

PERFORMANCE INDICES OF BSS FOR REAL-WORLD APPLICATIONS

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ABSTRACT

This paper deals with the independence measure problem. Over the last decade, many Independent Component Analysis (ICA) algorithms have been proposed to solve the blind source separation (BSS) of convolutive mixture. However few performance indices can be found in the literature. The most used performance indices are described hereafter and three new performance indices are also proposed.

1. INTRODUCTION

We are involved in Passive Acoustic Tomography (PAT) problem. It is well known that Acoustic Tomography can be applied in many civil or military applications as : Mapping underwater surfaces, meteorological applications, to improve sonar technology. Recently, the Passive Acoustic Tomography (PAT) has taken an increased importance mainly for the three following reasons : Submarine Acoustic Warfare applications, Ecological reasons (it doesn't perturb underwater ecological system) and Economical and logistical reasons.

In PAT applications, the emitted signals are natural or artificial signals of opportunity. Therefore, PAT applications can be considered as a serious challenge to the classical Active Acoustic Tomography (AAT), since the parameters (number, position, etc.) of emitted signals as well as these signals are unknown. In such scenario, the received signals are the mixture of some acoustic signals of opportunity. Blind Source Separation (BSS) algorithms obviously are of great importance to our project, see [1].

In the literature, one can find a huge number of Independent Component Analysis (ICA) algorithms to solve BSS problem. Most of them are dedicated to the separation of instantaneous (i.e. echo free) channel. In our application, the underwater acoustic propagation channel can be modeled by a convolutive mixture (i.e a multi path and a Multi-Input-Multi-Out FIR channel with huge filter order ≥ 6000). It is well known that the BSS of convolutive mixture can lead us to the original sources up to a permutation and scalar filter :

$$\hat{s}_1(n) = h_1(z) * s_1(n) + h_2(z) * s_2(n) \quad (1)$$

where $s_2(n)$ represents a mixture of all the sources except the first one $s_1(n)$. The filter $h_i(z) = h_i(0) + h_i(1)z^{-1} + \dots + h_i(m_i)z^{-m_i}$ are the residual separation filter. In the following, we denote by N_{sig} the source number and by N_s the number of available samples. The separation is considered achieved when ever the norm of the residual error $h_2(z) * s_2(n)$ becomes much less than the one of the separated signal $h_1(z) * s_1(n)$. In addition, we should mention that the identification or the classification of underwater acoustic signals is very hard because these signals are non-stationary

and non-intelligible Gaussian or close to Gaussian signals. In this context, the classification of ICA algorithms according to the separation quality becomes a difficult and important task. Previously, we proposed [2] a survey of the performance indices used in instantaneous mixture case. In this paper, the real acoustic convolutive model is considered. The most used performance indices are described hereafter and three new performance indices are also proposed.

2. MODIFIED CROSSTALK

The crosstalk is the inverse of Signal to Noise Ratio (SNR) and it is widely used as a performance index for the BSS algorithms of instantaneous mixture, see [2] and the references therein. By definition the crosstalk index of the first estimated signal, is given by :

$$D_r(\hat{s}_1, s_1) = 10 \log_{10} \left(\frac{E\{(\hat{s}_1 - s_1)^2\}}{E\{s_1^2\}} \right) \quad (2)$$

here E stands for the expectation. To apply the crosstalk, one should have the original source. Therefore this performance index cannot be applied in real situation where the source are unknown. However it is very useful in simulations.

It is clear that the last definition D_r is useless for the BSS convolutive mixture, see equation (1), since it doesn't take into consideration the power ratio between the filtered version of the signal $\xi_1 = h_1(z) * s_1(n)$ and the residual error $h_2(z) * s_2(n)$.

Hereafter, we suggest a modified definition for the crosstalk. At first, one should apply (2) as $D_r(\hat{s}_1, \xi_1)$. Secondly an estimated $h_1(z)$ should be obtained using $s_1(n)$ and the estimated signal \hat{s}_1 . To estimate $h_1(z)$, one can minimize the Least Mean Square (LMS) error ζ :

$$\hat{h}_1 = \min_h E(\hat{s}_1 - h * s_1)^2 = \min_h \zeta \quad (3)$$

Let $H_i = (h_i(0) \dots h_i(m_i))^T$ and $S_i = (s_i(n) \dots s_i(n - m_i))^T$, the convolutive product in equation (1) becomes a simple scalar product :

$$h_1(z) * s_1(n) = H_1^T S_1$$

Using the independence properties of the sources, one can easily prove that :

$$\begin{aligned} \zeta &= (H_1 - H)^T E(S_1 S_1^T) (H_1 - H) + H_2^T E(S_2 S_2^T) H_2 \\ &= \epsilon_H^T \Sigma_1 \epsilon_H + H_2^T \Sigma_2 H_2 \end{aligned} \quad (4)$$

$$= E(\hat{s}_1)^2 + H^T \Sigma_1 H - H^T E(S_1 \hat{s}_1) - E(\hat{s}_1 S_1^T) H \quad (5)$$

where $\varepsilon_H = H_1 - H$ and $\Sigma_i = E(S_1 S_1^T)$ is an invertible definite positive matrix. The second term of (4) doesn't depend on H . Therefore, one can prove that the optimal value of H is given by :

$$H_{opt} = \Sigma^{-1} E(S_1 \hat{s}_1) \quad (6)$$

Our experimental results show that for low order channel filter (less than 20) this performance index can be used efficiently. When the order of channel is larger than 20, computing time becomes more important. Unfortunately, we couldn't get good results using this performance index on our acoustic sounds and underwater channel.

3. MUTUAL INFORMATION

Mutual information is used as criteria in many ICA algorithms [3, 4]. According to [5], mutual information is one of the best independence indices. The mutual information is defined as following :

$$I(p_U) = \int p_U(V) \log \frac{p_U(V)}{\prod_{i=1}^N p_{u_i}(v_i)} dV \quad (7)$$

where $U = (u_1, \dots, u_n)^T$ is a random vector and $P_U(V)$ (resp. $p_{u_i}(v_i)$) are the joint (resp. marginal) probability density function (PDF). In the context of BSS problem, the joint and the marginal PDF are unknown but they can be estimated [6].

To estimate the mutual information in our project, we used a method proposed recently by Pham [7]. In his method, the integral is replaced by a discrete sum and the PDF are estimated using kernel methods. In [7], spline functions¹ of third order have been used as kernel function. Finally, the mutual information estimator is given by :

$$\hat{I}(u_1, \dots, u_n) = \sum_i \hat{\pi}_U(i) \log \left(\frac{\hat{\pi}_U(i)}{\prod_k \hat{\pi}_{u_k}(i_k)} \right) \quad (8)$$

Here $\hat{\pi}_U(i)$ is the joint PDF estimator and $\hat{\pi}_{u_k}(j)$ is the marginal PDF estimator. Even though we got good results with stationary signals, we couldn't get similar results for underwater acoustic signals.

4. QUADRATIC DEPENDENCE

Mutual information isn't the only independence index used in the literature. To measure the independence among the components of a random vector $\mathbf{X} = (x_1, \dots, x_n)^T$, the authors of [9] make a comparison between the joint PDF of the vector \mathbf{X} and the marginal PDF product of its components x_i . Using similar approach, Kankainen in [10] propose an independence index based on the quadratic dependence measure and the First Characteristic Function (FCF), i.e. $\Phi(\Omega) = E\{\exp(j\Omega^T \mathbf{X})\}$. In [8], Achard *et al.* proposed a method to apply the last independence index in the context of nonlinear blind source separation problem.

¹Spline function of order r is the PDF of the sum of r uniform independent random variables $u_i \in [-0.5, 0.5]$. For example, the spline function of third order is defined as :

$$\mathbb{K}_3(u) = \begin{cases} \frac{3}{4} - u^2 & \text{If } |u| \leq \frac{1}{2} \\ \frac{(1.5 - |u|)^2}{2} & \text{If } 0.5 \leq |u| \leq 1.5 \\ 0 & \text{Elsewhere} \end{cases}$$

The quadratic independence measure $D(\mathbf{X})$ is a comparison between the joint FCF and the product of the marginal FCF, [10] :

$$D(\mathbf{X}) = \int |\Phi(\Omega) - \prod_{k=1}^n \Phi(\Omega_i)|^2 h(\Omega) d\Omega \quad (9)$$

Here h is an integrable function from \mathbb{R}^n to \mathbb{R} . If the components of the vector \mathbf{X} are independent in their set than the joint FCF is equal to the product of the marginal FCF (i.e. $\Phi(\Omega) = \prod_{i=1}^n \Phi(\Omega_i)$) and $D(\mathbf{X}) = 0$. Function h should satisfy the following two conditions, see [10] :

- h is a non zero almost every where and a positive function.
- For analytical FCF $\Phi(\Omega)$, h should be positive around zero and vanish elsewhere.

Achard *et al.* in [8] proposed the following h :

$$h(\Omega) = \prod_{i=1}^n \left| \frac{\sqrt{\sigma_{X_i}} \Phi_{\mathbb{K}}(\sigma_{X_i} \Omega_i)}{\sqrt{2\pi}} \right|^2 \quad (10)$$

Here \mathbb{K} is a square integrable kernel function that its Fourier transform should be non zero almost every where and σ_{X_i} is a scale factor (i.e. a positive function only depends on the PDF of X_i). Using the energy conservation theorem of Parseval, Achard *et al.* in [8] shows that equation (9) can be replaced by the following function :

$$Q(\mathbf{X}) = \frac{1}{2} \int_{\mathbb{R}^n} D(\mathbf{T})^2 d\mathbf{T} \quad (11)$$

where $D(\mathbf{T}) = E \left[\prod_{i=1}^n \mathbb{K} \left(t_i - \frac{x_i}{\sigma_i} \right) \right] - \prod_{i=1}^n E \left[\mathbb{K} \left(t_i - \frac{x_i}{\sigma_i} \right) \right]$.

The authors of [8] prove that $Q(\mathbf{X}) = 0 \Leftrightarrow x_i$ are independent from each other. In [11], Achard estimates Q as following :

$$\hat{Q}(\mathbf{X}) = \frac{1}{2} \hat{E}\{F(\mathbf{X})\} + \frac{1}{2} \prod_{i=1}^n \hat{E}\{f(x_i)\} - \hat{E} \left\{ \prod_{i=1}^n f(x_i) \right\}$$

Here $f(x_k) = \frac{1}{N_s} \sum_{i=1}^{N_s} \mathbb{K} \left(\frac{x_k - X_k(i)}{\sigma_k} \right)$,

$F(\mathbf{X}) = \frac{1}{N_s} \sum_{i=1}^{N_s} \prod_{k=1}^n \mathbb{K} \left(\frac{x_k - X_k(i)}{\sigma_k} \right)$, $X_k(i)$ is the i th sample of the k th component of \mathbf{X} and \hat{E} is the empirical mean. Function \mathbb{K} can be chosen from the following functions, [11] :

1. Gaussian Kernel $\mathbb{K}_1(x) = \exp(-x^2)$
2. Square Gaussian Kernel $\mathbb{K}_2(x) = \frac{1}{(1+x^2)^2}$
3. The inverse of Square Gaussian Kernel second derivative function $\mathbb{K}_3(x) = -\frac{4-20x^2}{(1+x^2)^2}$

In our experimental studies, best results were obtained using the Gaussian Kernel. In fact, the Gaussian Kernel gives the largest possible difference between the quadratic independence measure applied on a vector A with i.i.d uniformly independent components and the quadratic independence measure applied on a vector $B = MA$, M is a full rank mixing matrix. Using 2000 samples and random signals, we found $D(\mathbf{A}) = -68$ and $D(\mathbf{B}) = -28$.

Signals	Mixture Model	NL-Decorrelation of Sources	NL-Decorrelation of Mixed Signals
i.i.d Uniform PDF	Instantaneous	Kernel 'Gaussian' -23.4	Kernel 'Gaussien' -5.8319
		Kernel 'poly' -25.5	Kernel 'poly' 8.1
		Kernel 'hermite' -22.4	Kernel 'hermite' -20.4
4 Acoustic Signals 2000 samples	Instantaneous Convulsive	Kernel 'poly' -33.4	Kernel 'poly' 3.2 Kernel 'poly' -14.9817
4 Acoustic Signals $4 * 10^5$ samples	Instantaneous Convulsive	Kernel 'poly' -31.3	Kernel 'poly' 8.8 Kernel 'poly' -13.2

TAB. 1 – NL-Decorrelation applied on source and mixed signals using different kernels, Gaussian, Polynomial and Hermite functions.

The main drawback of such performance index is the important computing time, few minutes are needed to get the results over a random signals of 2000 samples. In our application, the underwater acoustic signals are very close to Gaussian signals that means a huge number of samples (over a million samples) are needed to achieve the separation of such signals. Therefore, we couldn't consider this performance index in our project.

5. NON-LINEAR KERNEL DECORRELATION

The authors of [12, 13] propose an ICA algorithm as well as an independence measure based on the concept of Non-linear Decorrelation. To achieve the source separation, the authors minimize the following \mathbb{F} -correlation function $\rho_{\mathbb{F}}$:

$$\begin{aligned} \rho_{\mathbb{F}} &= \max_{f,g \in \mathbb{F}} \text{Corr}(f(X), g(Y)) \\ &= \max_{f,g \in \mathbb{F}} \frac{\text{Cov}(f(X), g(Y))}{\sqrt{\text{Var}(f(X)) \text{Var}(g(Y))}} \end{aligned} \quad (12)$$

We call $\text{Corr}(X, Y)$, $\text{Cov}(X, Y)$ and $\text{Var}(X)$ respectively the correlation, the covariance and the variance of X and Y . We should mention here that \mathbb{F} is a vectorial space of all functions applied from \mathbb{R} to \mathbb{R} . It is known that when \mathbb{F} contents all Fourier transform basis (i.e. the exponential functions $\exp(jwx)$ with $w \in \mathbb{R}$) than $\rho_{\mathbb{F}} = 0$ means the independence of the random variables X and Y .

The algorithm of [12] can be considered as Canonical Correlation Analysis (CCA) which is a generalized version of classical Principal Component Analysis (PCA). It is well known that PCA can be done using an EigenValue Decomposition (EVD) of decorrelation matrices. According to [12], CCA can be considered as the EVD of a huge $N_{sig}N_s \times N_{sig}N_s$ decorrelation matrix.

According to [12], the best choice of the two non-linear functions f and g can be done using Mercer Kernel functions². $K(X, Y)$ should also have the translation invariance property, the convergence property in $L^2(\mathbb{R}^m)$ and isotropic property. One possible kernel is the Gaussian kernel proposed by the authors of [12] :

$$K(x, y) = \exp\left(-\frac{1}{2\sigma^2} \|x - y\|^2\right) \quad (13)$$

²A bilinear function $K(X, Y)$ from a vectorial space X (for example \mathbb{R}^m) to \mathbb{R} is said to be a Mercer kernel iff its Gram matrix is a semi-positive matrix. By definition the Gram matrix of basis vectors X_1, \dots, X_m of a m dimensional vectorial space X with respect to a bilinear function $K(X, Y)$ is the matrix given by $G_{ij} = K(X_i, X_j)$.

Table 1 shows Experimental results obtained by applying NL-Decorrelation on source signals and mixed signals using three different kernels, Gaussian, Polynomial and Hermite functions. We should notice that for acoustic signals better results are obtained using polynomial kernel. Our experimental studies show that this performance index can be applied successfully in our project. However, computing time and needed memory become very important when the number of samples is over 500000 samples. Finally, we should mention that the difference between the NL-Decorrelation of the sources and the mixed signals depends on the original signals, the chosen kernel, as well as the mixing model and parameters.

6. SIMPLIFIED NON-LINEAR DECORRELATION

Using similar approach to the previous one [12, 13], we propose here a simplified performance index based on the concept of non-linear covariance matrix. Let us define the following matrix $Y = (\rho_{ij})$ as the non-linear covariance matrix :

$$\rho_{ij} = \frac{E(\langle f(x_i) \rangle_c \langle g(x_j) \rangle_c)}{\sqrt{E(\langle f(x_i) \rangle_c^2) E(\langle g(x_j) \rangle_c^2)}} \quad (14)$$

where $X = (x_i)$ is a random vector, $f(x)$ and $g(x)$ are two non-linear functions, and $\langle x \rangle_c = x - E\{x\}$. If the components of X are independent from each other than we can prove that Y becomes a diagonal matrix. using the last definition, we suggest the following performance index :

$$c = 20 \log \left(\frac{\|\text{Off}(Y)\|^2}{\|\text{diag}(Y)\|^2} \right) \quad (15)$$

Here $\text{diag}(M)$ is a diagonal matrix which has the same principal diagonal of matrix M and $\text{Off}(M) = M - \text{diag}(M)$. The two functions f and g are chosen from the following functions :

1. 'Gauss' : Gaussian kernel.
2. 'poly' : 6 order polynomial Kernel which the coefficients are the components of an unitary vector.
3. 'atan' : Saturation kernel using arc-tangent function.
4. 'tanh' : Saturation kernel using hyperbolic tangent function.

Our experimental studies (see table 2) show the effectiveness of this performance index to deal with underwater acoustic signals and channels. The main drawback of this performance index is that the obtained values depend on the

Signals	Mixture Model	NL-Decorrelation of Sources	NL-Decorrelation of Mixed Signals
i.i.d Uniform PDF uniform	Instantaneous	Kernel 'Gaussian' -66.3211	Kernel 'Gaussian' -40.6513
		Kernel 'poly' -49.2054	Kernel 'poly' -6.6205
		Kernel 'atan' -63.2202	Kernel 'atan' -0.0802
		Kernel 'tanh' -52.5625	Kernel 'tanh' 0.1597
4 Acoustic Signals 2000 samples	Instantaneous Convulsive	Kernel 'atan' -40.7142	Kernel 'atan' 1.5864 Kernel 'atan' -31.8532
4 Acoustic Signals 4 * 10 ⁵ samples	Instantaneous Convulsive	Kernel 'tanh' -86.6931	Kernel 'tanh' 1.0391 Kernel 'tanh' -57.4885

TAB. 2 – Simplified NL-Decorrelation applied on source and mixed signals using different kernels.

kind and number of the original independent signals. Therefore this performance index can only be used in simulations where the original sources are known.

7. INDEPENDENCE MEASURE BASED ON THE FIRST CHARACTERISTIC FUNCTION

In the last few decades, many signal processing researchers were involved in independence measurement problem. In [9] and to measure the independence among random signals, the authors proposed a joint PDF estimator. In [14], the authors propose a study and an estimator $\Phi_n(t)$ of First Characteristic Function (FCF) $\Phi(t)$:

$$\Phi_n(t) = \frac{1}{n} \sum_i \exp(\sqrt{-1}tX_i) \quad (16)$$

Here X is a random iid signal with n samples and X_i is the i th realization of X . The authors proved that : If $Y_n(t) = \{\Phi_n(t) - \Phi(t)\}\sqrt{n}$ is the residual estimation error than $Y_n(t)$ is a zero-mean complex Gaussian random variable. They also proved that $\text{Prob} \left\{ \lim_{n \rightarrow \infty} \sup_{|t| < T} |\Phi_n(t) - \Phi(t)| = 0 \right\} = 1 \quad \forall T \in \mathbb{R}$.

We mentioned before that the joint FCF of a random vector $X = (x_1, \dots, x_n)^T$ is equal to the product of the marginal FCF of its components iff these components are independent from each other. Using the previous property of the FCF, Feuerverger in [15] proposed an independence measure based on the FCF of two random signals X and Y :

$$\begin{aligned} T_n &= \frac{\pi^2}{n^2} \sum_{ij} g(X'_j - X'_i) g(Y'_j - Y'_i) \\ &\quad - \frac{2\pi^2}{n^3} \sum_{ijk} g(X'_j - X'_i) g(Y'_j - Y'_k) \\ &\quad + \frac{\pi^2}{n^4} \sum_{ijkl} g(X'_j - X'_i) g(Y'_k - Y'_l) \end{aligned} \quad (17)$$

where g is an adequately chosen function (see [15] for further details), $X' = \Phi^{-1} \left(\frac{8X-3}{8n+2} \right)$ is the approximation of the score function of X , and $\Phi(X)$ is the PDF of zero mean and unit variance Gaussian signal. Our experimental studies show that the computing time is the main drawback of this performance index. We should mention that for stationary signals, this performance index is a consistence one. Unfortunately, the last nice property is useless in our application since the acoustic signals are non-stationary signals.

Recently, Murata in [16] proposed a simplified test to measure the independence between two random signals. This independence measure is also based on the estimation of the cross FCF :

$$\Phi_n^{XY}(t, s) = \frac{1}{n} \sum_i \exp(jtX_i + jsY_i) \quad (18)$$

If X and Y are independent than $\Phi_{XY}(t, s) = \Phi_X(t)\Phi_Y(s)$. Murata's independence measure is defined by the following equation :

$$\int_{\mathbb{R}^2} \left\| \{ \Phi_n^{XY}(t, s) - \Phi_n^X(t)\Phi_n^Y(s) \} \sqrt{nk}(t, s) \right\| dt ds \quad (19)$$

$k(t, s)$ is a bounded estimation window. Let $\text{Re}(X)$ and $\text{Im}(X)$ denote the real and the imaginary part of X . Using the fact that $Z_n(t, s)$ becomes asymptotically Gaussian, Murata proved that the following random variable :

$$T(t, s) = (\text{Re}(Z_n(t, s)) \quad \text{Im}(Z_n(t, s))) \Sigma^{-1} \begin{pmatrix} \text{Re}(Z_n(t, s)) \\ \text{Im}(Z_n(t, s)) \end{pmatrix}$$

is a central chi-squared random variable of second order. Σ is a specific 2×2 symmetrical matrix based on the variance and the covariance of the components of $Z_n(t, s)$, further details can be founded in [16]. Using the fact that $T(t, s)$ is a central chi-squared random variable, one can easily prove that $\text{Prob} \{ T(t, s) \leq 5.9915 \} = 0.95$. Our experimental studies show that :

- The norm of $Z_n(t, s)$ often gives better results than the minimum value of $T(t, s)$.
- The obtained values depend on the original sources. This inconvenient is common to previous performance indices.
- For beta random variable, good results have been obtained. On the other hand, we noticed bad results for uniform random signals.
- For acoustic signals, we noticed good results for instantaneous mixture and bad ones for convolutive mixtures.
- Computing time is important.

8. CROSS-CUMULANTS

As we mentioned before that the previously described performance indices cannot be applied in real situations where the original signals are unknown because the performance values depend on the sources. Therefore, we develop here a real situation performance index based on cross-cumulant :

$$\text{Perfc} = \frac{\overline{\text{Cum}_{(1,3)}(X,Y)^2} + \overline{\text{Cum}_{(3,1)}(X,Y)^2}}{\text{Var}(X)\text{Var}(Y)} \quad (20)$$

Here $\overline{\text{Cum}_{(1,3)}(X,Y)^2}$ is the average of $\text{Cum}_{(1,3)}(X,Y)^2$ which is obtained using a sliding estimation window, see [17]. The index of equation (20) is limited to two signals. To generalize this index to the case of multi-signals, we can use the following :

$$\text{PerfCG}(X) = \text{Off}(\Gamma) \quad (21)$$

where $\Gamma = (\text{Perfc}(X_i, X_j))$ and $\text{Off}(\Gamma) = \sum_{i \neq j} \gamma_{ij}^2$. Finally, we should mention that good results have been obtained using this performance index on instantaneous or convolutive mixture of acoustic signals. However, the computing time is relatively important.

9. CONCLUSION

In this paper, a survey of major performance indices of blind source separation of convolutive mixture is addressed. Four well known and widely used performance indices are described here. Besides, three new performance indices have been developed. The advantages and the drawbacks of these performance indices are given here. All the simulations have been done using simulated signals and real underwater acoustic signals. These performance indices are used to classify the various Independent Component Analysis algorithms of the literature according to their separation achievement in passive acoustic tomography context. As it has been mentioned before, some of these indices show satisfactory results in our application. Many of the last mentioned indices are time consuming. Up to now, the simplified non-linear decorrelation is the most adapted performance index to our application.

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