

# A NEURAL NETWORK APPROACH TO IMPROVE RADAR DETECTOR ROBUSTNESS

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## ABSTRACT

A neural network (NN) based detector is proposed for approximating the ALR detector in composite hypothesis-testing problems. The case of detecting gaussian targets with gaussian ACF and unknown one-lag correlation coefficient,  $\rho_s$ , in AWGN is considered. After proving the dependence of the simple hypothesis-testing problem LR detector on the assumed value of  $\rho_s$ , and the extreme complexity of the integral that involves the ALR detector, NNs are proposed as tools to approximate the ALR detector. NNs not only are capable of approximating this detector and its more robust performance with respect to  $\rho_s$ , but the implemented approximation is expected to have lower computational cost than other numerical approximations, a very important characteristic in real-time applications. MLPs of different sizes have been trained using a quasi-Newton algorithm to minimize the cross-entropy error. Results prove that MLPs with one hidden layer with 23 neurons can implement very robust detectors for  $T\text{SNR}$  values lower than 10dB.

## 1. INTRODUCTION

In this paper, a neural network (NN) based approach is presented for improving the robustness of optimum detectors based on the Neyman-Pearson (NP) criterion. The NP detector maximizes the probability of detection ( $P_D$ ), while maintaining the probability of false alarm ( $P_{FA}$ ) lower than or equal to a given value [1]. In simple hypothesis tests, the probability density functions (pdfs) of the inputs conditioned to both hypothesis, the likelihood functions, are known. In these problems, a decision rule based on comparing the likelihood ratio (LR), or any other equivalent discriminant function, with a detection threshold fixed attending to  $P_{FA}$  requirements, is an implementation of the NP detector [1].

In a radar detection problem, while interference statistics can be estimated from measurements in the operating environment, the estimation of target statistics is not possible. Because of that, a target model has to be assumed. The robustness of this detector with respect to the assumed target model can be evaluated as the observed loss in  $P_D$  when some statistical characteristics of the target vary from those assumed when calculating the LR. In [2] the robustness of the LR detector is studied for the problem of detecting gaussian targets in gaussian interference, proving an important dependence of the LR detector on the assumed target model.

In practical situations, target parameters such as the signal-to-interference ratio or the doppler shift, are random variables, so the likelihood function for the alternative hypothesis,  $H_1$ , is a function of these parameters, and the detection problem can be formulated as a composite hypothesis test [1]. As the probability density functions of these parameters are unknown, a very conservative solution is to assume that they are uniform in the variation interval. In many situations, this approach leads to intractable integrals without a closed-form solution. This is the case of the problem of detection of targets with unknown doppler shift in colored gaussian noise, studied in [3].

NNs are known to be able to approximate the optimum Bayesian classifier [4, 5, 6], and they have been widely applied to

classification tasks. But there are less examples of their application to detection problems attending to the NP criterion.

The possibility of approximate this detector using adaptive systems trained in a supervised manner for minimizing an error function, has been proven in [13]. Multi-Layer Perceptrons (MLPs), [7, 8, 9, 10], and Radial Basis Function Networks (RBFNs), [11, 12], have been applied to approximate the Neyman-Pearson detector in simple hypothesis test. In this paper, NNs are proposed as tools to approximate the Average Likelihood Ratio (ALR) detector in composite hypothesis test as a solution to the low robustness of the NP detector designed assuming a simple hypothesis problem with a given target model.

## 2. PROBLEM FORMULATION

The basic components of a simple binary decision problem are the source, the probabilistic transition mechanism, the observation space and the decision rule. The source generates an hypothesis of two possible ones ( $H_0$  and  $H_1$ ), while the probabilistic transition mechanism generates a point,  $\mathbf{z}$ , in the observation space,  $Z$ , according to the likelihood functions,  $f(\mathbf{z}|H_0)$  and  $f(\mathbf{z}|H_1)$ . In this problem, the NP detector decision rule consists in comparing the likelihood ratio (LR), or any equivalent discriminant function, to a detection threshold fixed attending to  $P_{FA}$  requirements [1].

$$\Lambda(\mathbf{z}) = \frac{f(\mathbf{z}|H_1)}{f(\mathbf{z}|H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta_0(P_{FA}) \quad (1)$$

In a composite hypothesis-test, the output of the source is a point,  $\theta$ , in a parameter space,  $\chi$ , and the hypothesis are subspaces of  $\chi$ . The pdf governing the mapping from  $\chi$  to  $Z$  is denoted by  $f(\mathbf{z}|\theta)$  and is assumed to be known for all values of  $\theta$  in  $\chi$ . If  $\theta$  is a random variable with known pdfs under the two hypothesis,  $f(\theta|H_0)$  and  $f(\theta|H_1)$ , the LR is [1]:

$$\Lambda(\mathbf{z}) = \frac{f(\mathbf{z}|H_1)}{f(\mathbf{z}|H_0)} = \frac{\int_{\chi} f(\mathbf{z}|\theta, H_1) f(\theta|H_1) d\theta}{\int_{\chi} f(\mathbf{z}|\theta, H_0) f(\theta|H_0) d\theta} \quad (2)$$

In a radar detection problem, as the interference parameters can be estimated from measurements in the operating environment, the parameters governing  $f(\mathbf{z}|H_0)$  assume specific known values. But to obtain  $f(\mathbf{z}|H_1)$  the integral in the numerator of (2) must be calculated. In many practical cases, this integral is intractable and has not any closed solution.

Taking into consideration that NN can be trained to approximate a discriminant function equivalent to the likelihood ratio, [13], a novel approach based on a MLP is proposed. The problem of detecting gaussian targets with gaussian autocorrelation function (ACF) and zero doppler shift in additive white gaussian noise (AWGN) is considered.

The target echo is modelled as a zero mean, gaussian complex vector,  $\mathbf{z}$ , of dimension  $n$ , with gaussian ACF, and a covariance matrix  $\mathbf{M}_s$ :

$$(\mathbf{M}_s)_{h,k} = p_s \cdot \rho_s^{|h-k|} \quad (3)$$

(with  $h, k = 1, 2, \dots, n$ ) where  $p_s$  and  $\rho_s$  are, respectively, the target power and the one-lag correlation coefficient.  $\rho_s$  is defined in (4), where  $\sigma_s$  is the standard deviation of the target spectrum and  $PRF$  is the pulse repetition frequency, or sampling rate [2].

$$\rho_s = \exp\left(-2\pi^2 \left(\frac{\sigma_s}{PRF}\right)^2\right) \quad (4)$$

The interference is modelled as zero mean AWGN, whose covariance matrix is given by  $(\mathbf{M}_n)_{h,k} = p_n \delta_{hk}$ , where  $p_n$  is the noise power and  $\delta_{hk}$  is the Kronecker delta.

As a normalization criterion,  $p_n$  is assumed equal to 2, and the signal-to-noise ratio is calculated as:

$$SNR = 10 \log_{10}(snr) = 10 \log_{10}\left(\frac{p_s}{p_n}\right) = 10 \log_{10}\left(\frac{p_s}{2}\right) \quad (5)$$

Two cases of study are analyzed:

- The one-lag correlation coefficient ( $\rho_s$ ) governing the ACF of the target is known.
- The one-lag correlation coefficient ( $\rho_s$ ) governing the ACF of the target is unknown.

## 2.1 $\rho_s$ is known

When  $p_s$  and  $\rho_s$  have known specific values, the detection problem can be formulated as a simple hypothesis-test. From the expression of the general multivariate normal pdf of zero mean and covariance matrix  $\mathbf{M}$ , in  $\mathbb{C}^n$  (6), and considering that  $\mathbf{M} = \mathbf{M}_n$  under  $H_0$ , and  $\mathbf{M} = \mathbf{M}_n + \mathbf{M}_s$  under  $H_1$ , the log-likelihood ratio (LLR) can be easily calculated and the NP decision rule can be expressed as in (7).

$$f(\mathbf{z}) = \frac{1}{\pi^n \cdot \det(\mathbf{M})} \exp(-\mathbf{z}^T \mathbf{M}^{-1} \mathbf{z}^*) \quad (6)$$

$$\ln(\Lambda(\mathbf{z})) = \mathbf{z}^T [\mathbf{M}_n^{-1} - (\mathbf{M}_n + \mathbf{M}_s)^{-1}] \mathbf{z}^* + k \underset{H_0}{\overset{H_1}{\geq}} \ln(\eta_0(P_{FA})) \quad (7)$$

$\mathbf{z}^T$  denotes the transposed vector, while  $\mathbf{z}^*$  denotes the complex conjugate vector.

As  $p_s$  and  $\rho_s$  are known,  $k$  is an immaterial constant that can be subtracted from both sides of the rule (7) to obtain an equivalent one:

$$\mathbf{z}^T [\mathbf{M}_n^{-1} - (\mathbf{M}_n + \mathbf{M}_s)^{-1}] \mathbf{z}^* \underset{H_0}{\overset{H_1}{\geq}} \eta_1(P_{FA}) \quad (8)$$

The statistics of rules (7) and (8) depend on the  $SNR$  and  $\rho_s$  values, because matrix  $\mathbf{M}_s$  does, as stated before. These are the design  $SNR$  and  $\rho_s$ . On the other hand, the observation vector is generated in an environment with  $SNR$  and  $\rho_s$  values known as *simulation* values in this paper. Clearly, the detectors of expressions (7) and (8) are only optimum for input vectors with simulation  $SNR$  and  $\rho_s$  values equal to the design ones. If its ROC curve is calculated for simulation  $SNR$  and  $\rho_s$  values different from the design ones, a loss in  $P_D$  will be observed that will be a function of the desired  $P_{FA}$ . From now on, the design and simulation  $SNR$  values will be denoted as  $DSNR$  and  $SSNR$ , respectively, while the design and simulation values of  $\rho_s$  will be denoted as  $\rho_s^d$  and  $\rho_s^s$ , respectively.

In order to study the robustness of rules (7) and (8) with respect to the  $DSNR$ , given a  $SSNR$  value, ROC curves have been plotted for different  $DSNR$ s. As in Air Traffic Control radar, the usual number of collected pulses in a scan is  $n = 8$ , this is the dimension of the complex input vector  $\mathbf{z}$ . In all cases, the simulation and the design  $\rho_s$  values are the same, and because of that, both are denoted as

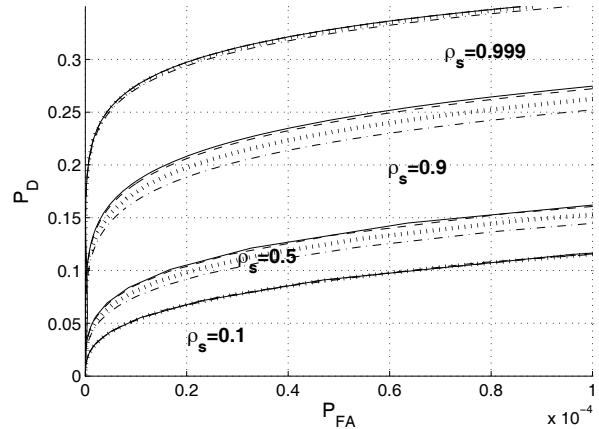


Figure 1: ROC curves for  $SSNR = 0dB$  and different  $\rho_s$  and  $DSNR$  values

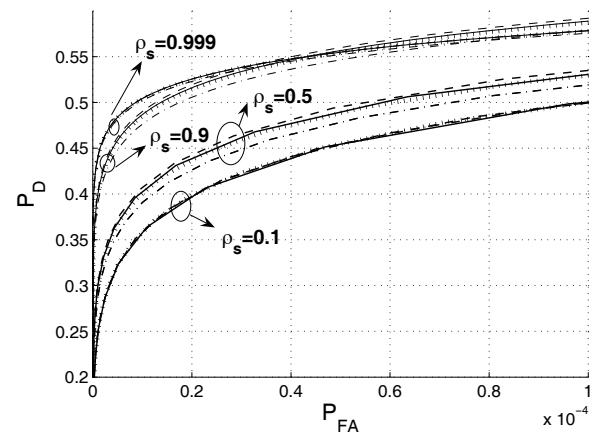


Figure 2: ROC curves for  $SSNR = 3dB$  and different  $\rho_s$  and  $DSNR$  values

$\rho_s$ . Results are presented in figures 1, 2 and 3, for  $SSNR = 0, 3$  and  $7dB$  and different  $DSNR$  and  $\rho_s$  values. Only  $P_{FA} \leq 10^{-4}$  are considered, because higher  $P_{FA}$  values have no interest in practical applications.

Results show that the dependence of detector performance on  $DSNR$  depends on  $\rho_s$  and  $SSNR$ . In general, it reduces as  $SSNR$  increases. For  $\rho_s$  values near 1 and 0, it is very low. For the rest of values, the variation in  $P_D$  has been evaluated for the considered  $SSNR$  values, and it is always lower than 2% for  $P_{FA} = 10^{-4}$ .

For  $SSNR$  values that produce low  $P_D$ , the detection capabilities are better for high values of  $\rho_s$  (figures 1 and 2). But when the obtained  $P_D$  is high, low  $\rho_s$  values produce better  $P_D$  ones (figure 3). This was observed for the envelope detector and is explained in [14].

$P_D$  values near 0.5 or lower have no interest in practical situations, and taking into consideration that for  $SSNR = 7dB$ , the obtained  $P_D$  values are suitable for all  $\rho_s$  ones, from now on, all the studies will be done for  $SSNR = 7dB$ .

To evaluate the robustness with respect to  $\rho_s$ , given a simulation value,  $\rho_s^s$ , ROC curves have been plotted for  $DSNR = SSNR = 7dB$ , and different  $\rho_s^d$  values. Results are shown in figures 4 and 5.

Although in order to save space, only some results are presented, we can conclude that for  $\rho_s^d \leq 0.5$  the dependence on  $\rho_s^d$  is not significant for  $\rho_s^d < \rho_s^s + 0.2$  (the observed reduction in  $P_D$  is lower than 1%), but the reduction in  $P_D$  is very important for detec-

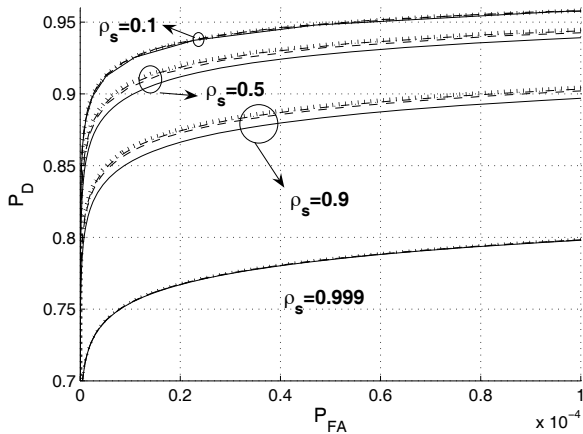


Figure 3: ROC curves for SSNR = 7dB and different  $\rho_s$  and DSIR values

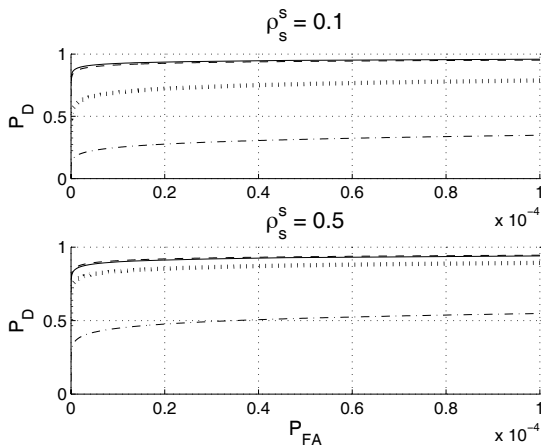


Figure 4: ROC curves for DSSN = SSNR = 7dB,  $\rho_s^s = 0.1$  (top) and  $\rho_s^s = 0.5$  (bottom) and different  $\rho_s^d$  values:  $\rho_s^d = 0.1$  (solid),  $\rho_s^d = 0.5$  (dashed),  $\rho_s^d = 0.9$  (dotted) and  $\rho_s^d = 0.999$  (dashdot).

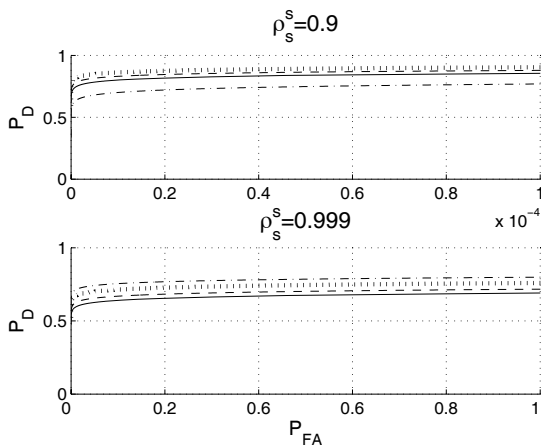


Figure 5: ROC curves for DSSN = SSNR = 7dB,  $\rho_s^s = 0.9$  (top) and  $\rho_s^s = 0.999$  (bottom) and different  $\rho_s^d$  values:  $\rho_s^d = 0.1$  (solid),  $\rho_s^d = 0.5$  (dashed),  $\rho_s^d = 0.9$  (dotted) and  $\rho_s^d = 0.999$  (dashdot).

tors with  $\rho_s^d$  near to 1 (figure 4). On the other hand the dependence increases for  $\rho_s^s > 0.5$ , except for  $\rho_s^d$  values near to 1, for which the reduction in  $P_D$  is smaller (figure 5).

### 2.2 $\rho_s$ is unknown

One solution to the problem of the dependence of the NP detector on the assumed value of  $\rho_s$ , is to formulate a composite hypothesis-testing problem where the likelihood function under  $H_1$  is a function of this parameter. Furthermore, in practical situations, the actual value of  $\rho_s$  is unknown and difficult to estimate. In our problem, the covariance matrix of the target component of the input vector will be a function of  $\rho_s$ , and will be denoted as  $M_s(\rho_s)$ . The optimum decision rule in the NP sense compares the ALR, or any equivalent discriminant function, to a detection threshold fixed attending to  $P_{FA}$  requirements. Assuming that  $\rho_s$  can be modelled as a random variable with uniform pdf in  $[0, 1]$ , and attending to (2), the numerator involves the calculus of the following integral:

$$\int_0^1 \frac{1}{\pi^n \cdot \det(M_s(\rho_s) + M_n)} \exp(-z^T (M_s(\rho_s) + M_n)^{-1} z^*) d\rho_s \quad (9)$$

Note that as  $M_s(\rho_s) + M_n$  is a function of  $\rho_s$ , its determinant and the argument of the exponential in (9) are complex functions of  $\rho_s$ . Because of that, the calculus of this integral is very complex. In the following section, a NN based approach is proposed to approximate an expression equivalent to the ALR.

### 3. NN BASED DETECTOR

In this section MLPs that use real arithmetic are trained for approximating the ALR for detecting gaussian targets with gaussian ACF, and unknown  $\rho_s$ . This approach is proposed as a solution to the dependence of the simple hypothesis-testing problem NP detector on  $\rho_s$ . For the NN to approximate the ALR test, the patterns under  $H_1$  which are used for training are generated with  $\rho_s$  values that vary uniformly in  $[0, 1]$ . Also, the approximation implemented by the NN is expected to have lower computational cost than other numerical approximations, a very important characteristic in real-time applications.

In all cases, the MLPs have a hidden layer and an output one. While the number of hidden units is varied, the output layer always has one neuron. As the  $n$ -dimension complex vectors are transformed in  $2n$ -dimension real ones, composed of the real parts (the first  $n$  samples) and the imaginary parts (the remaining  $n$  samples), MLPs with  $2n = 16$  inputs have been trained. A hard threshold detector has been placed at the MLP output with a threshold fixed attending to  $P_{FA}$  requirements: if the NN output is greater than the threshold,  $H_1$  is accepted, in other case,  $H_0$  is accepted.

#### 3.1 Design of the experiments

In this context, the design parameters previously defined are known as ‘training parameters’. If the DSSNR was the SNR value used for implementing the NP detector statistic, the Training Signal-to-Noise ratio (TSNR) is the value selected for generating the training set, while the SSNR is the value selected for generating the simulation sets for evaluating the performance of the trained NNs.

For simple hypothesis-testing problems and SSNR values that produce acceptable  $P_D$  for  $P_{FA}$  values of interest, the optimum detector has been proved to be very robust with respect to the DSSNR. Nevertheless, the dependence of the NN-based detector performance on the TSNR must be studied, because the statistical properties of the training set can determine the learning abilities of the NN [7, 8, 10, 12]. To study the dependence on TSNR, different values have been selected. For each TSNR, separated training and validation sets composed of 60,000 randomly distributed patterns from  $H_0$  and  $H_1$  have been generated.

Taking into consideration the results presented in [10, 12], the minimum number of hidden units that are necessary to enclose the

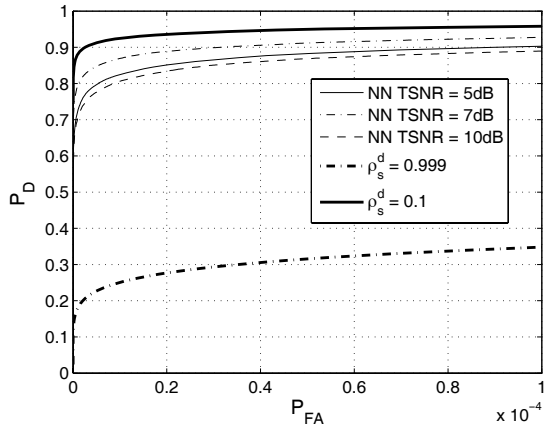


Figure 6: ROC curves for  $SSNR = 7dB$  and  $\rho_s^s = 0.1$ : detectors for the simple hypothesis-testing problem with  $\rho_s^d = 0.1$  and  $\rho_s^d = 0.999$  (wider lines) and NNs with 20 hidden units trained with  $TSNR = 5, 7$  and  $10dB$  (thinner lines).

decision boundary is expected to be between 3 and 17 (for  $\rho_s = 1$  and  $\rho_s = 0$ , respectively). NNs with different number of hidden units have been trained. Results show that although the detection performance of the trained NNs tends to increase as the number of hidden units increases, for more than 23 hidden neurons, the performance improvement is insignificant, while the associated computational cost continues growing. The ROC curves estimated for NNs with 20 and 23 hidden neurons are presented.

NNs have been trained for minimizing the cross-entropy error, using a quasi-Newton error minimization algorithm. This algorithm involves the estimation of the Hessian matrix, which can be computationally prohibitive for large NNs. In [15] a strategy is proposed for estimating the NN coefficients that reduces the computational burden, retaining the fast convergence properties of the quasi-Newton algorithm. A cross-validation technique has been used to avoid over-fitting and all NNs have been initialized using the Nguyen-Widrow method [16]. For each case, the training process has been repeated ten times. Only the cases where the performances of the ten trained networks were similar in average, have been considered to extract conclusions.

$P_{FA}$  values have been estimated using Importance Sampling techniques (relative error lower than 10% in the presented results) [17, 18].  $P_D$  values have been estimated using conventional Monte-carlo simulation.

### 3.2 Results

Results are presented in figures 6, 7, 8 and 9. Different size NNs trained with  $TSNR = 5, 7$  and  $9dB$ , and a  $\rho_s$  variable uniformly in  $[0, 1]$ , are compared to the optimum detectors for the simple hypothesis-test with  $\rho_s^d = 0.1$  and  $0.999$ , for  $SSNR = 7dB$ .

Figures 6 and 7 show the results obtained for  $\rho_s^s = 0.1$  and MLPs with 20 and 23 hidden units, respectively. As was expected, the NN based detectors are worse than the detector for the simple hypothesis-testing problem with  $\rho_s^d = \rho_s^s = 0.1$ , but much better than the detector for the simple hypothesis-testing problem with  $\rho_s^d = 0.999$ . The dependence of the performance of the NN based detectors on  $TSNR$  reduces when NN size increases, except for high  $TSNR$  values. The performance improvement observed for the best  $TSNR$ s as the number of hidden units increases is almost insignificant.

Figures 8 and 9 show the results obtained for  $\rho_s^s = 0.999$  and MLPs with 20 and 23 hidden units, respectively. Again, the NN based detectors are worse than the detector for the simple hypothesis-testing problem with  $\rho_s^d = \rho_s^s = 0.999$ , but better

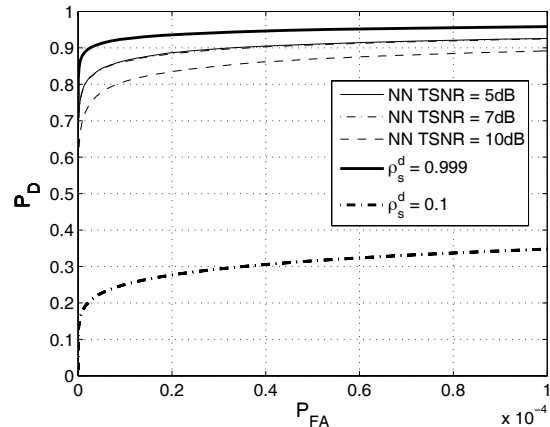


Figure 7: ROC curves for  $SSNR = 7dB$  and  $\rho_s^s = 0.1$ : detectors for the simple hypothesis-testing problem with  $\rho_s^d = 0.1$  and  $\rho_s^d = 0.999$  (wider lines) and NNs with 23 hidden units trained with  $TSNR = 5, 7$  and  $10dB$  (thinner lines).

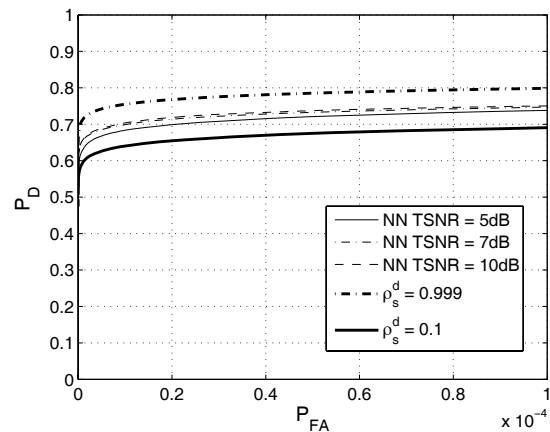


Figure 8: ROC curves for  $SSNR = 7dB$  and  $\rho_s^s = 0.999$ : detectors for the simple hypothesis-testing problem with  $\rho_s^d = 0.1$  and  $\rho_s^d = 0.999$  (wider lines) and NNs with 20 hidden units trained with  $TSNR = 5, 7$  and  $10dB$  (thinner lines).

than the detector for the simple hypothesis-testing problem with  $\rho_s^d = 0.1$ . The performance improvement with respect to the detector for the simple hypothesis-testing problem with  $\rho_s^d \neq \rho_s^s$  is lower than that observed in figures 6 and 7, because the dependence of the detector for the simple hypothesis-testing problem with respect to  $\rho_s^d$  decreases as  $\rho_s^s$  increases (figure 5). The dependence of the performance of the NN based detectors on  $TSNR$  is lower than that observed for  $\rho_s^s = 0.1$  and reduces when NN size increases for all the considered  $TSNR$  values. The performance improvement observed for the best  $TSNR$ s as the number of hidden units increases is almost insignificant.

## 4. CONCLUSIONS

Taking into consideration previous results where NNs are proved to be able to approximate the NP detector, in this paper, a NN based detector has been proposed for approximating the ALR detector in composite hypothesis-testing problems. In practical situations where only some statistical parameters of the hypothesis are known, this approach is proposed as a solution to the dependence of

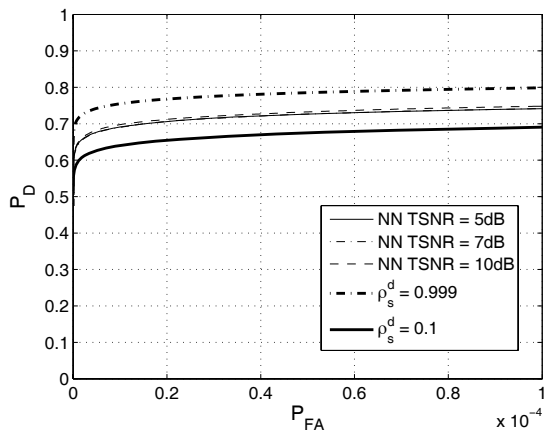


Figure 9: ROC curves for  $SSNR = 7dB$  and  $\rho_s^s = 0.999$ : detectors for the simple hypothesis-testing problem with  $\rho_s^d = 0.1$  and  $\rho_s^d = 0.999$  (wider lines) and NNs with 23 hidden units trained with  $TSNR = 5, 7$  and  $10dB$  (thinner lines).

the simple hypothesis-testing problem NP detector on the assumed parameters values. If the unknown parameters are random variables of known pdf, the optimum detector involves the calculus integrals that, in many cases, are intractable and rarely have closed solutions. In these cases, NNs are proposed as tools to approximate these integrals. Also, the approximation implemented by the NN is expected to have lower computational cost than other numerical approximations, a very important characteristic in real-time applications.

The case of detecting gaussian targets with gaussian ACF in AWGN is considered. As a first step, a study has been carried out to prove the dependence of the simple hypothesis-testing problem LR detector on the design one-lag correlation coefficient  $\rho_s^d$ . After that, the calculus of the ALR detector for  $\rho_s$  uniformly distributed in  $[0, 1]$  has been formulated, proving the extreme complexity of the required integral. Finally, MLPs of different sizes have been trained using a quasi-Newton algorithm to minimize the cross-entropy error. Although the ALR detector has not been calculated, the performance of the trained NNs is more robust than the simple hypothesis-testing LR detectors for  $\rho_s^d \neq \rho_s^s$ , as was expected.

Results prove that MLPs with one hidden layer with 23 neurons can implement very robust detectors for  $TSNR$  values lower than  $10dB$ .

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