density function of the achievable rates R_0, R_1 and R_2.

6. CONCLUSIONS

Motivated by the growing use of multiuser broadcast channels and the necessity of CSI in the transmitter side, we considered the problem of linear transmitter design in MISO compound channels with interference. We derived new efficient algorithms for the transmitter design with and without perfect CSI. Interestingly, even in this simplified linear setting there are many surprising results. For example, the existence of a saddle point and the optimality of beamforming are no longer necessarily true. These results motivate the continuation of research on this simple channel model. In addition, future work should also consider the application of non linear transmission schemes for the compound channel with interference.

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