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ESTIMATION OF MOTION BLUR POINT SPREAD FUNCTION FROM DIFFERENTLY EXPOSED IMAGE FRAMES

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ABSTRACT

In this paper we investigate the problem of recovering the motion blur point spread function (PSF) by fusing the information available in two differently exposed image frames of the same scene. The proposed method exploits the difference between the degradations which affect the two images due to their different exposure times. One of the images is mainly affected by noise due to low exposure whereas the other one is mainly affected by motion blur caused by camera motion during the exposure time. Assuming certain models for the observed images and the blur PSF, we propose a maximum a posteriory (MAP) estimator of the motion blur. The experimental results show that the proposed method has the ability to estimate the motion blur PSF caused by rather complex motion trajectories, allowing a significant increase in the signal to noise ratio of the restored image.

1. INTRODUCTION

The image degradation, known as motion blur, is caused by the relative motion between the camera and the scene during the exposure time. In the context of ongoing development and miniaturization of the consumer devices that have image acquisition capabilities, there is an imminent need for robust and efficient solutions able to correct or prevent the motion blur degradation. The main driven factors for this requirement are: (i) the difficulty to avoid camera motion during the integration time when using a small hand-held device (like a camera phone), and (ii) the need for longer integration times due to the small pixel area resulted from the miniaturization of the image sensors in conjunction with the increase in image resolution.

In general, if the point spread function (PSF) of the motion blur is known the original image could be restored up to some level of accuracy, by applying an image restoration approach [1]. However, the main difficulty is that in most practical situations the motion blur PSF is not known. Moreover, since the PSF depends of the camera motion during the exposure time, it is rather difficult to establish a universal model for the blur process. The lack of knowledge about the blur PSF suggests the use of blind deconvolution approaches in order to restore the motion blurred images [2, 3]. Unfortunately, most of these methods rely on rather simple motion models, e.g. linear constant speed motion, and hence their potential use in consumer products is rather limited. Another category of approaches consists of utilizing either additional hardware or special sensors [4] in order to estimate and correct for the motion of the camera during the exposure time. For instance in [5] the authors proposed the use of an additional camera in order to acquire motion information during the exposure time of the principal camera. The resulted motion information is subsequently used to estimate the motion blur PSF and to recover the blurred image of the main camera. Additional hardware is also used in optical image stabilization solutions that are present in high-end consumer devices. These solutions consists of moving either the image sensor or the optics in the opposite direction of the camera motion such that to maintain a stable image projected on the sensor during exposure.

In this paper we propose an approach to motion blur estimation by utilizing two image frames acquired at different exposure times. One frame is captured with a small exposure time in order to avoid motion blur, whereas a second frame is captured using the normal exposure time in the given conditions. When a long exposure time is required, like it is the case in low light conditions, the second image may be degraded by arbitrary motion blur. Our objective is to estimate the motion blur PSF in order to recover the original image by applying a deconvolution procedure onto the high exposed image frame.

2. THE PROPOSED METHOD

We formulate a model of the two observed images by taking into consideration their different degradations caused by the difference between their exposure times. Thus, the low exposed image is likely to be heavily corrupted by sensor noises [6], whereas the normal exposed image might be affected by motion blur due to camera motion during the exposure time.

In accordance to this model we have the following relations for the low and high exposed image frames denoted respectively by g_1 and g_2:

$$\begin{aligned} \alpha g_1(\mathbf{x}) &= f(\mathbf{x}) + n_1(\mathbf{x}), \\ g_2(\mathbf{x}) &= d(\mathbf{x}) * f(\mathbf{x}) + n_2(\mathbf{x}) \end{aligned} \tag{1}$$

where $\mathbf{x} = (x, y)$ denotes the coordinates of an image pixel, f denotes the original image, α accounts for the difference in illumination between the two observed images, $d(\mathbf{x})$ denotes the motion blur PSF, $n_i(\mathbf{x})$, for $i = 1, 2$, denote zero mean additive noise, and the symbol $*$ stands for the 2D convolution operation.

In addition to the model described in (1) we can also assume the energy conservation and positivity constraints on the PSF:

$$\sum_{\mathbf{x}\in\Psi} d(\mathbf{x}) = 1, \text{ and } d(\mathbf{x}) \geq 0, \ \mathbf{x} \in \Psi, \tag{2}$$

where Ψ denotes the 2D support of the blur PSF.

The posterior probability density function (p.d.f.) of the bur PSF given the two observed images can be expressed by:

$$p(d|g_1, g_2) = \frac{p(g_2|g_1, d)p(d)p(g_1)}{p(g_1, g_2)}, \tag{3}$$

from where, retaining only the terms which depend on d, we can write an objective function to be minimized by the maximum a posteriori (MAP) estimate of the blur PSF:

$$J(d, \alpha) = -\log p(g_2|g_1, d) - \log p(d). \tag{4}$$

We assume that the terms n_1 and n_2 in (1) are white Gaussian noises of variances σ_1^2 and σ_2^2 respectively, with $\sigma_1^2 \gg \sigma_2^2$. Consequently, the conditional p.d.f. $p(g_2|g_1, d)$, included in the first term of (4), is a multivariate Gaussian with mean $\alpha d * g_1$ and a non-diagonal covariance matrix. For tractability of the solution we will consider only the diagonal elements of the covariance matrix which are given by:

$$\sigma^2(d) = \sigma_2^2 + \sigma_1^2 \sum_{\mathbf{x}\in\Psi} d(\mathbf{x})^2 \tag{5}$$

resulting in the following simplified model for the conditional p.d.f

$$-\log p(g_2|g_1, d) \sim \frac{1}{2\sigma^2(d)} \sum_{\mathbf{x}\in\Omega} n(\mathbf{x})^2 + \frac{N}{2} \log \sigma^2(d), \tag{6}$$

where N denotes the number of image pixels and

$$n(\mathbf{x}) = g_2(\mathbf{x}) - \alpha d(\mathbf{x}) * g_1(\mathbf{x}). \tag{7}$$

The second term in (4) describes the model of the motion blur PSF. Assuming that the camera undergoes only translational motion during the exposure time we may consider that the PSF is space invariant. Thus, the motion blur PSF can be regarded as the projection of the camera motion trajectory onto the image plane, resembling thereby the appearance of a ridge that follows a curved trajectory inside the PSF support (e.g. Fig. 1). In our model of the motion blur PSF we impose this ridge appearance of the PSF by defining the prior p.d.f. as:

$$-\log p(d) \sim \frac{\lambda}{2} \sum_{\mathbf{x}\in\Psi} [1 - m(\mathbf{x})]\, d(\mathbf{x})^2, \tag{8}$$

where $m(\mathbf{x})$ denotes the indicator function for the PSF ridge path, i.e. $m(\mathbf{x}) = 1$ if $\mathbf{x}$ belongs to the PSF ridge, and $m(\mathbf{x}) = 0$ otherwise.

Due to physical constraints on camera motion speed and acceleration, the PSF ridge path trajectory can be assumed continuous and differentiable. Consequently, in most of its points $\mathbf{x}$, the direction $\theta(\mathbf{x})$, tangent to the ridge path is well defined. Based on this observation, and aiming for a ridge like appearance of the motion blur PSF, we define the path function

$$m(\mathbf{x}) = \begin{cases} 1 & \text{if } d(\mathbf{x}) \geq d(\mathbf{y}) \text{ for any } \mathbf{y} \in \mathcal{N}(\mathbf{x}) \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

where $\mathcal{N}(\mathbf{x})$ denotes a local neighborhood of $\mathbf{x}$ selected along the direction orthogonal to the local ridge orientation $(\theta(\mathbf{x}))$.

In practice we do not know the blur PSF in order to calculate m, as shown above. However we can apply the same approach on an intermediate estimate of the blur PSF, where the local ridge orientation $\theta(\mathbf{x})$ in the neighborhood of any point $\mathbf{x} \in \Psi$ can be calculated using a texture orientation estimator (e.g. [7]).

Joining (6) and (8) we obtain the final form of the objective function (4). This function can be minimized by employing an iterative minimization procedure. In our work we used a gradient descent approach, imposing the constraint (2) at each iteration.

The gradient of the objective function is given by:

$$\begin{aligned} J_d(d, \alpha) &= \frac{\partial J(d, \alpha)}{\partial d} \\ &= \frac{-\alpha g_1(-\mathbf{x}) * n(\mathbf{x})}{\sigma^2(d)} \\ &+ \frac{d(\mathbf{x})\sigma_1^2}{\sigma^2(d)} \left[N - \frac{\sum_{\mathbf{x}\in\Omega} n(\mathbf{x})^2}{\sigma^2(d)} \right] \\ &+ \lambda[1 - m(\mathbf{x})]d(\mathbf{x}), \end{aligned} \tag{10}$$

and

$$J_\alpha(d, \alpha) = \frac{\partial J(d, \alpha)}{\partial \alpha} = \frac{\sum_{\mathbf{x}\in\Omega} [g_1(\mathbf{x}) * d(\mathbf{x})]\, n(\mathbf{x})}{\sigma^2(d)}, \tag{11}$$

where $\Omega \subset R^2$ denotes the image support.

The parameter α is estimated at each iteration by equating with zero the equation (11). This is:

$$\alpha = \frac{\sum_{\mathbf{x}\in\Omega} g_2(\mathbf{x}) [g_1(\mathbf{x}) * d(\mathbf{x})]}{\sum_{\mathbf{x}\in\Omega} [g_1(\mathbf{x}) * d(\mathbf{x})]^2}. \tag{12}$$

and, in the first iteration, a first estimate of α can be obtained as the ratio between the means of the two images:

$$\alpha_0 = \sum_{\mathbf{x}\in\Omega} g_2(\mathbf{x}) / \sum_{\mathbf{x}\in\Omega} g_1(\mathbf{x}). \tag{13}$$

Noting that the prior term (8) is highly dependent of the current estimate of d, we start the minimization procedure with $\lambda = 0$. Next, after a number of iterations, λ is set to a high value in order to force the ridge like appearance of the current estimate. The local orientation in the neighborhood of each pixel $\mathbf{x} \in \Psi$, is calculated using the the least-square estimator proposed by Rao in [7].

The iterative minimization of the objective function could start from an arbitrary initial guess of the motion blur PSF. However, in order to speed up the process we can use an initial value of the blur PSF whose computation is described in the following section.

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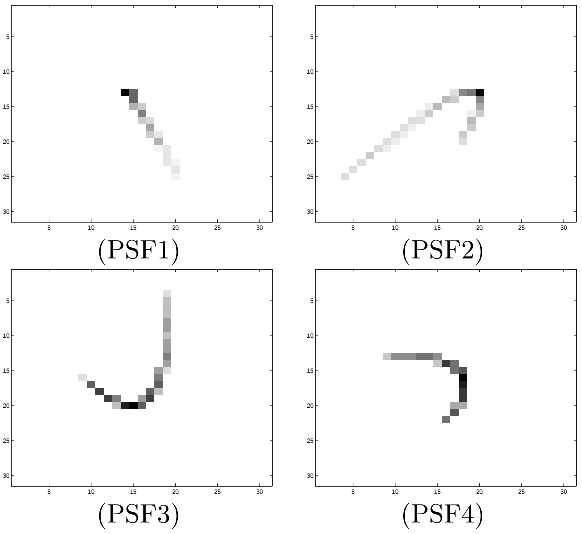

Figure 1: The motion blur PSFs used in the experiments.

2.1 The initial estimate of the motion blur PSF

Based on the model (1) we can write

$$g_2(\mathbf{x}) = \alpha d(\mathbf{x}) * g_1(\mathbf{x}) + n(\mathbf{x}), \tag{14}$$

where $n(\mathbf{x}) = n_2(\mathbf{x}) - d(\mathbf{x}) * n_1(\mathbf{x})$. Neglecting the non-diagonal terms of the covariance matrix of $n(\mathbf{x})$, and using the fact that $\sigma_2^2 \ll \sigma_1^2$, we obtain the Wienner filter estimate of the blur PSF

$$D(\omega) = \frac{\alpha G_1^*(\omega) G_2(\omega)}{\alpha^2 |G_1(\omega)|^2 + \sigma_1^2}, \tag{15}$$

where the capital letters stand for the Fourier transforms of the corresponding signals, and the value of α is calculated using the estimator (13).

An initial estimate of the blur PSF is thereby obtained from the inverse Fourier transform of (15). In the following we present an algorithm that describes a practical implementation of this estimate.

Input: The two images g_1 and g_2 and a rough estimate of the PSF support size, i.e. $S_1 \times S_2$.

Otput: The initial estimate of the blur PSF.

- Register the images g_1 and g_2, such that to minimize the translational displacement between them, and to cancel other motion parameters (e.g. rotation, scale). In order to obtain a good result the image registration approach used in this step must be robust to image degradations. Such methods have been proposed by several authors [8, 9] and could be employed for image registration in the context of this application. In our work we used the method proposed in [9].
- Select multiple image blocks of size $W_1 \times W_2$ (i.e. $W_1 > S_1$, and $W_2 > S_2$) from the blurred image (g_2). The selection process is based on the variance of each image block, being preferred blocks that have higher variance, and hence higher likelihood to contain significant transitions or prominent image details. Let us denote by g_2^k, for $k = 1, K$, the K image blocks selected from the image. Similarly, their corresponding blocks in the low exposed image are denoted by g_1^k.
- Average the K estimates (15) obtained from all pairs of corresponding image blocks g_1^k and g_2^k. In this operation, the Fourier transforms are calculated using Fast Fourier Transform algorithm, and the artifacts due to block boundary are reduced by windowing.
- Extract the blur PSF estimate by selecting the $S_1 \times S_2$ central part from the $W_1 \times W_2$ inverse Fourier transform of the average calculate in the previous point.

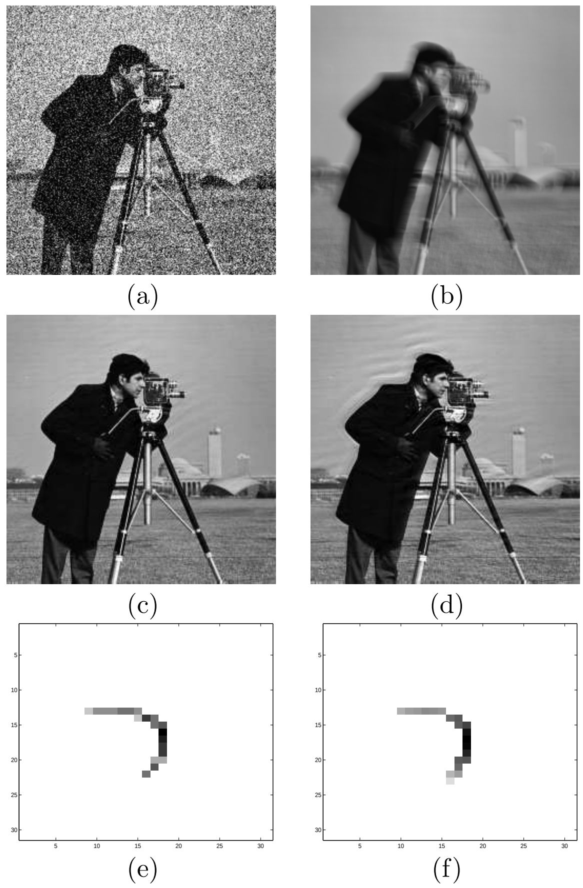

Figure 2: Example of motion blur estimation: (a) the noisy image frame, (b) the blurred image frame, (c) the image restored in the ideal case when the blur PSF is exactly known, (d) the restored image based on the estimated PSF, (e) the real PSF, and (f) the estimated PSF.

3. EXPERIMENTS

In this section we demonstrate the proposed algorithm through a series of motion blur identification experiments. The motion blur PSFs that are used in this simulations are shown in Fig. 1. These PSFs are more realistic than a simple linear and uniform motion, including non-uniform power distributions caused by variable motion speed, as well as curved trajectories.

SNR (dB) Noisy img.	SNR (dB) Blurred img.	SNR (dB) Restored img.
PSF1		
12.01 (0.03)	14.90	25.03 (0.53)
7.86 (0.02)	14.90	24.38 (0.98)
2.85 (0.03)	14.90	23.91 (0.76)
0.10 (0.02)	14.90	20.37 (1.09)
PSF2		
12.02 (0.02)	14.67	27.53 (0.38)
7.88 (0.02)	14.67	25.90 (0.57)
2.82 (0.02)	14.67	23.57 (0.88)
0.10 (0.03)	14.67	21.60 (1.03)
PSF3		
12.01 (0.03)	13.74	24.46 (0.30)
7.87 (0.04)	13.74	23.54 (0.53)
2.85 (0.01)	13.74	20.84 (1.28)
0.10 (0.02)	13.74	18.71 (0.74)
PSF4		
12.01 (0.02)	13.11	27.43 (1.58)
7.86 (0.03)	13.11	26.08 (1.06)
2.85 (0.03)	13.11	23.47 (1.88)
0.09 (0.02)	13.11	19.98 (1.32)

Table 1: Image restoration performance for different noise levels in the low exposed image, and different motion blur PSFs in the high exposed image. The first and last columns show the average and standard deviations of the corresponding image SNRs in multiple experiments.

In each experiment we create the two input images starting from the original "cameraman" image of size 256×256 on 256 gray levels. The image g_1 is obtained by adding white Gaussian noise of different variances plus Poisson noise. The level of noise in each example is then measured by means of the signal-to-noise ratio (SNR) with respect to the original image. The second image (g_2) is obtained by applying one of the the motion blur PSFs onto the original image. The SNR is also used in this case in order to evaluate the degradation of the image in comparison with the original image. In each experiment, the PSF estimated by the algorithm was used to restore the original image by applying the Richardson-Lucy algorithm [1] onto the image g_2.

The ability of the proposed algorithm to estimate the motion blur PSF is evaluated based on the improvement in SNR achieved after deconvolution. We consider that this is a more realistic evaluation of the algorithm than comparing directly the estimated and real PSFs. This is because small differences in PSF may result in significant degradations of the result due to inverse nature of the deconvolution problem. The results obtained in several experiments when using different motion blurs and noise levels are shown in Table 1. For each level of noise and PSF we performed a number of 10 experiments. The average and standard deviation of SNR results obtained in these experiments are shown in the table.

We note that the PSF estimated by the proposed algorithm allows a significant improvement of the restored image SNR. As expected the level of noise in g_1 is influencing the results by reducing the ability to recover the exact PSF. However we remark that the proposed algorithm estimates the blur PSF quite accurately even in very heavy noise conditions (e.g. $SNR < 0.1dB$) as long as it always resulted in an improvement of several decibels of the restored image.

A visual example of motion blur identification is shown in Fig. 2, where the blur PSF4 was used in conjunction with a noisy image of SNR 2.8dB. The estimated PSF is quite close to the real one as it is reveal also by the visual inspection of the restored image which achieves an SNR of 23dB in comparison to the original image.

4. CONCLUSIONS

We proposed a method to recover the motion blur PSF by exploiting the difference between the degradation models which affect two differently exposed images of the same scene. The proposed method is able to estimate more realistic motion blur PSFs than simple linear motion, including non-uniform power distributions caused by variable motion speed, as well as curved trajectories. The algorithm was evaluated through a series of experiments that reveal its ability to detect the motion blur PSF even in the presence of heavy degradations of the two observed image frames.

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