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LOW COMPLEXITY POST-CODED OFDM COMMUNICATION SYSTEM : DESIGN AND PERFORMANCE ANALYSIS

S. F. A. Shah and A. H. Tewfik

Department of Electrical & Computer Engineering
University of Minnesota, Minneapolis, MN 55455, USA
{sfaisal, tewfik}@umn.edu

ABSTRACT

Orthogonal frequency division multiplexing (OFDM) provides a viable solution to communicate over frequency selective fading channels. However, in the presence of frequency nulls in the channel response, the uncoded OFDM faces serious symbol recovery problems. As an alternative to previously reported error correction techniques in the form of pre-coding for OFDM, we propose the use of post-coding of OFDM symbols in order to achieve frequency diversity. Our proposed novel post-coded OFDM (PC-OFDM) comprises of two steps: 1) upsampling of OFDM symbols and 2) subsequent multiplication of each symbol with unit magnitude complex exponentials. It is important to mention that PC-OFDM introduces redundancy in OFDM symbols while precoded OFDM introduces redundancy in data symbols before performing the IFFT operation. The main advantages of this scheme are reduction in system complexity by having a simple encoder/decoder, smaller size IFFT/FFT (inverse fast Fourier transform/fast Fourier transform) modules, and lower clock rates in the receiver and transmitter leading to lower energy consumption. The proposed system is found to be equally good over Gaussian and fading channels where it achieves the maximum diversity gain of the channel. Simulation results show that PC-OFDM performs better than existing precoded OFDM and Pulse OFDM systems.

1. INTRODUCTION

Orthogonal frequency-division multiplexing (OFDM) offers several advantages like resilience to multipath fading, intersymbol interference, low complexity and others. It is believed to be a promising technique for future broadband wireless communications. It has been adopted in many wireless standards, such as digital audio/video broadcasting (DAB/DVB), the HIPERLAN/2 standard, the IEEE 802.11a standard for wireless local area networks [1].

While OFDM systems convert a multipath fading channel into a series of equivalent flat fading channels, they lack the inherent diversity available in multipath channels. When there are frequency nulls in the channel response, the uncoded OFDM performance decays gravely [2]. It is therefore necessary to introduce an explicit diversity in the transmitted symbols to improve system performance. Different coded OFDM systems have been reported that employ some form of channel coding or precoding [3, 2, 4]. Ding et. al in [4] designed minimum bit error rate precoders for wireline channels. In [2], it was shown that complex field coding is better than Galois field coding as it produces the codes that are better suited for fading channels. While most of the present literature [2, 4] concentrates on precoding of OFDM symbols to provide diversity, we explore the use of post-diversity in this work. We are interested in low complexity systems at the expense of bandwidth. An example of this scenario is an ultra wideband (UWB) wireless personal area networks where the bandwidth constraint is not much of an issue as compared to transceiver complexity and/or power consumption. Different from precoded systems that introduce redundancy in data symbols, we introduce redundancy in OFDM symbols after performing the IFFT operation to save computations and power. In the sequel, we will refer to this system as *post-coded OFDM* or PC-OFDM in short.

Our goal in this paper is to establish a general framework for PC-OFDM systems to show low complexity implementation and analyze their performance. After discussing the necessary details of uncoded OFDM system in Section 2 we design the encoder in Section 3. To facilitate the analysis of the system, we introduce a hypothetical equivalent precoding matrix. We observe a close resemblance between PC-OFDM encoder and *signal space encoders* used to rotate the signal constellation in fading channels [5]. Section 4 discusses a low complexity alternative to implement the decoder by exploiting the polyphase decomposition of the channel. Before analyzing PC-OFDM, we compare the complexity in Section 5 and found that the unique design of PC-OFDM results in a lower complexity coded OFDM system that consumes less energy due to lower clock rate. We perform probability of error analysis in Section 6 and observe that though PC-OFDM is designed primarily for fading channels, it performs as good as uncoded OFDM over Gaussian channels. For uncorrelated Rayleigh fading channels, it achieves the maximum available diversity gain of the channel that is further confirmed through simulations in Section 7. Simulation results show the superiority of PC-OFDM over previously proposed precoded OFDM [2] and Pulse OFDM systems [6].

2. SYSTEM DETAILS AND PROBLEM FORMULATION

Consider an uncoded OFDM system that is implemented by using an N-point IFFT/FFT. The information symbols are mapped to the signal space according to the modulation scheme. The serial stream of modulated data symbols $b(n)$ are grouped in blocks of size N such that the ith block is expressed as $\mathbf{b}(i) := \left[b(iN),\, b(iN+1)\, \cdots b(iN+N-1)\right]$. Let $\mathbf{F}_N$ be the $N \times N$ FFT (fast Fourier transform) matrix with (n,k)th entry as: $[\mathbf{F}_N]_{n,k} = (1/\sqrt{N})\exp\{-j2\pi(n-1)(k-1)/N\}$. Ignoring the block index i, the output of IFFT (inverse fast Fourier transform) block is an OFDM symbol in

This work is partially supported by NSF grant CCR-0313224

the form of $N \times 1$ vector and is given by

$$\mathbf{x} = \mathbf{F}_N^{\mathcal{H}} \mathbf{b}. \quad (1)$$

The insertion of the cyclic-prefix (CP) at the transmitter and CP-removal at the receiver, renders the channel matrix $\mathbf{H}$ an $N \times N$ circulant matrix $\widetilde{\mathbf{H}}$. The received OFDM symbol can therefore be expressed as $\mathbf{r} = \widetilde{\mathbf{H}}\mathbf{x} + \tilde{\eta}$, where $\tilde{\eta}$ represents the $N \times 1$ additive Gaussian noise vector. At the receiver, multiplication with the FFT matrix $\mathbf{F}_N$ diagonalizes the channel matrix $\widetilde{\mathbf{H}}$ such that it contains the N point discrete frequency response of the channel given by [7]:

$$\mathbf{F}_N \widetilde{\mathbf{H}} \mathbf{F}_N^{\mathcal{H}} = \mathbf{H}_D = \operatorname{diag}\left[\mathbf{F}_N \tilde{\mathbf{h}}\right], \quad (2)$$

where $\tilde{\mathbf{h}}$ is $N \times 1$ vector obtained from the concatenation of L_h channel taps, $\{h_l\}_{l=1}^{L_h}$, and $N - L_h$ zeros. Thus, the demodulated OFDM symbols can be simply written as:

$$\mathbf{u} = \mathbf{H}_D \mathbf{b} + \eta. \quad (3)$$

The diagonalization of $\widetilde{\mathbf{H}}$ converts an ISI channel into an ISI free channel and eliminates the need for a complex receiver. Although OFDM systems provide a means to have simple receivers, the system performance deteriorates severely in the presence of channel frequency nulls. This deterioration can be avoided by employing explicit diversity or redundancy (coding) in the OFDM symbols, which is the subject of this paper.

3. THE PROPOSED PC-OFDM SYSTEM

3.1 Encoder Design

Here we consider frequency diversity in OFDM symbols that can be fairly easily achieved by upsampling the output of IFFT. In an apparently similar approach, [8] introduced fractionally sampled OFDM (FS-OFDM) where upsampling is done at the receiver but not at transmitter as in our case. Since upsampling the signal in time domain creates multiple replicas of the signal in frequency domain, this operation is equivalent to repeating the modulated source symbols prior to IFFT. Since repetitive coding cannot harness the full coding advantage in general, we therefore multiply the upsampler output with unit magnitude complex number sequence. Again, equivalent to this operation in time domain will be a linear combination of modulated source symbols. The block diagram of PC-OFDM system is shown in Fig. 1. While PC-OFDM performs these operations in digital domain, a similar scheme for fast hopping UWB-OFDM is proposed in [9]

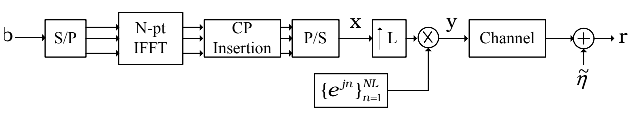

Figure 1: PC-OFDM transmitter block diagram

3.2 Analytical Model of PC-OFDM

Using matrix notation, we can write the transmitted OFDM symbol with post-coding as given by $\mathbf{y} = \mathbf{A}\mathbf{x} = \mathbf{A}\mathbf{F}_N^{\mathcal{H}}\mathbf{b}$ where $\mathbf{A}$ is an $NL \times N$ constructed as

$$\mathbf{A} = \begin{cases} [\mathbf{A}]_{n,k} = e^{jn} & \text{for } (n,k) = (iL-1, i) \text{ and } i = 1, \cdots, N \\ 0 & \text{otherwise.} \end{cases} \quad (4)$$

It is easy to verify that $\mathbf{A}$ is unitary matrix, i.e., $\mathbf{A}^{\mathcal{H}}\mathbf{A} = \mathbf{I}_N$. This property helps us to establish an important result later.

Example 1: Consider the design of PC-OFDM encoder for $N = 2$ and $L = 2$. The encoding matrix $\mathbf{A}$ is given by:

$$\mathbf{A} = \begin{bmatrix} e^{j1} & 0 \\ 0 & 0 \\ 0 & e^{j3} \\ 0 & 0 \end{bmatrix},$$

that jointly accounts for upsampling of $N = 2$ OFDM symbols by factor of $L = 2$ and multiplication with sequence $\{e^{jn}\}_{n=1}^{4}$.

In precoded OFDM, the transmitted OFDM symbols can be written as:

$$\mathbf{y} = \mathbf{F}_{NL}^{\mathcal{H}} \underline{\mathbf{A}} \mathbf{b}. \quad (5)$$

While in case of PC-OFDM, we encode the OFDM symbols after the IFFT as:

$$\mathbf{y} = \mathbf{A}\mathbf{x} = \mathbf{A}\mathbf{F}_N^{\mathcal{H}}\mathbf{b}. \quad (6)$$

In both cases, we consider complex field coding *i.e.,* $\mathbf{A}$ (or $\underline{\mathbf{A}}$) $\in \mathbb{C}^{K \times N}$ with $K \geq N$, instead of Galois field as it provides more degrees of freedom [2]. It is important to note that any postcoding scheme can be made equivalent to a precoding scheme by selecting

$$\underline{\mathbf{A}} = \mathbf{F}_{NL} \mathbf{A} \mathbf{F}_N^{\mathcal{H}}. \quad (7)$$

However, the converse is not true as PC-OFDM corresponds to precoded OFDM with a constrained precoding structure.

Example 2: Using (7), the equivalent precoding matrix for Example 1 is given by:

$$\underline{\mathbf{A}} = \frac{1}{2\sqrt{2}} \begin{bmatrix} e^{j1} + e^{j3} & e^{j1} - e^{j3} \\ e^{j1} - e^{j3} & e^{j1} + e^{j3} \\ e^{j1} + e^{j3} & e^{j1} - e^{j3} \\ e^{j1} - e^{j3} & e^{j1} + e^{j3} \end{bmatrix}, \quad (8)$$

This appears similar to redundant diversity codes that are generally used to rotate the signal constellation to achieve better performance in fading channels [5]. Thus in a sense, the PC-OFDM system does perform signal constellation rotation through the multiplication with unit amplitude phasors and improves the system performance over fading channels.

4. LOW COMPLEXITY IMPLEMENTATION MODEL OF PC-OFDM DECODER

While the analytical model reveals some important relationship between postcoded and precoded OFDM systems, the low complexity advantage of PC-OFDM will be clearer if we apply the multirate signal processing concepts to the system. To develop this model, assume that the ith OFDM symbol can be expressed as: $\mathbf{x}(i) := [x(i,0), \cdots, x(i, N-1)]$, where $x(i,n)$ is a component of the ith OFDM symbol along nth subcarrier. In a similar manner, if $\mathbf{y}(i) :=$

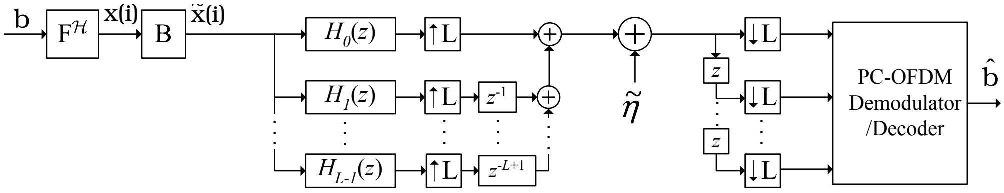

Figure 2: Equivalent model of PC-OFDM system with polyphase decomposition of channel

$[y(i,0),\cdots,y(i,NL-1)]$ is the output of PC-OFDM encoder then,

$$y(i,p) = \begin{cases} e^{jp}\,x(i,p-1) & \text{for } p = 1, L, \cdots, NL \\ 0 & \text{otherwise.} \end{cases}$$

For analytical simplicity, let us change the order of upsampling and multiplication in Fig. 1 such that the input to the upsampler can be expressed as $\tilde{\mathbf{x}}(i) = \mathbf{B}\mathbf{x}(i)$ where $\mathbf{B}$ is an $N \times N$ diagonal matrix constructed as: $\mathbf{B} = \mathrm{diag}\left[\{e^{jp}\}\right]$ for $p = 1, L, \cdots, NL$. This simplification results in a cascade of upsampling and filtering (transmission through the channel) operation that can be equivalently expressed as a polyphase decomposition of the channel. Writing the z-transform of the channel with coefficients $\{h(l)\}_{l=0}^{L_h-1}$ in the form

$$H(z) = \sum_{p=0}^{L-1} z^{-p} H_p(z^L),$$

where $H_p(z^L) := \sum_{l=0}^{L_h-1} h(lL+p)z^{-lL}$ represents the upsampled polyphase decomposition of $H(z)$. Thus, the equivalent of cascade of upsampling and multiplication operation in Fig. 1 can be expressed as an L branch multirate system as shown in Fig 2 where each branch contains a polyphase decomposition $H_p(z) = \sum_{l=0}^{L_h-1} h(lL+p)z^{-l}$ of the channel followed by an upsampling operation of factor L. Note that this decomposition also shows that PC-OFDM effectively implements a frequency domain coding scheme with very low complexity.

The polyphase decomposition of channel leads us to design a dual system with downsampling and delay operations at the receiver such that the L branches can be separated at the receiver as shown in Fig. 3. This is possible due to the factorization property of the FFT matrix. For example, a $2N \times 2N$ FFT matrix ($\mathbf{F}_{2N}$) can be factored as follows [10]:

$$\mathbf{F}_{2N} = \underbrace{\begin{bmatrix} \mathbf{I}_N & \mathbf{W}_N \\ \mathbf{I}_N & -\mathbf{W}_N \end{bmatrix}}_{\mathbf{D}_N} \begin{bmatrix} \mathbf{F}_N & \mathbf{O}_N \\ \mathbf{O}_N & \mathbf{F}_N \end{bmatrix} \begin{bmatrix} \text{even-odd} \\ \text{permutation} \end{bmatrix}, \quad (9)$$

where $\mathbf{I}_N$ is the $N \times N$ identity matrix, $\mathbf{W}_N = \mathrm{diag}[1\ e^{-j2\pi/N}\ \cdots\ e^{-j2\pi(N-1)/N}]$ and $\mathbf{O}_N$ is the null matrix of order $N \times N$. From (9), it is clear that at the receiver we need a downsampler to separate different phases and then we can use N point FFT followed by multiplication with a sparse matrix of the form $\mathbf{D}_N$.

For the sake of mathematical convenience, we can rewrite the channel input $\tilde{\mathbf{x}}(i) = \mathbf{B}\mathbf{F}_N^{\mathcal{H}}\mathbf{b}$ as $\tilde{\mathbf{x}}(i) = \mathbf{F}_N^{\mathcal{H}}\underline{\mathbf{B}}\mathbf{b}$, where $\underline{\mathbf{B}}$ is an equivalent precoded matrix that can be obtained as $\underline{\mathbf{B}} = \mathbf{F}_N\mathbf{B}\mathbf{F}_N^{\mathcal{H}}$. The use of cyclic prefix will render the channel matrix $\mathbf{H}_p$ in the pth branch as circulant that in turn becomes a diagonal matrix, $\mathbf{H}_{pD}$, after pre- and post-multiplication with DFT matrix. Thus the demodulated OFDM symbol at the pth branch of the receiver is given by:

$$\mathbf{u}_p = \mathbf{H}_{pD}\mathbf{b} + \eta. \quad (10)$$

If maximum likelihood (ML) detector is used at the receiver, then it combines the output from all the branches and searches for the most likelihood symbol according to the following minimization:

$$\hat{\mathbf{b}} = \arg\min_{\mathbf{b}_i} \sum_{p=0}^{L-1} ||\mathbf{u}_p - \mathbf{H}_{pD}\underline{\mathbf{B}}\mathbf{b}_i||. \quad (11)$$

5. COMPLEXITY AND POWER COMPARISON WITH PRECODED OFDM SYSTEMS

The proposed PC-OFDM system is capable of lowering the implementation cost of coded OFDM system. For instance, a PC-OFDM transmitter with N source symbols requires an N-point IFFT module with computational complexity of $\mathcal{O}(N\log N)$ per N data symbols. In contrast, a redundant precoded OFDM transmitter [2] with $NL \times N$ (where $L \in \mathbb{R}$ and $L \geq 1$) encoding has a computational complexity of $\mathcal{O}(NL\log NL)$. Similarly, the polyphase decomposition of channel in PC-OFDM will allow us to use N-point FFTs in all the L branches that results in total complexity of $\mathcal{O}(NL\log N)$ at the receiver.

In addition to the savings in FFT modules, the unique encoding scheme of PC-OFDM is a low cost operation and requires only $\mathcal{O}(N)$ complex multiplications as compared to $\mathcal{O}(N^2L)$ complex multiplications/additions in precoded OFDM. Table 1 compares the computation cost of FFT/IFFT modules and encoding/decoding operations for precoded and post-coded OFDM systems. The reduced complexity of PC-OFDM system makes it suitable for wireless personal area networks.

It is important to mention that the IFFT/FFT operations in PC-OFDM are performed at information symbol data rate, however, in precoded OFDM these operations are performed after encoding and at higher sampling rate. Since

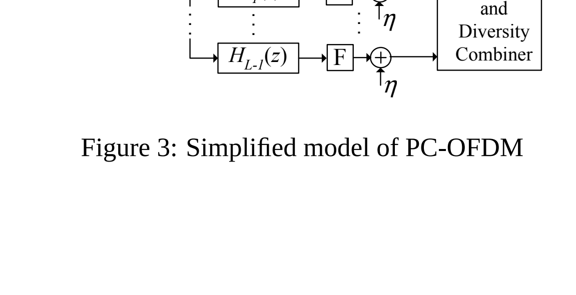

Figure 3: Simplified model of PC-OFDM

power consumption of these DSP modules is proportional to clock frequency, PC-OFDM saves power by computing the IFFT/FFT operations at lower rate. The comparison of required clock rate for different modules in precoded OFDM and PC-OFDM systems is shown in Table 2.

	Pre-coded OFDM	PC-OFDM
IFFT	$\mathcal{O}(NL\log NL)$	$\mathcal{O}(N\log N)$
FFT	$\mathcal{O}(NL\log NL)$	$\mathcal{O}(NL\log N)$
Encoding	$\mathcal{O}(N^2L)$	$\mathcal{O}(N)$
Decoding	$\mathcal{O}(N^2L)$	$\mathcal{O}(NL\log N)$

Table 1: Comparison of computation cost of different operations in precoded and post-coded OFDM systems

	IFFT	Transmitter DAC	FFT
Pre-coded OFDM	Lf_c	Lf_c	Lf_c
PC-OFDM	f_c	Lf_c	f_c

Table 2: Comparison of required clock rate for different modules ($f_c =$ clock rate in Hz)

6. PROBABILITY OF ERROR ANALYSIS

It has been shown in the recent research that the criteria commonly used to design codes for additive white Gaussian noise (AWGN) channels have to be adjusted when dealing with a fading channel (see [11] and references therein). As we shall see soon, the performance of a code over fading channels depends on the minimum Hamming distance and not on the Euclidean distance between codewords. In this paper, our main goal is to design the codes for fading channels. Nevertheless, it is important to see the system performance over AWGN channels. Therefore, we consider the probability of error for AWGN and Rayleigh fading channels separately.

6.1 AWGN Channels

It is well known that for AWGN channels the minimum Euclidean distance of the codewords determines the probability of error [11]. Considering ML detection, the probability of error (P_e) can be expressed as

$$P_e = \frac{1}{2}\text{erfc}\left(\frac{d_{\min}}{2\sqrt{(N_o)}}\right), \tag{12}$$

where erfc is the Gaussian tail function defined as $\text{erfc}(x) := 1/(\sqrt{2\pi})\int_x^\infty e^{-t^2/2}dt$ and $d_{\min}$ is the minimum Euclidean distance. If $\mathcal{A}$ is the set of codewords, then it is defined as:

$$d_{\min} = \min_{\mathbf{x}\neq\mathbf{x}'} ||\mathbf{x}-\mathbf{x}'|| = \min_{\mathbf{b}\neq\mathbf{b}'} ||\underline{\mathbf{A}}(\mathbf{b}-\mathbf{b}')||. \tag{13}$$

Simplifying the square of the norm in (13), we obtain $||\underline{\mathbf{A}}(\mathbf{b}-\mathbf{b}')||^2 = (\mathbf{b}-\mathbf{b}')^{\mathcal{H}}\underline{\mathbf{A}}^{\mathcal{H}}\underline{\mathbf{A}}(\mathbf{b}-\mathbf{b}')$. Thus, $d_{\min}$ which is the minimum Euclidean distance between the coded symbols can be different from the minimum Euclidean distance between the uncoded symbols. However, if the coding matrix forms a unitary transform pair, i.e., $\underline{\mathbf{A}}^{\mathcal{H}}\underline{\mathbf{A}} = \mathbf{I}_N$, the minimum Euclidean remains unchanged [12]. In this situation, the codes do not perform poorly in AWGN channels. The PC-OFDM system encoder follows this important property as stated in the following proposition:

Proposition 1. *In a PC-OFDM system, the equivalent precoding matrix $\underline{\mathbf{A}}$ is indeed a unitary matrix.*

Proof: From (7), we have $\underline{\mathbf{A}} = \mathbf{F}_{NL}\mathbf{A}\mathbf{F}_N^{\mathcal{H}}$. The result follows by evaluating $\underline{\mathbf{A}}^{\mathcal{H}}\underline{\mathbf{A}}$ and using the fact that $\mathbf{F}_N^{\mathcal{H}}\mathbf{F}_N = \mathbf{I}_N$ for all N and $\mathbf{A}^{\mathcal{H}}\mathbf{A} = \mathbf{I}_N$.

6.2 For Uncorrelated Fading Channels

To assess the performance of PC-OFDM over uncorrelated fading channels, we adopt the average pairwise error probability (PEP) technique that has been derived in similar context in [2, 13]. By definition, the PEP is the the probability of erroneously detecting $\mathbf{b}'$ when $\mathbf{b}$ was transmitted. In order to find the PEP (see [2] for details), we need to define a matrix $\mathbf{A}_e := (\mathbf{D}_e\mathbf{V})^{\mathcal{H}}\mathbf{D}_e\mathbf{V}$ where $\mathbf{V}$ is truncated FFT matrix with $[\mathbf{V}]_{(k,l)} = e^{-j2\pi kl/NL}$ and $\mathbf{D}_e = \underline{\mathbf{A}}(\mathbf{b}-\mathbf{b}')$. Now, for Rayleigh fading channels with uncorrelated paths, the PEP is given by:

$$\Pr(\mathbf{b}\to\mathbf{b}') \leq \left(\frac{1}{4\mathrm{N_o}}\right)^{-\mathrm{G_d}}\left(\prod_{\mathrm{l}=1}^{\mathrm{G_d}}\alpha_{\mathrm{l}}\lambda_{\mathrm{e,l}}\right)^{-1}, \tag{14}$$

where $N_o/2$ is the power spectral density of additive white Gaussian noise, $\alpha_l = \mathrm{E}[|h(l)|^2]$ is the channel correlation and λ_e are the eigenvalues of $\mathbf{A}_e$. It can be seen from (14) that the PEP depends on the following two factors:

- *Diversity gain (G_d):* Roughly speaking, the diversity gain represents the slope of the PEP curve especially at high SNR. It is related to the rank of $\mathbf{A}_e$ [13].
- *Coding gain (G_c):* The coding gain controls the shift in the PEP curve and depends on the product of eigenvalues $\{\lambda_{e,l}\}_{l=1}^{L_h}$ of $\mathbf{A}_e$ such that $G_c = \left(\prod_{l=1}^{G_d}\lambda_{e,l}\right)^{1/G_d}$

It was shown in [2] that the rank of $\mathbf{A}_e$ is related to the minimum Hamming distance of the codewords. If $\underline{\mathcal{A}}$ is the set of codewords such that $\mathbf{Ab}, \mathbf{Ab}' \in \underline{\mathcal{A}}$ then the Hamming distance $\delta(\mathbf{Ab}, \mathbf{Ab}')$ between these codewords is the number of non-zero entries in $\mathbf{A}(\mathbf{b}-\mathbf{b}')$. The minimum Hamming distance of the codeset $\underline{\mathcal{A}}$ is defined as :

$$\delta_{\min}(\underline{\mathcal{A}}) = \min\{\delta(\mathbf{Ab},\mathbf{Ab}')|\mathbf{Ab},\mathbf{Ab}' \in \underline{\mathcal{A}}\}. \tag{15}$$

As stated in [2], the diversity order G_d is upper bounded by the number of paths L_h in the channel, i.e., $G_d = \min\{\delta_{\min}(\underline{\mathcal{A}}), L_h\}$. The proposed encoder follows the following proposition:

Proposition 2. *The PC-OFDM system achieves the maximum available diversity gain.*

Proof: Assume $\underline{\mathbf{A}}$ is $NL\times N$. The particular structure of $\underline{\mathbf{A}}$ guarantees that all of its elements are non-zero.In addition, all the elements in a row of $\underline{\mathbf{A}}$ are different. Thus, each block of N data symbols is mapped to a unique point in NL dimensional space such that any two points differ in all the components. This results in $\delta_{\min} = NL$ which is generally larger than the channel length and thus $G_d = L_h$.

The second parameter that controls the shift in the PEP curve is the coding gain. To see this dependence, we obtain a series of PEP curves from (14) for two different values of G_d and three different values of G_c as shown in Fig. 4. It is obvious from Fig. 4 that the increase in diversity gain from $G_d = 2$ to $G_d = 4$ causes a drastic improvement in system performance by reducing the PEP while the increase in coding gain from $G_c = 10$ to $G_c = 25$ does not improve the system performance significantly.

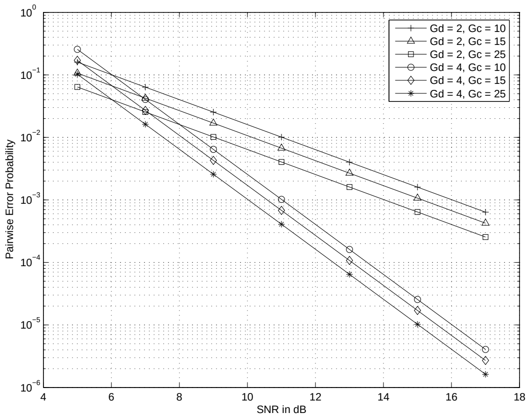

Figure 4: Upper bound on the PEP

7. SIMULATION RESULTS

We perform simulations to compare the bit error rate (BER) of different coded OFDM systems as shown in Fig. 5. The information symbols are BPSK modulated to yield $\mathcal{B} = \{+1, -1\}$ and transformed to OFDM symbols with $N = 4$. For all the coded OFDM systems considered in Fig. 5, we use $L = 2$ that results in code rate of $1/2$. The simulations are performed over Rayleigh fading channel with five taps that are generated according to the Jakes model. To compare with precoded OFDM systems, we employ the real (referred as precoded OFDM-a) and complex (referred as precoded OFDM-b) precoders proposed in [2]. The BER results of such precoders are obtained using ML detection and the results are shown in Fig. 5. As seen from Fig. 5, the BER performance depends on the choice of the precoder. We also obtain the BER performance of pulsed-OFDM [6] and the results are shown in Fig. 5. The slope of the curve shows that pulsed-OFDM could not achieve the full diversity order available in the system. The BER curve of PC-OFDM using ML detector is also shown in Fig. 5 verifies the superiority of PC-OFDM in terms of BER performance.

8. CONCLUSIONS

We proposed a novel coded OFDM system in the form of PC-OFDM that enjoys the benefits of low complexity and power consumption, and retains the benefits of other similar precoded schemes. Due to the unitary nature of encoding matrix, PC-OFDM performs equally good over Gaussian channels. For fading channels, it achieves the maximum available diversity gain of the channel while operating at a significantly low complexity.

REFERENCES

[1] T. Jiang and G. Zhu, "Complement block coding for reduction in peak-to-average power ratio of OFDM signals," *IEEE Commun. Mag.*, vol. 43, no. 9, pp. S17 – S22, Sept. 2005.

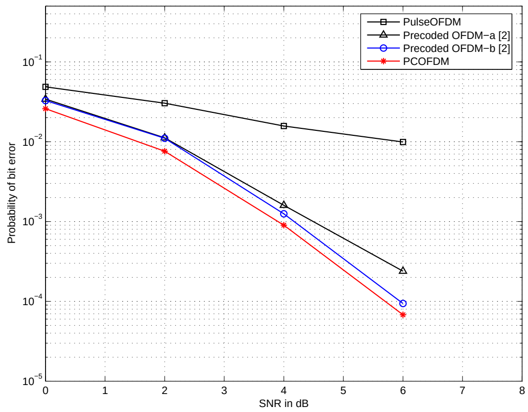

Figure 5: BER of different coded OFDM systems

[2] Z. Wang and G. Giannakis, "Complex-field coding for OFDM over fading wireless channels," *IEEE Trans. on Infor. Theory*, vol. 49, no. 3, pp. 707 – 720, March 2003.

[3] W. Zou and Y. Wu, "COFDM: an overview," *IEEE Trans. Broadcast*, vol. 41, pp. 1–8, Mar. 1995.

[4] Y. Ding, T. N. Davidson, Z. Luo, and K. M. Wong, "Minimum BER block precoders for zero-forcing equalization," *IEEE Trans. Signal Process.*, vol. 51, no. 9, pp. 2410–2423, September 2003.

[5] J. Boutros and E. Viterbo, "Signal space diversity: a power- and bandwidth-efficient diversity technique for the Rayleigh fading channel," *IEEE Trans. on Infor. Theory*, vol. 44, no. 4, pp. 1453 – 1467, July 1998.

[6] E. Saberinia, J. Tang, A. Tewfik, and K. Parhi, "Pulsed OFDM modulation for ultra wideband communications," in *Proc. of IEEE ISCAS'04*, May 2004, pp. V369 – V3972.

[7] Z. Wang and G. B. Giannakis, "Wireless multicarrier communications," *IEEE Commun. Mag.*, vol. 17, no. 3, pp. 29–48, December 2000.

[8] C. Tepedelenlioglu and R. Challagulla, "Low-complexity multipath diversity through fractional sampling in OFDM," *IEEE Trans. Signal Process.*, vol. 52, pp. 3104 – 3116, Nov 2003.

[9] A. H. Tewfik and E. Saberinia, "High bit rate ultra-wideband OFDM," in *Proc. of IEEE GLOBECOM'02*, Nov. 2002, pp. 2260–2264.

[10] G. Strang, *Introduction to Linear Algebra*, 3rd ed. Wellesley Cambridge Press, 2003.

[11] E. Biglieri, J. Proakis, and S. Shamai, "Fading channels: information-theoretic and communications aspects," *IEEE Trans. on Infor. Theory*, vol. 44, no. 6, pp. 2619 – 2692, Oct. 1998.

[12] X. Ma and G. Giannakis, "Complex field coded MIMO systems: performance, rate, and trade-offs," *Wireless Communications and Mobile Computing*, vol. 2, pp. 693–717, March 2002.

[13] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: performance criterion and code construction," *IEEE Trans. on Infor. Theory*, vol. 44, pp. 744–765, March 1998.