

## BEARING AND RANGE ESTIMATION OF BURIED CYLINDRICAL SHELL IN PRESENCE OF SENSOR PHASE ERRORS

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### ABSTRACT

Localization of buried cylindrical shells in presence of sensor phase errors is presented. This method is based on a new technique [5] that we have developed for buried object localization using a combination of acoustical model, focusing operator and the MUSIC method. The main assumption in [5] is the well known of the sensor positions that are not always satisfied. Thus, in practice the sensors move from their original positions during the experimentation (deformed sensor array) which introduce phase error in each sensor. Correction of phase errors is necessary to solve the object localization problem. The method provides estimation of the bearings and ranges of all buried objects as well as the phase error of each sensor in the observing array. This problem is reduced to minimization problem function using the DIRECT algorithm (DIviding RECTangles) to seek the minimum of this function. Finally, the performances of the proposed method are validated on experimental data recorded during an underwater acoustics experiments.

### 1. INTRODUCTION

Most array processing techniques are based on the assumption that the shape of the array remains unchanged. This assumption permits the use of simplified array signal processing techniques. Recently we have developed a method [5] based on the MUSIC method, modified by using the focusing matrix instead of the conventional spectral matrix of the observations, and by including the acoustic scattering model of the objects instead of the plane wave model. These well known methods are modified in order to adapt them to solve the buried object localization problem. This method is validated using experimental data and the obtained results are satisfying. However this method needs the well known sensor positions. Unfortunately in practice, the shape of a sensor array, such as flexible towed array and bottom mounted array, is deformed due to the fluctuations in ship maneuver-

ing, underwater currents and swells, and so on. These cause phase errors in the received signals. Several methods, to estimate the phase errors related to the use of deformed sensor array, were proposed in the literature [2], [4]. The proposed method in [4] uses the sources with known locations as reference sources to correct the positions of the sensors. In [1] the authors assume that the shape of the sensor array is known. Furthermore, those methods consider that the objects are located in the farfield region of sensor array. In such case the range objects are infinite and are not taken into account. This assumption is not valid in the nearfield region of the array.

In this study, we address the problem of estimating simultaneously the bearing and the range objects in presence of sensor random phase errors. Thus, here we extend the method developed in [5] to the case where the sensors have unknown (or imprecisely known) phases. The idea is to define an objective function with multiple variables which represent the phase errors and the bearing and the range objects. This objective function is based on the orthogonality property between the object subspace and the noise subspace and to minimize this function, we propose to use the DIRECT (DIviding RECTangle) algorithm.

The organization of this study is as follows : problem formulation is presented in Section 2. In Section 3, the objective function of multiple variables is defined. In Section 4, the experimental setup is presented. Experimental results supporting our conclusions and demonstrating our method are provided in section 5. Finally, conclusions are presented in Section 6.

Throughout the paper, lowercase boldface letters represent vectors, uppercase boldface letters represent matrices, and lower and uppercase letters represent scalars. The symbol "T" is used for transpose operation and the superscript "+" is used to denote complex conjugate transpose.

## 2. PROBLEM FORMULATION

We consider a flexible array of  $N$  sensors which received the wideband signals scattered from  $P$  objects ( $N > P$ ) in the presence of an additive Gaussian noise and sensor phase errors. The received signals are grouped in the vector  $\mathbf{r}$  given, in the frequency domain, by

$$\mathbf{r}(f_n, \theta, \rho, \phi) = \mathbf{A}_p(f_n, \theta, \rho, \phi)\mathbf{s}(f_n) + \mathbf{b}(f_n), \quad (1)$$

where,  $n = 1, \dots, L$ ,  $\mathbf{r}(f_n, \theta, \rho, \phi)$  is the Fourier transforms of the array output vector,  $\mathbf{s}(f_n)$  is the vector of object signals,  $\mathbf{b}(f_n)$  is the vector of white Gaussian noise of variance  $\sigma^2(f_n)$ ,  $\mathbf{A}_p(f_n, \theta, \rho, \phi)$  is the transfer matrix (propagation matrix) given by

$$\mathbf{A}_p(f_n, \theta, \rho, \phi) = [\mathbf{a}(f_n, \theta_1, \rho_1), \dots, \mathbf{a}(f_n, \theta_P, \rho_P)], \quad (2)$$

$\mathbf{a}(f_n, \theta_k, \rho_k)$  is given by,

$$\mathbf{a}(f_n, \theta_k, \rho_k) = [1a(f_n, \theta_{k1}, \rho_{k1}), \dots, e^{-j(\phi_{N-1})}a(f_n, \theta_{kN}, \rho_{kN})], \quad (3)$$

where  $k = 1, \dots, P$ ,  $\phi_i$  represents the phase error associated to the random moving of the  $i^{th}$  sensor from its original position (we assume that the first sensor does not move).  $\theta_k$  and  $\rho_k$  are the bearing and the range of the  $k^{th}$  object to the first sensor of the array, thus,  $\theta_k = \theta_{k1}$  and  $\rho_k = \rho_{k1}$ .  $a(f_n, \theta_{ki}, \rho_{ki})$  is the exact solution of the acoustic scattered field by a cylindrical shell, given by

$$a(f_n, \theta_{ki}, \rho_{ki}) = p_{c0} \sum_{m=0}^{\infty} j^m \epsilon_m b_m H_m^{(1)}(K_{n1} \rho_{ki}) \cos(m(\theta_{ki} - \theta_{inc})), \quad (4)$$

where  $p_{c0}$  is a constant,  $\epsilon_0 = 1, \epsilon_1 = \epsilon_2 = \dots = 2$ ,  $b_m$  is a coefficient depending on limits conditions and  $m$  is the number of modes,  $H_m$  represents the Hankel function,  $c_1$  is the sound velocity and the wavenumber  $K_{n1} = \frac{2\pi f_n}{c_1}$ . The spectral matrix  $\mathbf{\Gamma}(f_n, \theta, \rho, \phi)$  is formed for each narrowband data in each frequency bin and given by

$$\mathbf{\Gamma}(f_n, \theta, \rho, \phi) = \mathbf{A}_p(f_n, \theta, \rho, \phi)\mathbf{\Gamma}_s(f_n) \mathbf{A}_p^+(f_n, \theta, \rho, \phi) + \sigma^2(f_n)\mathbf{I}, \quad (5)$$

where,  $\mathbf{\Gamma}_s(f_n)$  is the spectral matrix associated to the object signals and  $\mathbf{I}$  is the identity matrix.

The frequency diversity is employed, in order to decorrelate the signals [6]. The idea is to use the bilinear focusing operator [7], to transform the narrowband data in each frequency bin into a single reference frequency bin  $f_0$ . Then, the average of the focused matrices is given by

$$\bar{\mathbf{\Gamma}}(f_0, \theta, \rho, \phi) = \frac{1}{L} \sum_{n=1}^L \mathbf{T}(f_0, f_n)\mathbf{\Gamma}(f_n, \theta, \rho, \phi)\mathbf{T}^+(f_0, f_n), \quad (6)$$

where  $\mathbf{T}(f_0, f_n)$  is the bilinear focusing operator [7] and  $f_0$  is the focusing frequency chosen in the frequency band on interest. Finally, the spatial spectrum is given by

$$Z_p(\theta_k, \rho_k) = \|\mathbf{a}(f_0, \theta_k, \rho_k)\mathbf{C}(\phi)\bar{\mathbf{V}}_b(f_0)\|^{-2}, \quad (7)$$

where  $\mathbf{C}$  is a diagonal matrix containing the sensor phase errors, given by,

$$\mathbf{C}(\phi) = \text{diag}[1, e^{-j(\phi_1)}, \dots, e^{-j(\phi_{N-1})}],$$

$\bar{\mathbf{V}}_b(f_0)$  is the eigenvector matrix of  $\bar{\mathbf{\Gamma}}(f_0, \theta, \rho, \phi)$  associated to the smallest eigenvalues. The goal of our method is to estimate simultaneously these phase errors and the object bearings and ranges. Thus, we use the orthogonality property between the source subspace and the noise subspace to form an objective function to minimize using the DIRECT algorithm.

## 3. THE OBJECTIVE FUNCTION

The objective function, that we minimize using the DIRECT (DIviding RECTangles) optimization algorithm [3], is based on the orthogonality between the two subspaces defined above and it is given by

$$F(\theta, \rho, \phi_1, \dots, \phi_{N-1}) = \|\bar{\mathbf{V}}_b^+ \mathbf{a}(f_0, \theta, \rho)\mathbf{C}(\phi_1, \dots, \phi_{N-1})\|^2, \quad (8)$$

where, Note that Eq. (8) is Lipchizian [3] and satisfies the following condition

$$|F(\Phi) - F(\Phi')| \leq \beta |\Phi - \Phi'|, \quad (9)$$

where  $\Phi = [\theta, \rho, \phi_2, \dots, \phi_N]^T$  and  $\Phi' = [\theta', \rho', \phi'_2, \dots, \phi'_N]^T$  and  $0 < \beta < 1$ . The proposed method allows us to estimate both the source bearings and the phase errors by minimizing the objective function defined above.

The following is the step-by-step description of the developed method:

- dividing the bearing axis into  $L_\theta$  overlapping intervals of length  $2\Delta\theta$ , where each interval  $i$  is defined by :  
 $[\theta_i - \Delta\theta, \theta_i + \Delta\theta]$ ,
- dividing the range axis into  $L_\rho$  overlapping intervals of length  $2\Delta\rho$ , where each interval  $j$  is defined by :  
 $[\rho_j - \Delta\rho, \rho_j + \Delta\rho]$ ,
- form the objective function for each  $[\theta_i - \Delta\theta, \theta_i + \Delta\theta] \times [\rho_j - \Delta\rho, \rho_j + \Delta\rho]$  interval,

$$F(\theta_i, \rho_j, \phi_{1ij}, \dots, \phi_{N-1ij}) = \|\bar{\mathbf{V}}_b^+ \mathbf{a}(\theta_i, \rho_j)\mathbf{C}_{ij}(\phi_{1ij}, \dots, \phi_{N-1ij})\|^2,$$

where,  $i = 1, \dots, L_\theta$  and  $j = 1, \dots, L_\rho$ ,

- find  $\theta_i, \rho_j, \phi_{1ij}, \dots, \phi_{N-1ij}$  that minimize the objective function in each interval, using the DIRECT algorithm.

The minimum of the objective function in an interval containing sources is smaller than the minimum of that function in an interval which not containing objects thus the idea is to calculate it for all the intervals and to make a thresholding to keep only the smallest objective functions.

#### 4. EXPERIMENTAL SETUP

The data has been recorded using an experimental water tank ( Fig. 1) in order to evaluate the performances of the developed method.

The transmitter sensor is fixed at an incident angle  $\theta_{inc} = 60^\circ$  and has a beamwidth equal to  $5^\circ$ . The receiver sensor is omnidirectional and moves horizontally along the  $XX'$  axis, step by step, from the initial to the final position ( Fig. 2) with a step size  $d = 0.002$  m and takes ten positions in order to form an uniform linear array of sensors with  $N = 10$ . The transmitted signal has the following properties; impulse duration is  $15 \mu s$ , the frequency band is  $[f_{min} = 150, f_{max} = 250]$  kHz and the sampling rate is 2 MHz. The duration of the received signal is  $700 \mu s$ . This tank is filled of water with  $W_h = 0.5$  m (Fig. 2)

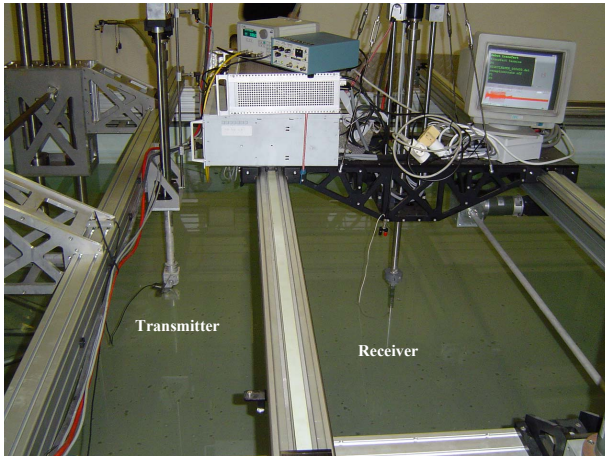


Fig. 1. Experimental tank

and its bottom is filled with homogeneous fine sand with  $c_s = 1700$  m/s, where are buried three infinitely long cylinder couples  $((O_1, O_2), (O_3, O_4), (O_5, O_6))$  ( Fig. 3). Table 1 summarizes the characteristics of these objects. The considered objects are made of dural aluminum with density  $D_2 = 1800$  kg/m<sup>3</sup>, the longitudinal and transverse-elastic wave velocities inside the shell medium are  $c_l = 6300$  m/s and  $c_t = 3200$  m/s, respectively. The external fluid is water with density  $D_1 = 1000$  kg/m<sup>3</sup> and the internal fluid

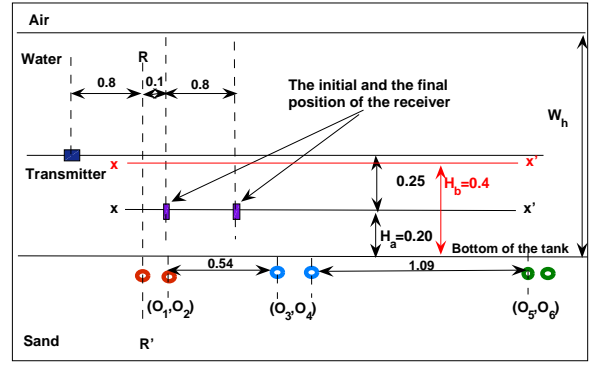


Fig. 2. Experimental setup



Fig. 3. Objects

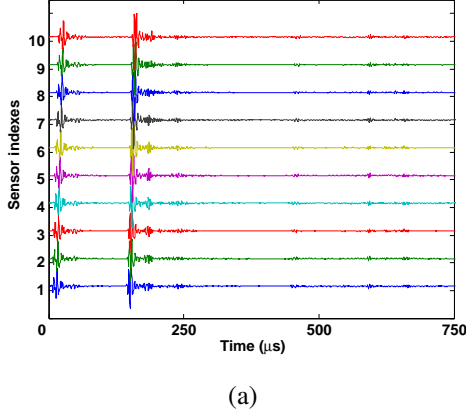
Couple	$(O_1, O_2)$	$(O_3, O_4)$	$(O_5, O_6)$
Outer radius (m)	0.01	0.018	0.02
Length (m)	$l_{O_1} = 0.258$ $l_{O_2} = 0.69$	$l_{O_3} = 0.372$ $l_{O_4} = 0.396$	$l_{O_5} = 0.63$ $l_{O_6} = 0.24$
Filled of	air	water	air
Separated by (m)	0.13	0.16	0.06

Table 1. characteristics of the various objects (the inner radius=the outer radius–0.001 m)

is water or air with density  $D_{3air} = 1.2 \cdot 10^{-6}$  kg/m<sup>3</sup> or  $D_{3water} = 1000$  kg/m<sup>3</sup>.

The experimental setup is shown in Fig.2 where all the dimensions are given in meter. First, we have buried the considered objects in the sand at 0.005 m. Then, we have done six experiments that we have called  $E_{i(O_{ii}, O_{ii+1})}$ , where  $i = 1, \dots, 6$  and  $ii = 1, 3, 5$ . Two experiments are performed for each couple: One, when the receiver horizontal axis  $XX'$  is fixed at  $H_a = 0.2$  m ( $E_{1(O_1, O_2)}, \dots, E_{3(O_5, O_6)}$ ), the other when this axis is fixed at  $H_b = 0.4$  m ( $E_{4(O_1, O_2)}, \dots, E_{6(O_5, O_6)}$ ).  $RR'$  is a vertical axis which goes through the center of the first object of each couple. Thus, for each ex-

periment, only one object couple is radiated by the transmitter sensor. At each sensor, time-domain data corresponding only to target echoes are collected with signal to noise ratio equal to 20 dB. The typical sensor output signals recorded during one experiment are shown in Fig. 4.



**Fig. 4.** Observed sensor output signals associated to experiment  $E_{1(O_1, O_2)}$ .

## 5. RESULTS AND DISCUSSION

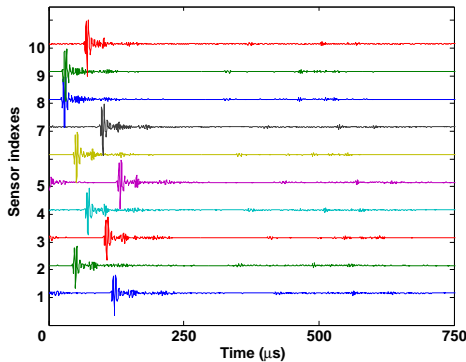
The first step is to add random phase errors on each sensor output signal. Thus, for the three first experiments ( $E_{1(O_1, O_2)}$ ,  $E_{2(O_3, O_4)}$  and  $E_{3(O_5, O_6)}$ ) we have used,

$$\phi_a = [0, 30, 48, 35, -15, -42, -60, -12, 6, 10]^\circ,$$

For the three others ( $E_{4(O_1, O_2)}$ ,  $E_{5(O_3, O_4)}$  and  $E_{6(O_5, O_6)}$ ) we have used

$$\phi_b = [0, 12, 31, 5, -55, -90, -63, -14, 2, 38]^\circ.$$

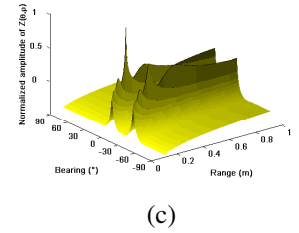
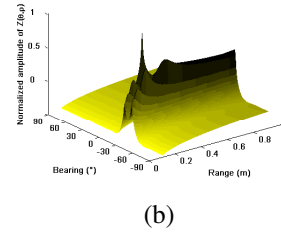
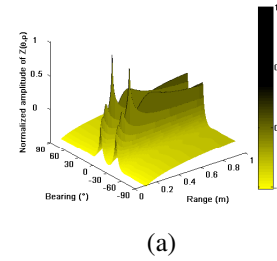
The experimental data affected by  $\phi_a$  associated to the experiment  $E_{1(O_1, O_2)}$  is shown in Fig. 5 Furthermore, the



**Fig. 5.** Observed sensor output signals affected by sensor phase errors  $\phi_a$  and associated to experiment  $E_{1(O_1, O_2)}$ .

average of the focused matrices is calculated using  $L = 50$  frequencies chosen in the frequency band of interest  $[150, 250]$  kHz and the middle frequency is chosen as the focusing frequency  $f_0 = 200$  kHz. Furthermore, a sweeping on the bearing and the range have been applied ( $[-90^\circ, 90^\circ]$  for the bearing with a step  $0.1^\circ$  and  $[0.15, 1]$  m (or  $[0.15, 1.4]$  m) for the range with a step  $0.002$  m). The obtained bearings and ranges before the correction of the random phase errors are shown in Figs. 6 and 8. Then, after correction of sensor random phase errors using the minimisation of the objective function, the obtained bearings and ranges are shown in Figs. 7 and 9.

Furthermore, we have obtained an  $\text{RMSE}_\theta = 1.48^\circ$  and



**Fig. 6.** Spatial spectrum in presence of sensor phase errors  $\phi_a$ : before correction of the phase errors. (a)  $E_{1(O_1, O_2)}$ . (b)  $E_{2(O_3, O_4)}$ . (c)  $E_{3(O_5, O_6)}$ .

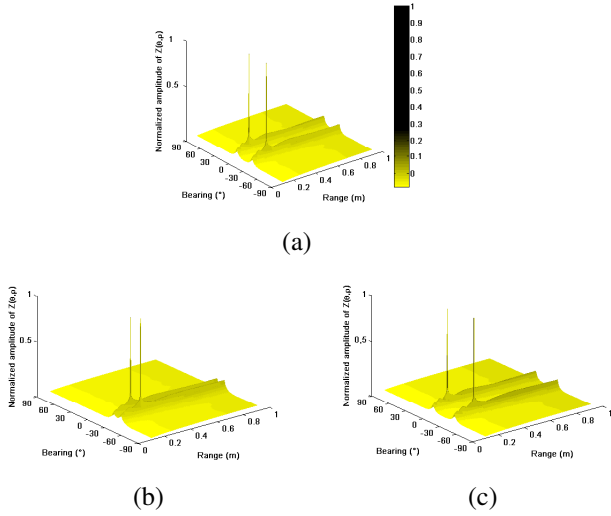
$\text{RMSE}_\rho = 0.04$  m between the expected ( $[\cdot]_{exp}$ ) and the estimated ( $[\cdot]_{est}$ ) bearings and ranges by the proposed method using the following equation,

$$\text{RMSE}_X = \sqrt{\frac{\sum_{i=1}^6 \left[ (X_{exp1} - X_{est1})_i^2 + (X_{exp2} - X_{est2})_i^2 \right]}{12}},$$

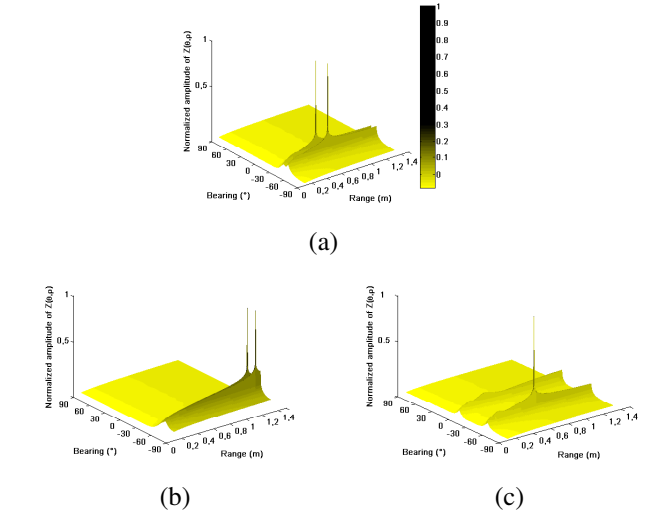
where  $X$  represents  $\theta$  or  $\rho$ ,  $i$  is the experiment and the indexes 1 and 2 represent the first and the second objects of each couple, respectively.

## 6. CONCLUSION

In this study, we have proposed a novel method to estimate both the range and the bearing of buried objects in presence of sensor phase errors. This approach, does not require any prior knowledge of the sensor array shape to be able to correct the data. For that, one combined the method developed



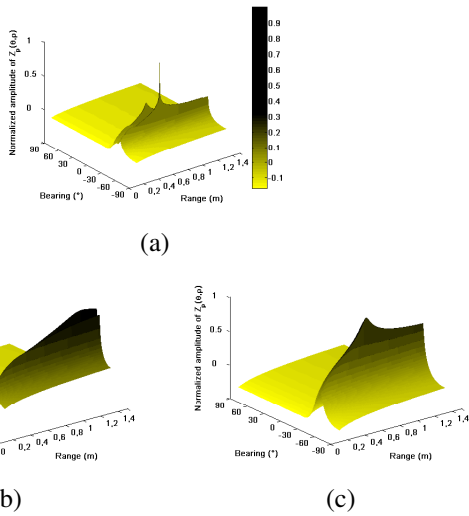
**Fig. 7.** Spatial spectrum in presence of sensor phase errors  $\phi_a$ : after correction of the phase errors. (a)  $E_{1(O_1, O_2)}$ . (b)  $E_{2(O_3, O_4)}$ . (c)  $E_{3(O_5, O_6)}$ .



**Fig. 9.** Spatial spectrum in presence of sensor phase errors  $\phi_b$ : after correction of the phase errors. (a)  $E_{4(O_1, O_2)}$ . (b)  $E_{5(O_3, O_4)}$ . (c)  $E_{6(O_5, O_6)}$ .

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**Fig. 8.** Spatial spectrum in presence of sensor phase errors  $\phi_b$ : before correction of the phase errors. (a)  $E_{4(O_1, O_2)}$ . (b)  $E_{5(O_3, O_4)}$ . (c)  $E_{6(O_5, O_6)}$ .

in [5] with the DIRECT algorithm to estimate simultaneously the bearing and the range objects in presence of sensor phase errors. The performances of this method are investigated through experimental data affected by random phase errors and associated to many cylindrical shells buried under the sand. The proposed method is superior in terms of performance to the conventional method.

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