A NEW 3D SHAPE-DEPENDANT SKELETONIZATION METHOD. APPLICATION TO POROUS MEDIA

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ABSTRACT

This communication presents a new method to compute a precise shape-dependant skeleton and its application to porous media. The local geometry of the object's structure is taken into account in order to switch between curve and surface thinning criterions. The resulting skeleton combines 2D surfaces and 1D curves to represent respectively the plate-shaped and rod-shaped parts of the object. First, methods to compute the shape-dependant skeleton are described: rod and plate classification, surface and curve thinning. Then, applications of the technique are presented in the field of biomedical imaging (trabecular bone) and geology (sandstone). A clinical study is led on 2 sets of bone samples. It shows the ability of the skeleton to characterize the trabecular bone microarchitecture.

1. INTRODUCTION

Since the early work of Morgenthaler [1], 3D skeletonization has shown a strong potential in porous media analysis. The curve skeleton provides efficient topological information, and its oversimplified geometry makes it easy to create structural models. However, parasite curves appear in the non-beam-shaped zones, as it is sensible to the surface irregularities. On the contrary, the surface skeleton better respects the plates geometry, but is not efficient in rod-shaped zones.

In the case of disordered porous media like sand, soil or trabecular bone, the material is usually heterogeneous, composed of rod-shaped and plate-shaped elements. Curve thinning and surface thinning are efficient respectively for one or the other, but not for both. The aim of this work is to provide a tool for skeletonizing hybrid-shaped media composed of rods and plates.

The hybrid skeletonization method relies on an original combination of 3 techniques: a recently published shape classification to distinguish rods and plates [2], the MESPTA (Modified Extended Safe Point Thinning Algorithm) [3] to compute the surface skeleton and a sequential Betti numbers based 3D thinning algorithm [1] to compute the curve skeleton.

After validating the new hybrid skeleton on a 3D synthesis image, hybrid skeletons of trabecular bone and sandstone samples are presented to illustrate different applications of the algorithm in the field of biomedical imaging and geology. A clinical study of trabecular bone samples is then led to show the efficiency of the hybrid skeleton based analysis. Skeletons of 2 pathologically-different sets of bone samples are computed and morphological and topological parameters are extracted.

2. THE HYBRID SKELETON

Most of thinning-based skeletons preserve topology. Thus, the geometry usually governs the choice of the suitable skeleton type to characterize a porous medium. For example, curve thinning is efficient for modeling rod-shaped zones by 1D medial curves. Surface thinning can generate 2D medial surfaces in plate-shaped zones. The hybrid skeleton approach gathers the advantages of both techniques by combining them using a shape classifier. Figure 1 illustrates this approach by organizing the different processing units of the algorithm.

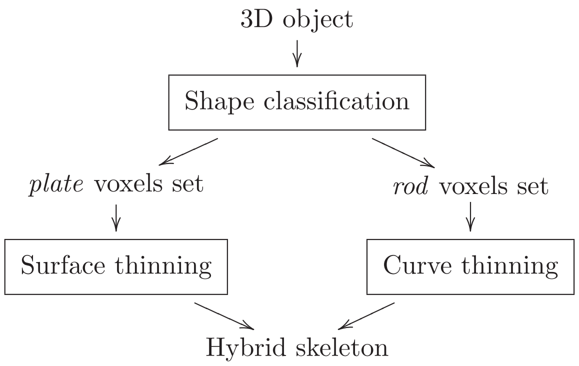

Figure 1: Global chart of the hybrid skeletonization algorithm.

The hybrid thinning algorithm is a non reversible iterative process. It determines *rod* and *plate* zones with a shape classification technique [2]. Then, the MESPTA surface thinning algorithm [3] is applied on the *plate* set. Similarly, a curve thinning algorithm [1] is applied on the *rod* set. The following subsections give an overview of these 3 methods.

2.1 Shape classification

The recent work of A. Bonnassie [2] is used to determine which voxels are to be classified as *rod* or *plate* in the analyzed object. This classification method consists in affecting a class identifier to each voxel of the object's solid phase, depending on the local shape of the structure. The 4 standard classes obtained in [2] are *arc*, *boundary*, *branching* or *regular*. In the hybrid thinning case, there can only be as much classes as variants of thinning techniques. For this reason, the labels are gathered using a region growth process in order to separate the voxels into 2 classes : *rod* and *plate*. Figure 2 exposes the hybrid-shaped test vector used in this communication (with 3 *plates* and 3 *rods*) and the result of the shape classifier. *Rod* and *plate* zones have been successfully classified, as shown with different grey levels.

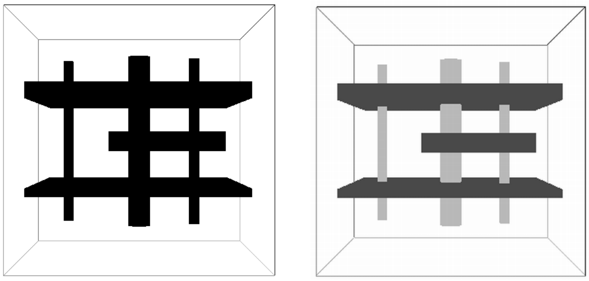

Figure 2: Test vector and result of the shape classification (*rods* in light grey and *plates* in dark grey).

2.2 Surface thinning

The MESPTA surface thinning algorithm [3] consists in an iterative parallel topology-preserving process that transforms an object into its medial surface. The main advantage of this technique is that it generates a simple surface geometry, which is convenient to model the plate-shaped parts of an object. In the hybrid thinning technique, all *plate* voxels that match the safe point condition as explained in [3] are deleted at the end of an iteration. The process is iterated until no voxel is deleted. Figure 3 shows the result of the MESPTA on the test vector of figure 2, which has been thinned to simple 2D subsets.

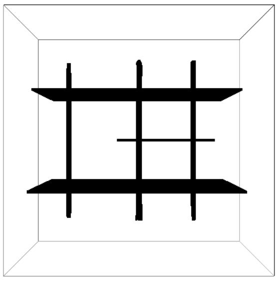

Figure 3: Result of the surface thinning process on the test vector of figure 2.

2.3 Curve thinning

The curve thinning algorithm used in the hybrid skeletonization method is based on the well-established work of Morgenthaler [1]. This iterative process relies on the evaluation of the Betti numbers in each voxel's neighbourhood. A voxel is said to be removable if its deletion does not change these numbers. Voxels are removed sequentially if they match the deletion criterions, which ensures a perfect topology preservation. The process is repeated until only the curve skeleton remains. Figure 4 shows the result of the curve thinning algorithm on the test vector of figure 2. In this example, plate zones have fully disappeared, which confirms that curve thinning is not sufficient to describe the entire porous medium. However, it efficiently models the rod-shaped parts of the object.

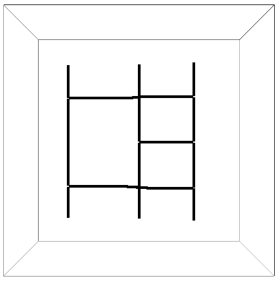

Figure 4: Result of the curve thinning process on the test vector of figure 2.

2.4 Hybrid thinning

The hybrid thinning algorithm combines the 3 previous methods as explained in figure 1. It has been validated on a test vector by checking the coherence between the morphological and topological properties of the skeleton and those of the original object. The number of *rod* and *plate* zones, the number of interfaces between 1D-curves and 2D-surfaces, and the connectivity density have been verified. Figure 5 exposes the hybrid skeleton of the test vector of figure 2.

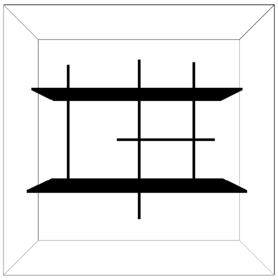

Figure 5: Hybrid skeleton of the test vector of figure 2.

3. APPLICATIONS

The hybrid skeletonization algorithm is an efficient disordered porous media analysis tool. It can be used to study biphasic objects, especially when their structure is composed of different shapes. On this kind of media, the hybrid skeleton is more precise than classical skeletons, as it takes into account the shape information. The following subsections illustrate different applications of the hybrid thinning algorithm: in the biomedical field with trabecular bone then in geology with sandstone.

3.1 Trabecular bone

As many other porous media, trabecular bone has a disordered structure which is often difficult to model. The hybrid thinning technique has been designed especially to perform precise structure analysis of such complex shapes. Figure 6 compares the hybrid skeleton of a trabecular bone sample with its equivalent curve and surface skeletons.

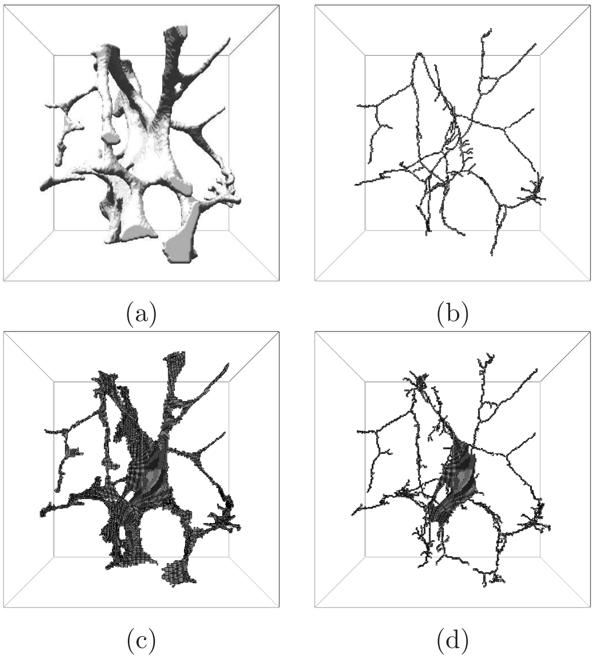
(a) (b)
(c) (d)

Figure 6: A trabecular bone sample (a), its curve skeleton (b), surface skeleton (c) and hybrid skeleton (d).

The hybrid skeleton models the *plate* zones by 2D-surfaces, which prevent the generation of parasite pathes as can be seen on the curve skeleton. These imperfections are due to the curve skeleton's sensitivity to the object surface irregularities. The hybrid skeleton also models *rod* zones by 1D-curves, on the contrary to the surface skeleton which does not erode *rods* enough. Interesting features can be extracted directly from the skeleton, characterizing the morphology and the topology of the sample: *rod* and *plate* ratio, connectivity density, numbers of nodes, line-ends or skeleton voxels. A clinical study which uses these skeleton features is exposed in the following section.

The hybrid skeleton can also be complemented with volumetric information, for example using hybrid skeleton based region-growth segmentation. Figure 7 illustrates this kind of volumetric representation for the bone sample of figure 6. As for trabecular bone characterization, multiple fields may have an interest in these skeleton-based analysis methods to get volume, section, length or shape information of each element of the porous medium's microarchitecture.

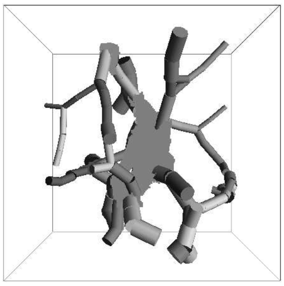

Figure 7: Volumetric representation of the bone sample of figure 6 (each element has a different grey level).

3.2 Sandstone

Papers have been published about using skeleton-based techniques in geology to study the properties of porous materials like sandstone [4] or soil [5]. In this field, the pore phase organisation usually gives information about the behavior of the material. The hybrid skeleton can be applied either on the solid phase or the pore phase, depending on the type of information needed.

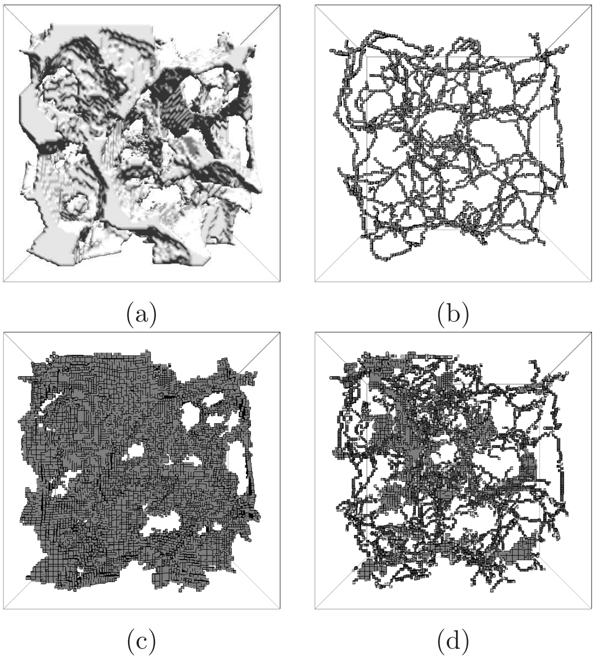
(a) (b)
(c) (d)

Figure 8: Pore phase of a sandstone sample (a) and its curve (b), surface (c) and hybrid (d) skeletons.

Figure 8 illustrates an application of the hybrid thinning process on the pore phase of a Fontainebleau sandstone sample from W. B. Lindquist [4]. The presence of *plates* in the hybrid skeleton confirms that the structure is complex and contains both *rod* and *plate* elements. Typically, this geometrical information is lost in the curve skeleton case, and hard to extract from the surface skeleton.

4. CLINICAL STUDY

The hybrid thinning technique has been applied to biomedical data in order to show its efficiency. 2 sets of trabecular bone samples have been studied by computing the hybrid skeleton of each item and extracting microarchitectural features. The 2 populations are composed of 9 coxarthric (OA) and 9 osteoporotic (OP) post-mortem bone samples extracted from frozen human femoral heads and acquired with a high-resolution μCT (12 μm). Each numerical sample is a 400^3 8-bits voxels isotropic volume which is preprocessed as follows: after applying a median filter to remove acquisition noise, it is binarized using the local minimum threshold between the 2 modes of its histogram. Then clustering using the Hoshen-Kopelman algorithm [6] is applied in order to keep only a single solid phase and remove all internal holes. These preprocessing steps are to obtain a reality-compliant volume. Indeed, there can neither be isolated material nor isolated holes in a real trabecular bone. Figure 9 shows 2 preprocessed extracts from respectively an OA and an OP sample.

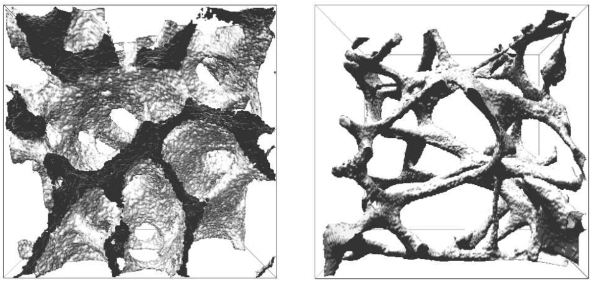
OA OP

Figure 9: Extracts of OA and OP bone samples.

Osteoporosis is characterized by the deterioration of the bone microarchitecture which leads to bone fragility and fracture risk. On the contrary, a coxarthric patient suffers from a hypertrophy of the bone structure which increases its density. Goulet *et al.* have pointed out that the bone density covers about 70 % of the diagnosis, but can be complemented with microarchitectural information to get an almost complete classification [7]. This study aims to show that the hybrid skeleton is able to detect and measure the microarchitectural differences between the 2 populations. The hybrid skeletons of the bone extracts of figure 9 are exposed on figure 10.

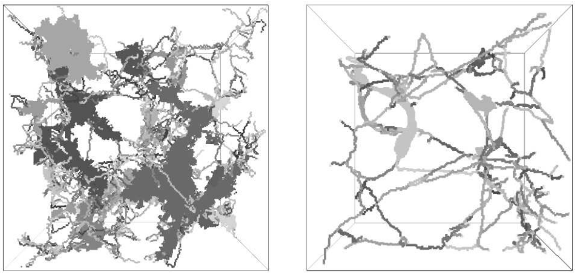
OA hybrid skeleton OP hybrid skeleton

Figure 10: Hybrid skeletons of the samples of figure 9.

After computing the hybrid skeleton of each sample, the following features are extracted:

- Conn.D, skeleton Connectivity Density in mm^{-3}
- SV/TV, skeleton density (i.e. Skeleton Voxels / Total Volume)
- Le.N, Number of skeleton Line-ends
- No.N Number of skeleton Nodes
- Ro/Pl, skeleton *rods* and *plates* ratio

These features are compared when possible to their equivalent extracted from the curve and the surface skeletons. The bone density (BV/TV) is also computed for each sample. A bilateral hypothesis test [8] is then used to evaluate the discriminative power of the different parameters for each of the 3 skeletons. Any parameter is said to separate the 2 populations if the Student's |t| value exceeds 2.92 for a significance level of 0.01 with a freedom degree of 16.

Feature	Student |t| value		
	Curve skeleton	Surface skeleton	Hybrid skeleton
Conn.D	2.08	2.08	2.08
SV/TV	3.05	4.19	4.19
Le.N	2.56	n.a.	3.07
No.N	2.27	n.a.	3.00
Ro/Pl	n.a.	n.a.	1.46

Table 1: Results of the comparative study on 2 sets of trabecular bone samples

Table 1 displays the results obtained for this data set. The |t| values show that the connectivity density (Conn.D) is the same for the 3 skeletons. This remark is obvious, as they all preserve topology. But on the contrary to the bone density (BV/TV) which gives |t|= 4.06, neither the connectivity nor the skeleton *rods* and *plates* ratio (Ro/Pl) seems to be a significative feature.

The skeleton density (SV/TV) better separates the 2 populations in the surface and hybrid cases than in the curve case. This is a consequence of the information loss in curve thinning. The more plate zones the object contains, the more the hybrid skeleton outclasses the curve skeleton on a geometrical point of view. The number of line-ends (Le.N) and the number of nodes (No.N) separate the 2 populations in the hybrid case but not in the curve case. For all parameters, the hybrid skeleton is always more discriminative than the curve skeleton, since *plates* preserve from generating parasite curves.

To sum up, 3 of the 5 hybrid skeleton features discriminate the 2 populations. This demonstrates that the hybrid skeleton gathers the advantages of both curve and surface skeletonization techniques:

- the discriminative power of the surface skeleton's density (SV/TV)
- optimized curve-specific features like number of line-ends (Le.N) and number of nodes (No.N)

5. CONCLUSION

2D and 3D thinning are widely used in pattern recognition and shape analysis. In the field of disordered porous media analysis, curve and surface skeletons provide interesting morphological and topological information but suffer from a lack of geometrical precision. Due to the shape heterogeneity inside their structure, many porous materials can neither be modelled with only 1D curves or 2D surfaces.

This communication presented a new tool for 3D thinning of porous media, using a recent shape classification method to judiciously switch between curve and surface skeletons. The resulting hybrid skeleton has been shown to match the geometry of the medium, choosing the best way to model the local shapes of its structure. Furthermore, it was proved to preserve topology.

Applications of this technique to the fields of biomedical imaging and geology have been investigated. The hybrid skeleton of the pore phase of a sandstone sample has been computed. A clinical study has then been carried out on trabecular bone samples. Microarchitectural features extracted from the hybrid skeletons of 2 pathologically-different populations have been compared. The results demonstrated that the hybrid skeleton gives better results than classical curve and surface skeletons. Moreover, the 2 bone sets have been statistically separated which shows the clinical efficiency of the method.

6. PROSPECTS

In the applications presented in this communication, porous medium features are directly extracted from the hybrid skeleton. But this algorithm can also be considered as a basis for building more complex models. Skeleton-based segmentation and thickness mapping are efficient methods to get volumetric information about each element of the object's structure. In the biomedical field, this provides volume, length and section of each bone trabecula. L. Pothuaud proposed the Line Skeleton Graph Analysis (LSGA) technique [9] which was a very innovative and promising work for trabecular bone analysis. Future Hybrid Skeleton Graph Analysis (HSGA) is expected to give better clinical results, as the precision of the skeleton is improved. This is a new opportunity to complement the classical density measurement for the study of bone alteration due to osteoporosis.

In the case of sandstone and soil analysis, the volumetric information can be combined with mathematical local behaviors to obtain hydrodynamical models [5].

Last but not least, there is an interesting prospect in the field of biomechanical simulation. Finite Element (FE) models can be generated from the skeleton to evaluate the stiffness of the porous medium. Papers have been published emphasizing the awesome computational efficiency of such curve skeleton based models [10]. Work on beam-chain elements has been carried out to improve this approach [11]. Further work is to be led on hybrid skeleton based FE models for biomechanical simulation.

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