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Distributed Algorithms for Maximum Throughput in Wireless Networks

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I. Introduction

The optimal control of multi-hop wireless networks is a major research and design challenge due, in part, to the interference between nodes, the time-varying nature of the communication channels, the energy limitation of mobile nodes, and the lack of centralized coordination. This problem is further complicated by the randomness of data traffic arrivals. Although a complete solution to the problem is still elusive, a major advance is made in the seminal work of [1], which obtains a *throughput optimal* routing and link activation policy without *a priori* knowledge of arrival statistics. The policy operates on the Maximum Differential Backlog (MDB) principle, which essentially seeks to achieve load-balancing in the network. The MDB policy has been extended to multi-hop networks with general capacity constraints in [2].

There remains, however, a significant difficulty in applying the MDB policy to practical wireless networks. The mutual interference between wireless links implies that the evaluation of the MDB policy involves a centralized optimization. On the other hand, effective control strategies for large-scale wireless networks require distributed implementations with low control messaging overhead. Motivated by this concern, we investigate the distributed implementation of the MDB algorithm within interference-limited CDMA wireless networks, where transmission on any given link potentially interferes with transmissions on all other active links.

We represent the network by a directed and connected graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$. For convenience, let $\mathcal{O}(i) \triangleq \{j : (i,j) \in \mathcal{E}\}$ and $\mathcal{I}(i) \triangleq \{j : (j,i) \in \mathcal{E}\}$ denote the sets of node i's next-hop and previous-hop neighbors, respectively. Let $\boldsymbol{h} = (h_{ij})_{(i,j)\in\mathcal{E}}$ represent the (constant) channel gains on all links. Denote the transmission power used on link (i,j) by P_{ij}, and the service rate of link (i,j) by $R_{ij} \le C_{ij}$, where C_{ij} is the (approximate[1]) information-theoretic capacity of link (i,j) given by

$$\log SINR_{ij} = \log \frac{K h_{ij} P_{ij}}{h_{ij}(P_i - P_{ij}) + \sum\limits_{m \ne i} h_{mj} P_m + N_j},$$

where K is the processing gain, $P_m = \sum_{k\in\mathcal{O}(m)} P_{mk}$ is the total transmission power of node m, and N_j represents the noise power of receiver j.

Let $\mathcal{K}$ be the set of all data traffic types. When type k traffic reaches any node in its destination set $\mathcal{N}_k \subset \mathcal{N}$, it exits the network. The new arrivals of type k traffic at node i in the tth slot is a nonnegative random variable $B_i^k[t]$. Node $i \notin \mathcal{N}_k$ provides a (separate) infinite buffer i^k for each type k of traffic. Denote the unfinished work in i^k at the beginning of the tth slot by $U_i^k[t]$.

The following Maximum Differential Backlog (MDB) policy has been shown to be *throughput optimal* [1], [2] in the sense that it stabilizes all input processes with average arrival rate vectors which can be stabilized by any feasible control policy. Let $b_{ij}^*[t] = \max\{0, U_i^{k_{ij}^*[t]}[t] - U_j^{k_{ij}^*[t]}[t]\}$, where $k_{ij}^*[t] = \arg\max_{k\in\mathcal{K}}\{U_i^k[t] - U_j^k[t]\}$. The MDB policy then finds the rate vector $\boldsymbol{R}^*(\boldsymbol{U}[t])$ which solves

$$\max_{\boldsymbol{R}\text{ feasible}} \sum_{(i,j)\in\mathcal{E}} b_{ij}^*[t] \cdot R_{ij}. \qquad (1)$$

The service rate provided by link (i,j) to queue i^k is determined by: $R_{ij}^k[t] = R_{ij}^*[t]$ if $k = k_{ij}^*[t]$, and $R_{ij}^k[t] = 0$ for all other k.

II. Distributed MDB Control

We consider solving the optimization problem in (1) at a fixed time slot in a power-constrained CDMA network, i.e., for each node i, the total transmission power $\sum_{j\in\mathcal{O}(i)} P_{ij} \le \hat{P}_i$. Let the transmission powers be controlled by the following node-based *power*

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[1]We assume a high-SINR situation, which is typical in CDMA systems, and normalize the channel symbol rate to be one.

allocation variables $\eta_{ij} \equiv P_{ij}/P_i$ and *power control variables* $\gamma_i \equiv \ln P_i / \ln \hat{P}_i$. With appropriate scaling, we can always let $\hat{P}_i > 1$ for all $i \in \mathcal{N}$ so that the above variables are constrained by $\eta_{ij} \geq 0$, $\forall (i,j) \in \mathcal{E}$, and $\sum_{j \in \mathcal{O}(i)} \eta_{ij} = 1$, $\gamma_i \leq 1$, $\forall i \in \mathcal{N}$. We show that a power configuration is optimal if and only if for all $i \in \mathcal{N}$, there exists a constant ν_i such that $\delta\eta_{ij} = \nu_i$, $\forall j \in \mathcal{O}(i)$, $\delta\gamma_i = 0$, if $\gamma_i < 1$, and $\delta\gamma_i \geq 0$, if $\gamma_i = 1$. Here, $\delta\eta_{ij}$ and $\delta\gamma_i$ are the *power allocation indicator* and the *power control indicator* capturing the gradients in η_{ij} and γ_i, respectively. We show that $(\delta\eta_{ij})$ can be computed by node i based entirely on local measures. Although the computation of γ_i involves global information, it turns out that a simple message exchange protocol requiring each node to broadcast one local measure suffices to let all nodes determine their γ_i.

To achieve the optimal configuration, we design scaled gradient projection algorithms PA and PC which iteratively update the nodes' power allocation and power control variables in a distributed manner. At each iteration k, the variables are updated in the direction of $\delta\boldsymbol{\eta}_i$ or $\delta\boldsymbol{\gamma}$, scaled by a positive definite matrix Q_i^k or V^k. When an update leads to a point outside the feasible set, the point is projected back into the feasible set. More specifically, at the kth iteration at node i, the local power allocation vector $\boldsymbol{\eta}_i^k = (\eta_{ij}^k)$ is updated by

$$\boldsymbol{\eta}_i^{k+1} = PA(\boldsymbol{\eta}_i^k) = \left[\boldsymbol{\eta}_i^k + (Q_i^k)^{-1} \cdot \delta\boldsymbol{\eta}_i^k\right]_{Q_i^k}^+ .$$

The power control vector $\boldsymbol{\gamma}^k = (\gamma_i^k)$ is updated by

$$\boldsymbol{\gamma}^{k+1} = PC(\boldsymbol{\gamma}^k) = \left[\boldsymbol{\gamma}^k + (V^k)^{-1} \cdot \delta\boldsymbol{\gamma}^k\right]_{V^k}^+ .$$

Note that PC is decomposable into node-based computations if and only if V^k is diagonal. We show that there exist valid scaling matrices $\{Q_i^k\}$ and V^k such that the update sequences generated by the algorithms PA and PC converge to an optimal power configuration.

For convergence, we choose the scaling matrices to approximate the relevant Hessians such that the objective value is increased by every iteration until the optimum is achieved. This allows the scaled gradient projection algorithms to approximate constrained Newton algorithms, which are known to have fast convergence rates. Furthermore, we show that the scaling matrices are easily computable at each node using limited control messaging [3].

III. Throughput Optimality of Delayed MDB

Since the PA and PC algorithms need a certain number of iterations before reaching a neighborhood of the optimum, the MDB policy must now be implemented with *delayed queue state information*. This issue is studied in a queueing network with Poisson arrivals and exponential service rates by Tassiulas and Ephremides [4]. Here, we analyze the MDB algorithm with delayed queue state information in general multi-hop networks with i.i.d. random arrival processes and general rate regions. We show that the throughput optimality of the MDB policy is preserved for any finite delay in the queue state information if the second moments of the random arrivals are bounded.

We assume a general convex feasible service rate region $\mathcal{C}$. Due to the iterative nature of the distributed MDB algorithms, the actual service rates are always in transience, shifting from the previous optimum to the next optimum. Without loss of generality, assume the convergence time of the MDB algorithms in Section II is the length of a time slot (normalized to 1), i.e., at the beginning of slot $t+1$, the optimal service rate vector for $\boldsymbol{U}[t]$ is achieved. With i.i.d. arrivals, the process $\{(\boldsymbol{U}[t], \boldsymbol{U}[t-1])\}_{t=1}^{\infty}$ forms a Markov chain with state $\boldsymbol{W}[t] \equiv (\boldsymbol{U}[t], \boldsymbol{U}[t-1])$.

We use the Lyapunov function from [4]:

$$V(\boldsymbol{W}[t]) = \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{N}} U_i^k[t]^2 + (U_i^k[t] - U_i^k[t-1])^2$$

and the geometric approach from [3] to conveniently relate the position of the average arrival rate vector $\boldsymbol{a} \in \mathrm{int}(\mathcal{C})$ to the expected Lyapunov drift. We show that if the second moments of arrival rates are finite, we can always find a compact subset $\mathcal{W}_0 = \{\boldsymbol{w} \in \mathbb{R}_+^M \times \mathbb{R}_+^M : V(\boldsymbol{w}) \leq \Omega\}$ of the state space such that the Foster's criterion for recurrent Markov chains is satisfied. Therefore, the delayed MDB policy remains throughput optimal.

We conclude that our algorithms in Section II find the optimal power and rate control required by the MDB policy in a distributed manner, and preserve the throughput optimality of the MDB policy even though the optimal control is implemented with delay relative to the queue state information. Therefore, our work yields a distributed throughput optimal control policy for CDMA wireless networks with random traffic arrivals.

References

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