NOVEL EFFICIENT WEIGHTING FACTORS FOR PTS-BASED PAPR REDUCTION IN LOW-POWER OFDM TRANSMITTERS

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ABSTRACT

Increased Peak-to-Average Power Ratio (PAPR) in an Orthogonal Frequency Division Multiplexing (OFDM) signal poses serious challenges in wireless telecom system design, since increased PAPR stresses the performance of analog and RF parts of a wireless modem. The Partial Transmit Sequence (PTS) algorithm is a flexible and distortionless peak power reduction scheme of low computational complexity. In this paper we experimentally derive a new set of weighting factors which improve the performance of the PTS algorithm while the computational complexity remains the same. Furthermore, this paper examines the complexity of the VLSI implementation of PTS versus the power savings in the analog part due to the PAPR reduction. It is here shown that with the new set of weighting factors the power consumption of PA is reduced by 21.1% in comparison to OFDM without PAPR reduction.

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a very attractive technique for high-bit-rate transmission in wireless applications, as it provides greater immunity to multipath fading and impulse noise. OFDM-based systems are popular and have been adopted in several International Standards for Digital Audio Broadcasting, Digital Terrestrial Television Broadcasting, and Wireless Local and Metropolitan Area Networks. Two major drawbacks of OFDM signals are the great sensitivity to time and frequency synchronization errors, and the high PAPR. In OFDM systems the output of IFFT consists of a sum of N modulated subcarriers; therefore OFDM signals exhibit large dynamic range.

When passed through nonlinear devices, such as a Power Amplifier (PA), the amplified signal suffers from distortion and out-of-band noise. In order to combat the problem, highly linear PAs are required [1]. High linearity normally implies low efficiency, and great power dissipation which prohibs use in portable wireless applications [2]. For that reason the use of PAPR reduction techniques is essential.

A number of PAPR reduction schemes have been proposed to alleviate this problem. The simplest approach is clipping the OFDM signal [3]. However clipping may cause significant in-band distortion and out-of-band noise. Another solution is to use appropriate block coding [4]. Using this approach a 3-dB PAPR can be achieved. Nevertheless, these schemes require large look-up tables both at the transmitter and the receiver, limiting their usefulness to applications with a small number of subcarriers. Two promising and distortionless techniques for improving the statistics of the PAPR are the Selective Mapping approach [5] and the Partial Transmit Sequence approach [6][7].

The PTS approach is based on combining signal subblocks, phase-shifted by constant weighting factors. PTS-based PAPR reduction can be exploited to achieve system-level low-power operation in cases of practical interest [14]. The major challenges on the application of PTS are: (1) how the receiver would be informed of the values of the weighting factors and (2) the computation of more efficient weighting factors, resulting in the bigger PAPR reduction. Using differential encoding or embedding side information within the transmitted data [9], no explicit side information has to be transmitted. In this paper we focus on the derivation of good weighting factors.

In the related literature [6][7][8] the most common choices of weighting factors are the sets $\{\pm 1\}$ and $\{\pm 1, \pm j\}$, because no actual multiplications are required for the formation of alternative PTSs. Assume that the initial OFDM block has 128 carriers and is partitioned into 4 subblocks. Using the set of weighting factors $\{\pm 1, \pm j\}$, the probability $Pr(PAPR > 10^{-4})$ can be reduced by more than 4 dB. Tellambura has proposed an algorithm for computing weighting factors that achieves better performance than the previous approach, but it requires more arithmetic operations for small number of subblocks (i.e.; less than 8) [10].

This paper experimentally derives a new set of weighting factors for the PTS approach, which increases the achieved PAPR reduction in comparison with the set $\{\pm 1\}$, without any impact on the implementation complexity. Due to further PAPR reduction, the power consumed by the PA decreases. Hence the power consumption of the complete digital-analog system is reduced in comparison to previously reported results [14]. Furthermore, this paper presents the digital implementation of the PTS algorithm when the proposed set of weighting factors is employed. Finally, the additional processing required to reduce PAPR, in terms of area, latency, and power consumption and the corresponding power savings in the analog part of the transmitter are quantitatively explored. In particular the affect of PAPR reduction on the power consumption of a class-A PA is studied.

The remainder of the paper is as follows: Section 2 discusses the basics of OFDM transmission, defines PAPR and relates PAPR reduction with PA efficiency. Then, in Section 3 the PTS approach is presented. In section 4 the proposed architecture is presented. Subsequently, in Section 5 the theoretical results are verified via extended simulation, while section 6 discusses conclusions.

2. BASIC NOTATION

An OFDM signal is the sum of many independent signals modulated onto subchannels of equal bandwidth. Let us denote the collection of all data symbols X_n, $n = 0, 1, \ldots, N-1$, as a vector $X = [X_0, X_1, \ldots, X_{N-1}]^T$. The complex baseband OFDM signal, consisting of N subcarriers is repre-

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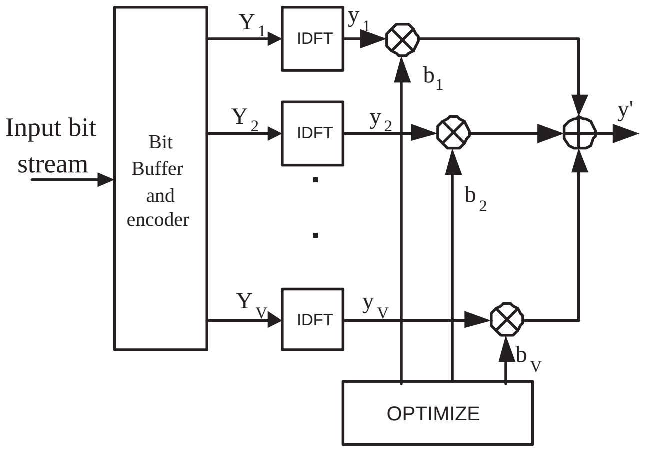

Figure 1: General Architecture of PTS.

sented by

$$x_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{\frac{j2\pi nk}{N}}, \qquad k = 0, 1, \ldots, N-1. \quad (1)$$

The symbol-related PAPR of the corresponding signal is defined as the ratio of the peak power value to the square root of the average power of the signal; i.e.,

$$PAPR = \frac{\max\{|x_k|^2\}}{E[|x_k|^2]}, \quad (2)$$

where $E[\cdot]$ denotes expected value.

In general, the average DC-input power is defined as

$$n = P_{outAVG}/P_{inAVG}. \quad (3)$$

In multicarrier communications systems, the average efficiency of a class-A amplifier, which is the most linear amplifier, is given by

$$n_{AVG} = n_{PEP}/\xi, \quad (4)$$

where ξ is the corresponding PAPR value, and n_{PEP} is the average efficiency if the signal had constant envelope and equal with the peak value [12]. Assume that a PAPR reduction method is employed. The achieved average efficiency is $n'_{AVG} = n_{PEP}/\ \xi'$, where ξ' is the reduced PAPR value and in comparison to n_{AVG}, is

$$n'_{AVG} = \frac{\xi}{\xi'} n_{AVG}. \quad (5)$$

According to (3) and (5) the average DC-input power, required to give the same output power, is

$$P'_{inAVG} = \frac{\xi'}{\xi} P_{inAVG}. \quad (6)$$

3. PARTIAL TRANSMIT SEQUENCES

In the PTS approach [6][7], the input data vector $X = [X_0, X_1, \ldots, X_{N-1}]$, is partitioned into V pairwise disjoint subblocks of equal size, each consisting of a different set of subcarriers. PTS forms V sequences of length N, represented by the vectors $Y_v,\ v = 1, \ldots, V$, such as $X = \sum_{v=1}^{V} Y_v$. Every used subcarrier of the initial OFDM symbol is included in exactly one of these sequences. In every sequence, elements at positions which corresponds to subcarriers X_i included in an other PTS, are set to zero.

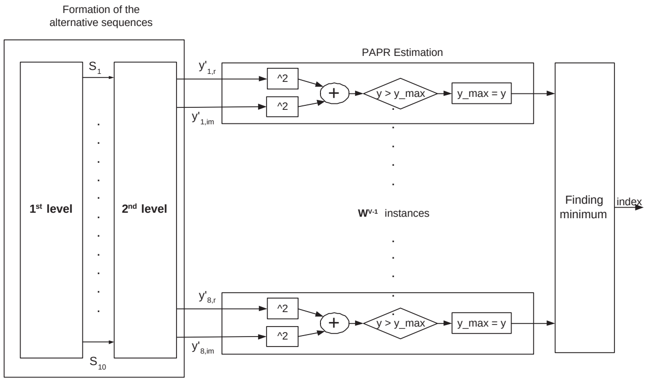

Figure 2: Block Diagram of the optimize block.

The objective of the PTS approach (Fig. 1) is to form a weighted combination of the V sequences, Y_v, as follows

$$Y'_s = \sum_{v=1}^{V} b_w Y_v, \quad (7)$$

where $b_w, w = 1, 2, \cdots, W$ are the weighting factors appropriately selected to minimize the PAPR of $y'_s = IDFT\{Y'_s\}$, and $s = 1, 2, \ldots, V^W$.

The implementation complexity of PTS algorithm depends on the number V of the subblocks and the number W of the different possible values of weighting factors [10]. The first subblock may remain unrotated; i.e., $b_1 = 1$ without any loss of performance [6]. Therefore an exhaustive search is required to choose the remaining $(W-1)$ weighting factors. Hence, V^{W-1} alternative sequences y' are evaluated to find the optimum set of weighting factors in terms of lowest PAPR. A restriction to four subblocks and two or four weighting factors yield high reduction of PAPR, while the implementation complexity remains moderate. Exploring more weighting factors requires increased complexity, not justified by the additional gains in PAPR reduction [6].

In order to calculate y'_s the linearity of the IFFT is exploited. Accordingly, the subblocks are transformed by V separate and parallel IFFTs yielding

$$y'_s = IDFT\{\sum_{v=2}^{V} b_w Y_v\} = \sum_{v=2}^{V} b_w y_v, \quad (8)$$

where $y_v = IDFT\{Y_v\}$. An iterative algorithm is used to derive all the alternative y'_s and subsequently to identify $\mathbf{b}$ that satisfies [6]:

$$\mathbf{b} = [b_1, b_2, \ldots, b_W] = \arg\min\left(\max\left|y'_s\right|\right). \quad (9)$$

The particular choice achieves the sequence y'_s of lowest PAPR, chosen to be transmitted.

If the block partitioning of the initial OFDM symbol is performed with respect to the computational structure of FFTs and taking into account the fact that most of the points of the sequence, on which the IFFT is applied, are zero, the implementation complexity of the V parallel IFFTs is reduced to the complexity of the classic OFDM (OFDM without the application of PTS) [13].

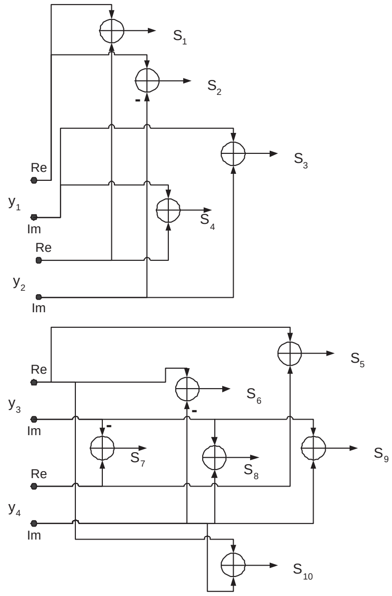

Figure 3: Architecture of level 1.

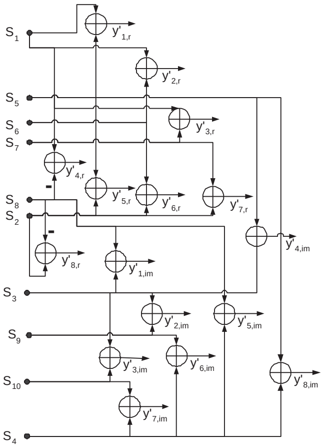

Figure 4: Architecture of level 2.

4. PROPOSED ARCHITECTURE

In [14] a VLSI architecture for the PTS algorithm using the set $\{\pm 1\}$ as weighting factors is proposed. Applying the particular PTS architecture in the digital part of the transmitter, the power consumption of the complete digital-analog system is reduced by 12.6%, due to the PAPR reduction. In this section we present the VLSI architecture of PTS algorithm where the initial OFDM symbol is partitioned into 4 subblocks, and the proposed weighting factors; i.e., $\{1, j\}$ are used. The application of PTS algorithm, employing the new weighting factors reduce PAPR further and the corresponding power savings increase, as quantified from the experimental results.

The complexity of the VLSI implementation for both cases is almost the same. Both require the exploration of $2^{4-1} = 8$ alternative sequences. Their only difference is in the formation of the alternative sequences. In [14], the sums for the combination of the partial sequences are not calculated directly. The four common partial sums are initially calculated and subsequently combined to derive the elements of the eight alternative sequences. In that case, for all the alternative sequences $12N$ complex additions (i.e., $24N$ real additions) are required.

n	PAPR Reduction (dB)	DC-input Power Reduction (%)
7	1.7	14.8
8	2.2	19.6
9	2.4	21.1

Table 1: Achieved DC-input Power reduction.

With the proposed set of weighting factors the required complex additions do not have such a straightforward symmetry. However a similar symmetry can be found exploring the real arithmetic operations. Taking into account the common partial sums $26N$ real additions are required for the formation of all the alternative sequences. Here it should be noted that still no multiplication is necessary. Fig. 2 depicts the optimize block; i.e., the VLSI architecture of PTS.

The optimize block takes as input consecutively the $y_v(k), k = 0, 1, \ldots, N-1$. Initially the y'_s are created according to (8). This is implemented by two levels of adders, depicted in Figs. 3 and 4. Subsequently, for each PTS the sample with the maximum magnitude is identified. The maximum-magnitude sample signifies the PAPR value, as average power is assumed constant. Finally the minimum of the maximum-magnitude values is derived. The corresponding sequence to which the minimum maximum-magnitude value belongs, is the one of lowest PAPR. It requires $N+3$ clock cycles to execute the algorithm; i.e., $N+2$ clock cycles to compute the PAPR of each PTS and one cycle to derive the PTS with minimum PAPR. The optimize block produces as output the b_w of (9) which optimize the transmitted sequence in terms of PAPR.

In the proposed architecture, magnitude and hence PAPR, is estimated by [14]

$$|y|^2 = y_1^2 + y_2^2. \tag{10}$$

According to the results derived in [14] two's-complement fixed-point representations with more than 7 bits for the real and imaginary part, respectively, are explored. Table 1 tabulates PAPR reduction in comparison to normal OFDM, achieved when a fixed-point representation using n bits for the real and imaginary part, respectively, is employed, for an OFDM system with 64 subcarriers modulated by QPSK. Furthermore, Table 1 depicts the reduction of the power consumption of the PA when the PTS approach is used. From the experimental results, it follows that 9 bits are enough for the VLSI implementation of PTS algorithm. In that case the DC-input power consumption of the PA, in comparison to normal OFDM, is reduced by 21.1% when the proposed weighting factors $\{1, j\}$ are used instead of 15.8% achieved by the weighting factors $\{\pm 1\}$ [14].

Table 2 tabulates the complexity of the VLSI implementation of PTS algorithm. For different wordlengths the circuit area, delay, and power consumption are calculated. The corresponding results are obtained using an $0.18\mu m$ ASIC library. In comparison to the VLSI architecture of [14], the architecture proposed in this paper requires slightly less area and smaller power consumption for the case of interest; i.e., 9-bits data and for the same latency.

5. SIMULATION RESULTS

In an OFDM symbol the signal amplitude is approximately Rayleigh distributed and the large peaks occur with very low

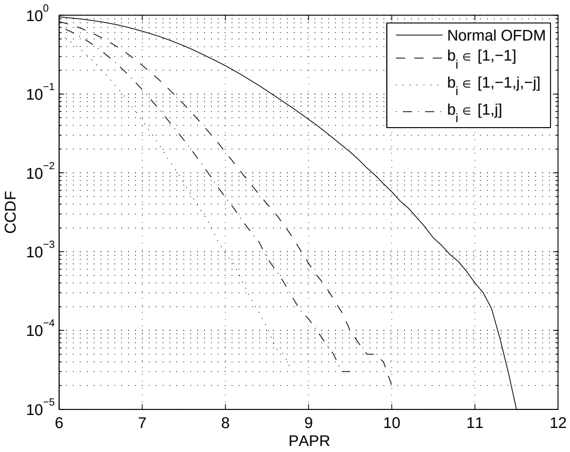

Figure 5: CCDF for an OFDM signal with 64 carriers and 64-QAM symbols.

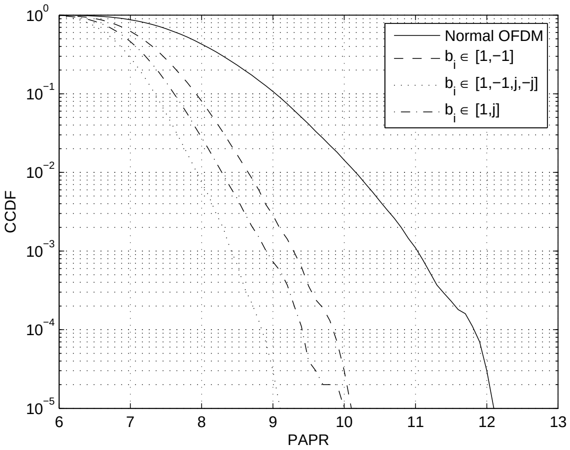

Figure 6: CCDF for an OFDM signal with 128 carriers and QPSK symbols.

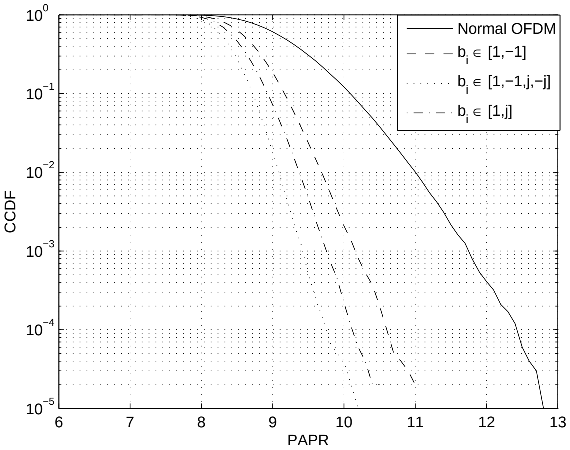

Figure 7: CCDF for an OFDM signal with 1024 carriers and QPSK symbols.

n	area (μm^2)	power (mW)	delay (ns)
7	159158	42.19	3.3
8	178397	47.68	3.4
9	211596	50.72	3.5

Table 2: Implementation complexity.

carriers	constellation	$b_i \in S_1$	$b_i \in S_2$	$b_i \in S_4$
64	QPSK	1.9	2.4	3.0
	64-QAM	1.7	2.1	2.7
128	QPSK	2.0	2.4	3.0
	64-QAM	1.8	2.1	2.6
256	QPSK	2.0	2.2	2.7
	64-QAM	1.7	2.2	2.7
512	QPSK	1.8	2.2	2.7
	64-QAM	1.8	2.2	2.6
1024	QPSK	1.8	2.3	2.5
	64-QAM	1.8	2.2	2.6

Table 3: PAPR reduction in comparison to normal OFDM.

probability. Therefore, the upper bound may not be meaningful for characterizing the PAPR of OFDM signals; instead the statistical distribution of the PAPR should be taken into account [11]. For that reason the complementary cumulative distribution function $CCDF = Pr(PAPR > PAPR_o)$ is used. $CCDF$ expresses the probability that the PAPR of an OFDM symbol exceeds a particular value $PAPR_o$. Cyclic Prefix is restricted to the 25% of the OFDM symbol. The transmitted signal is oversampled by a factor of four [7], in order to better approximate the continuous-time PAPR. Throughout this section, OFDM systems with $2^n, n = 6, 7, \ldots 10$ carriers are studied. In each case, 10^5 OFDM signals are simulated. Every carrier is modulated by QPSK or 64-QAM symbols. The performance of PTS approach is evaluated for three different sets of permissible weighting factors, namely $S_1 = \{1,\ -1\}$, $S_2 = \{1,\ j\}$, and $S_4 = \{\pm 1,\ \pm j\}$. Sets S_1 and S_4 are used in the related literature [6][8] while S_2 is the set, proposed in this paper.

Figs. 5–7 depict the achieved PAPR reduction for the various weighting factors. In particular Fig. 5 refers to an OFDM signal with 64 carriers modulated by 64-QAM, while Figs. 6 and 7 depict the cases of 128 and 1024 carriers modulated by QPSK respectively. Table 3 concludes the achieved, by the application of PTS algorithm, PAPR reduction for the various sets of weighting factors, in comparison to normal OFDM. For all the cases, PTS is more efficient when weighting factors take values from set S_2 instead of S_1, while the implementation complexity for both cases is the same.

The application of PTS with set S_4 as weighting factors, reduces PAPR further. For the case of an OFDM system with 128 carriers modulated by QPSK, the achieved PAPR reduction is 2, 2.4, and 3 dB respectively when set S_1, S_2, or S_4 is used respectively. The corresponding power savings at the PA are 17.0%, 20.4%, and 25.5%. However, the computational complexity in the first two cases is the same, while the number of arithmetic operations are increased by a factor of eight when the set S_4 is employed. When PTS algorithm uses the set S_4, $4^3 = 64$ different sequences must be formulated and subsequently explored for the lowest PAPR sequence. On the other hand, when S_1 or S_2 is used, PTS explores only eight alternative sequences.

6. CONCLUSIONS

This paper quantifies the impact of PTS PAPR reduction scheme adopted at the digital part of a transmitter, versus the corresponding efficiency increase expected at the analog part. Via extended simulation, a new set of weighting factors is derived. The set $\{1, j\}$ increases the performance of PTS, in comparison to the set $\{\pm 1\}$, used widely in the literature. Measurements of design, implemented using Synopsys Design Compiler using and $0.18 \mu m$ technology reveal that the digital implementation of PTS algorithm with the proposed set of weighting factors, requires slightly less area, while the power consumption of PTS for the proposed set is $2.5 mW$ less. Furthermore, the power consumption of the PA decreases by 21.1% in comparison to normal OFDM and 6.3% in comparison to the case where PTS employs the set $\{\pm 1\}$ as weighting factors.

REFERENCES

[1] R. Van Nee and R. Prasad, *OFDM Wireless Multimedia Communications*. Address: Artech House Publishers, 2000.

[2] B. Razavi, *RF Microelectronics*. Address: Prentice Hall, 1998.

[3] X. Li and L. J. Cimini Jr.,"Effects of clipping and filtering on the performance of OFDM," *IEEE Comm. Letts.*, vol. 2,pp. 131-133, May 1998.

[4] A. E. Jones, T. A. Wilkinson and S. K. Barton, "Block Coding Scheme for Reduction of Peak to Mean Envelope Power Ratio of Multicarrier Transmission Scheme," *Elec. Letts.*, vol. 30, pp. 2098-2099, Dec. 1994.

[5] R. W. Bäuml, R. F. H. Fischer and J. B. Hüber, "Reducing the Peak-to-Average Power Ratio of Multicarrier Modulation by Selective Mapping", *Elec. Letts.*, vol. 32, pp. 2056-2057, Oct. 1996.

[6] S. H. Müller, R. W. Bäuml and R. F. H. Fischer and J. B. Huber, "OFDM with Reduced Peak-to-Average Power Ratio by Multiple Signal Representation," *Annals of Telecommunications*, vol. 52, pp. 58-67, Feb. 1997.

[7] L.J. Cimini Jr. and N. R. Sollenberger, "Peak-to-average power ratio by optimum combination of partial transmit sequences," in *Proc. of ICC'99*, June 1999, pp. 511–515.

[8] S. H. Müller and J. B. Huber,"OFDM with Reduced Peak-to-Average Power Ratio by Optimum Combination of Partial Transmit Sequences," *Electron. Lett.*, vol. 33, pp. 368-369, Feb. 1997.

[9] L. J. Cimini and N. R. Sollenberger, "Peak-to-Average Power Ratio of an OFDM Signal Using Partial Transmit Sequences With Embedded Side Information, " in *Proc. of IEEE GLOBECOM '00*, 27 Nov.-1 Dec. 2000, pp. 2056-2057.

[10] C. Tellambura,"Improved phase factor computation for the PAR reduction of an OFDM signal using PTS," *IEEE Communications Letters*, vol. 5. pp. 135–137, Apr. 2001.

[11] H. Ochiai and H. Imai, "On the distribution of the Peak-to-average power ratio in OFDM Signals," *IEEE Trans. on Comm.*, vol. 49, pp. 282–289, Feb 2001.

[12] F. H. Raab, P. Asbeck, S. Cripps, P. B Kennington, Z. P. Popovic, N. Pothecary, J. F. Sevic and N. O. Sokal, "Power Amplifiers and Transmitters for RF and Microwave," *IEEE Transactions on Microwave Theory*,vol. 50, pp. 814–826, Mar. 2002.

[13] Th. Giannopoulos and V. Paliouras, "An efficient architecture for Peak-to-Average Power Ratio Reduction in OFDM Systems in the Presence of Pulse-Shaping Filtering, " in *Proc. of ISCAS'04*, May 2004, pp. 85–88.

[14] Th. Giannopoulos and V. Paliouras,"Low-power VLSI Architectures for OFDM Transmitters based on PAPR Reduction," in *Proc. of 15th International Workshop on Power and Timing Modeling, Optimization and Simulation, PATMOS 2005*, Leuven, Belgium, September 2005, pp. 177-186.