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LOW COMPLEXITY ZERO-PADDING ZERO-JAMMING DMT SYSTEMS

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ABSTRACT

Discrete multitone (DMT) systems have been widely adopted in broadband communications. When the transmission channel is frequency selective, there will be interblock interference (IBI). IBI can be avoided by *zero-padding* (ZP) [1]. Another solution is to allow IBI during transmission, and at the receiver the samples that contain IBI are removed by *zero-jamming* (ZJ) [2]. The ZP DMT system employs the ZP technique whereas the CP DMT system where a cyclic prefix is added at the transmitter uses the ZJ technique. In both the ZP DMT and CP DMT systems, the number of redundant samples are larger than or equal to the channel order. In this paper, we propose a ZP-ZJ DMT system. By combining ZP and ZJ techniques, we are able to reduce the number of redundant samples needed for IBI elimination by as much as one half. The transmitter of the ZP-ZJ DMT system involves only one IFFT operation and its receiver can be implemented efficiently using a small number of FFT/IFFT operations. Simulation shows that the bandwidth efficient ZP-ZJ DMT system can sometimes outperform the CP DMT system.

1. INTRODUCTION

The block transmission scheme has found many applications in broadband communications. For frequency selective channels, there will be interblock interference (IBI) between successive transmission blocks. One can avoid IBI by adding a sufficient number of zeros at the end of each transmission block and such a technique is called zero-padding (ZP) [1][2]. Another solution is to allow IBI during transmission, and at the receiver the samples that contain IBI are removed by zero-jamming (ZJ) [2]. The ZP DMT system employs the ZP technique whereas the CP DMT system where a cyclic prefix is added at the transmitter uses the ZJ technique.

Suppose that the channel order is L. Then for both the ZP and ZJ systems, the minimum number of redundant samples needed for IBI elimination is L. As the redundant samples do not carry any information, this will reduce the bandwidth efficiency. Thus there have been studies on transmission schemes with a reduced number of redundant samples [3] [4] [5]. In many DMT applications, some subchannels or tones are not used for data transmission. These unused tones are known as null tones or virtual subcarriers. In [4] [5], the authors show that by exploiting these null tones, zero-forcing (ZF) equalization can be obtained even when no redundant sample is added at the transmitter. However, in order to achieve zero-forcing transmission, there should be at least L null tones in the system. In [3], the authors proposed a block based transceiver with minimum redundancy. By jointly designing the transmitter and receiver, the required number of redundant samples can be reduced to $\lceil L/2 \rceil$, where $\lceil x \rceil$ denotes the smallest integer $\geq x$. However, these minimum redundancy systems have a high implementation cost and the design of transmitter requires the knowledge of channel impulse response.

In this paper, we will combine the ZP and ZJ techniques to reduce the redundancy of DMT systems. For the proposed ZP-ZJ DMT system, we will show that ZF equalization can be achieved if the number of redundant samples is at least $\lceil L/2 \rceil$. Moreover, we will also study the least-squares and MMSE receivers for ZP-ZJ system. A direct implementation of these receivers in general has a high complexity. Thus efficient implementation based on FFT operations will be derived. We also carry out the numerical simulation of the bit error rate (BER) performance of the ZP-ZJ DMT systems. Simulation results show that the ZP-ZJ DMT system with a smaller number of redundant samples can outperform the CP DMT system.

Notations: Boldfaced lower and upper case letters represent vectors and matrices respectively. The notation $\mathbf{A}^\dagger$ denotes transpose-conjugate of $\mathbf{A}$ and $\mathbf{A}^T$ denotes the transpose of $\mathbf{A}$. $\mathbf{W}$ is the $M \times M$ normalized DFT matrix, whose (k,l)th entry is $[\mathbf{W}]_{k,l} = \frac{1}{\sqrt{M}} e^{\frac{-j2\pi kl}{M}}$.

2. SYSTEM DESCRIPTION

Let $\mathbf{x}(n)$ be the $M \times 1$ vector containing M input modulation symbols of the transmitter. Without much loss of generality, it is assumed that $\mathbf{x}(n)$ is zero-mean and uncorrelated with symbol energy equal to E_s. That is,

$$\mathscr{E}\{\mathbf{x}(n)\} = \mathbf{0}, \;\; \mathscr{E}\{\mathbf{x}(n)\mathbf{x}^\dagger(n-l)\} = E_s \mathbf{I} \delta(l). \qquad (1)$$

Let the transmitting matrix $\mathbf{G}$ be an $N \times M$ matrix. For every $M \times 1$ input vector, we transmit an $N \times 1$ vector given by

$$\mathbf{u}(n) = \mathbf{G}\mathbf{x}(n). \qquad (2)$$

In this paper, we assume that $N \geq M$ so that zero-forcing equalization is possible. Let $K = N - M$. Then K is the number of redundant symbols added at the transmitter. We assume that the channel does not change during the transmission of one input block and it can be modeled as an FIR LTI system with order L. The channel transfer function is

$$C(z) = c_0 + c_1 z^{-1} + c_2 z^{-2} + \cdots + c_L z^{-L}. \qquad (3)$$

In this paper, we will consider the case of even L for notational simplicity. One can easily extend the results to the

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case of odd L. In most applications, M is usually a large integer and $N \geq L$. Under this condition, the received vector at the receiver is given by

$$\mathbf{r}(n) = \mathbf{C}_0\mathbf{u}(n) + \mathbf{C}_1\mathbf{u}(n-1) + \mathbf{q}(n), \tag{4}$$

where $\mathbf{q}(n)$ is a noise vector. The matrix $\mathbf{C}_0$ is an $N \times N$ lower triangular Toeplitz matrix with the first column $[\ c_0 \ \ c_1 \ \ \cdots \ \ c_L \ \ 0 \ \ \cdots \ \ 0\ ]^T$ and $\mathbf{C}_1$ is an $N \times N$ upper triangular Toeplitz matrix with first row $[\ 0 \ \ \cdots \ \ 0 \ \ c_L \ \ c_{L-1} \ \ \cdots \ \ c_1\ ]$. When $\mathbf{C}_1\mathbf{G} \neq \mathbf{0}$, there is IBI in the received vector $\mathbf{r}(n)$. Let the $M \times N$ receiving matrix at the receiver be $\mathbf{S}$. Then the output of the receiver is given by

$$\hat{\mathbf{x}}(n) = \underbrace{\mathbf{S}\mathbf{C}_0\mathbf{G}}_{\mathbf{T}_0}\mathbf{x}(n) + \underbrace{\mathbf{S}\mathbf{C}_1\mathbf{G}}_{\mathbf{T}_1}\mathbf{x}(n-1) + \mathbf{S}\mathbf{q}(n). \tag{5}$$

To eliminate the IBI, the transmitting and receiving matrices should satisfy $\mathbf{T}_1 = \mathbf{S}\mathbf{C}_1\mathbf{G} = \mathbf{0}$. Two well-known designs of $\mathbf{G}$ and $\mathbf{S}$ for IBI elimination are briefly reviewed below.

2.1 ZP System

The first solution is the ZP system with the redundancy $K = L$. In this case, the $(M+L) \times M$ transmitting matrix has the following form

$$\mathbf{G} = \begin{bmatrix} \mathbf{G}' \\ \mathbf{0} \end{bmatrix}, \tag{6}$$

where $\mathbf{G}'$ is an $M \times M$ matrix. One can verify that $\mathbf{C}_1\mathbf{G} = \mathbf{0}$; there is no IBI. In the ZP DMT system, $\mathbf{G}'$ is chosen as the IDFT matrix $\mathbf{W}^\dagger$, and the receiving matrix is

$$\mathbf{S} = \mathbf{E}^{-1}\mathbf{W}\begin{bmatrix} \mathbf{I}_M & \begin{matrix} \mathbf{I}_L \\ \mathbf{0} \end{matrix} \end{bmatrix}, \tag{7}$$

where $\mathbf{E}$ is a diagonal matrix consisting of the M-point DFT coefficients of the channel impulse response c_k [1]. With these choices of transmitting and receiving matrices, one can verify that $\mathbf{T}_0 = \mathbf{S}\mathbf{C}_0\mathbf{G} = \mathbf{I}$ and the ZP DMT system achieves zero-forcing. Note that the receiver $\mathbf{S}$ in (8) has a low implementation cost as $\mathbf{E}^{-1}$ is a diagonal matrix and the DFT matrix $\mathbf{W}$ can be efficiently implemented using FFT.

2.2 ZJ System

The second solution is the ZJ system with redundancy $K = L$. In this case, the $M \times (M+L)$ receiving matrix $\mathbf{S}$ has the following form

$$\mathbf{S} = [\ \mathbf{0} \ \ \mathbf{S}'\ ], \tag{8}$$

where $\mathbf{S}'$ is an $M \times M$ matrix. For each received block $\mathbf{r}(n)$, the first L samples are discarded by the receiver. One can verify that $\mathbf{S}\mathbf{C}_1 = \mathbf{0}$; IBI is eliminated. In the special case of CP DMT system, the transmitting matrix is chosen as

$$\mathbf{G} = \begin{bmatrix} \mathbf{W}_L^\dagger \\ \mathbf{W}^\dagger \end{bmatrix}, \tag{9}$$

where $\mathbf{W}_L^\dagger$ consists of the last L rows of $\mathbf{W}^\dagger$, and the matrix $\mathbf{S}'$ is chosen as

$$\mathbf{S}' = \mathbf{E}^{-1}\mathbf{W}. \tag{10}$$

Using (9) and (10), we can show that $\mathbf{T}_0 = \mathbf{S}\mathbf{C}_0\mathbf{G} = \mathbf{I}$ and the CP DMT system is zero-forcing. The receiver $\mathbf{S}'$ in (10) also has a low implementation cost.

3. ZP-ZJ DMT SYSTEM

Both the ZP and ZJ systems in Section 2 add L redundant samples at the transmitter. In this section, we propose a DMT system that uses both the ZP and ZJ techniques. Let the number of redundant samples added at the transmitter be K and $K \leq L$. The transmitting matrix $\mathbf{G}$ is an $(M+K) \times M$ matrix given by

$$\mathbf{G} = \begin{bmatrix} \mathbf{W}^\dagger \\ \mathbf{0}_{K\times M} \end{bmatrix}. \tag{11}$$

That is, K zeros are added to each transmission block. When $K < L$, the first $(L-K)$ rows of $\mathbf{C}_1\mathbf{G}$ are nonzero, there is IBI due to insufficient number of zeros between successive blocks. One can remove the residual IBI by discarding the first $(L-K)$ samples of each received block. This can be attained by selecting an $M \times (M+K)$ receiving matrix of the form

$$\mathbf{S} = \begin{bmatrix} \mathbf{0}_{M\times(L-K)} & \mathbf{S}' \end{bmatrix}. \tag{12}$$

Note that $\mathbf{S}'$ is an $M \times (M+2K-L)$ matrix. By substituting (11) and (12) into (5), it is easy to see that $\mathbf{T}_1 = \mathbf{0}$. IBI is eliminated and the output of the receiving matrix is given by

$$\mathbf{y}(n) = \mathbf{S}'\mathbf{B}\mathbf{W}^\dagger\mathbf{x}(n) + \mathbf{S}'\mathbf{q}(n), \tag{13}$$

where $\mathbf{B}$ is an $(M+2K-L) \times M$ Toeplitz matrix given by

$$\mathbf{B} = \begin{bmatrix} c_{L-K} & \cdots & c_0 & 0 & \cdots & \cdots & 0 \\ \vdots & \ddots & & & & & \vdots \\ c_K & \ddots & & \ddots & & & \vdots \\ \vdots & & \ddots & & & & c_0 \\ c_L & & & \ddots & & & \vdots \\ 0 & \ddots & & & \ddots & & c_{L-K} \\ \vdots & \ddots & \ddots & & & & \vdots \\ 0 & \cdots & \cdots & 0 & c_L & \cdots & c_K \end{bmatrix}. \tag{14}$$

For a zero-forcing solution, we need $\mathbf{S}'\mathbf{B}\mathbf{W}^\dagger = \mathbf{I}_M$, which implies that $(M+2K-L) \geq M$. Thus the number of redundant samples should satisfy $K \geq L/2$. When $K = L/2$, it is called a minimum redundancy ZP-ZJ DMT system. In the following, we will find the zero-forcing and MMSE (minimum mean square error) receivers for the ZP-ZJ DMT system.

Zero-Forcing Receivers: When $K = L/2$, $\mathbf{B}$ is an $M \times M$ matrix, and its inverse is unique (if it exists). From (13), we can obtain the zero-forcing receiver as

$$\mathbf{S}'_{zf} = \mathbf{W}\mathbf{B}^{-1}. \tag{15}$$

When $K > L/2$, the $(M+2K-L) \times M$ matrix $\mathbf{B}$ is a tall matrix and its left inverse (if it exists) is not unique. One of the solutions is the least-squares receiver given by:

$$\mathbf{S}'_{ls} = \mathbf{W}(\mathbf{B}^\dagger\mathbf{B})^{-1}\mathbf{B}^\dagger. \tag{16}$$

It is known that the least-squares receiver is the least-norm solution. Thus when $\mathbf{q}(n)$ is AWGN, the least-squares receiver $\mathbf{S}'_{ls}$ also minimizes the output noise variance.

MMSE Receivers: One can also solve for the MMSE receiver from (13). Assume that $\mathbf{q}(n)$ is AWGN with variance N_0. Then using the orthogonality principle, it is straightforward to verify that the MMSE receiver is given by

$$\mathbf{S}'_{mmse} = \mathbf{W}\mathbf{B}^\dagger(\rho\mathbf{I}_N + \mathbf{B}\mathbf{B}^\dagger)^{-1}, \quad (17)$$

where $\rho = N_0/E_s$ and $N = M+K$. Using the singular value decomposition of $\mathbf{B}$, $\mathbf{S}'_{mmse}$ can be rewritten as

$$\mathbf{S}'_{mmse} = \mathbf{W}(\rho\mathbf{I}_M + \mathbf{B}^\dagger\mathbf{B})^{-1}\mathbf{B}^\dagger, \quad (18)$$

Comparing (16) and (18), we see that except for the term $\rho\mathbf{I}_M$, both the least-squares receiver $\mathbf{S}'_{ls}$ and the MMSE receiver $\mathbf{S}'_{mmse}$ have the same expressions. When the system is noise free, that is $\rho = 0$, the MMSE receiver $\mathbf{S}'_{mmse}$ reduces to the least-squares receiver.

Unlike the ZP and CP DMT systems in Section 2, the receiving matrices of the ZP-ZJ system are no longer a product of a diagonal matrix and a DFT matrix. Their implementation cost is in general in the order of M^2.

4. LOW COMPLEXITY RECEIVERS

Though the ZP-ZJ DMT system can reduce the required redundancy for IBI elimination by as much as one half, its receiver has a high implementation cost. Below we will derive low cost implementations of the zero-forcing, least-squares and MMSE receivers for the ZP-ZJ DMT system. The basic idea is to exploit the structure in the matrix $\mathbf{B}$ given in (14). Recall that for a ZP-ZJ DMT system with redundancy $K \geq L/2$, the $(M+2K-L)\times M$ matrix $\mathbf{B}$ is a tall matrix. We can partition $\mathbf{B}$ as

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \end{bmatrix}, \quad (19)$$

where $\mathbf{B}_1$ is an $M\times M$ matrix and $\mathbf{B}_2$ is an $(2K-L)\times M$ matrix. We can rewrite $\mathbf{B}_1$ as

$$\mathbf{B}_1 = \mathbf{C}_{cir} - \mathbf{\Delta}, \quad (20)$$

where $\mathbf{C}_{cir}$ is a right circulant matrix whose first row is

$$\begin{bmatrix} c_{L-K} & \cdots & c_L & 0 & \cdots & 0 & c_0 & \cdots & c_{L-K-1} \end{bmatrix}.$$

The $M\times M$ matrix $\mathbf{\Delta}$ is a sparse matrix of the form:

$$\mathbf{\Delta} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{U}_1 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{L}_1 & \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad (21)$$

where $\mathbf{U}_1$ is an $K\times K$ upper triangular Toeplitz matrix with first row $\begin{bmatrix} c_L & \cdots & c_{L-K+2} & c_{L-K+1} \end{bmatrix}$ and $\mathbf{L}_1$ is an $(L-K)\times(L-K)$ lower triangular Toeplitz matrix with the first column $\begin{bmatrix} c_0 & c_1 & \cdots & c_{L-K-1} \end{bmatrix}^T$. It is known [1] that a circulant matrix satisfies

$$\mathbf{W}\mathbf{C}_{cir}\mathbf{W}^\dagger = \mathbf{D}, \quad (22)$$

where $\mathbf{D}$ is a diagonal matrix. The M diagonal entries are the M-point DFT coefficients of the first column of $\mathbf{C}_{cir}$. Using this diagonalization property and the decomposition of $\mathbf{B}_1$ in (20), we are now ready to derive the low cost implementation of the receivers. We will first consider the zero-forcing receiver $\mathbf{S}'_{zf}$ for the minimum redundancy case of $K = L/2$ and then the MMSE receiver $\mathbf{S}'_{mmse}$. The derivation for the least-squares receiver $\mathbf{S}'_{ls}$ can be obtained by setting $\rho = 0$ in the expression of $\mathbf{S}'_{mmse}$.

4.1 Zero-forcing Receiver $\mathbf{S}'_{zf}$

When $K = L/2$, the matrix $\mathbf{B} = \mathbf{B}_1$ is an $M\times M$ matrix, and the matrices $\mathbf{U}_1$ and $\mathbf{L}_1$ are $L/2\times L/2$ matrices. Recall that the zero-forcing receiver $\mathbf{S}'_{zf}$ is given in (15). Substituting (20) and (22) into the expression of $\mathbf{S}'_{zf}$, we get

$$\begin{aligned} \mathbf{S}'_{zf} &= \mathbf{W}(\mathbf{W}^\dagger\mathbf{D}\mathbf{W} - \mathbf{\Delta})^{-1} & (23)\\ &= (\mathbf{D} - \mathbf{W}\mathbf{\Delta}\mathbf{W}^\dagger)^{-1}\mathbf{W}. & (24) \end{aligned}$$

Let us partition the DFT matrix as $\mathbf{W} = \begin{bmatrix} \mathbf{W}_1 & \mathbf{W}_2 & \mathbf{W}_3 \end{bmatrix}$ where $\mathbf{W}_1$ and $\mathbf{W}_3$ consist of the first $L/2$ and the last $L/2$ columns of $\mathbf{W}$ respectively. Then we can write

$$\mathbf{W}\mathbf{\Delta}\mathbf{W}^\dagger = \begin{bmatrix} \mathbf{W}_1 & \mathbf{W}_3 \end{bmatrix} \begin{bmatrix} \mathbf{0} & \mathbf{U}_1 \\ \mathbf{L}_1 & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{W}_1^\dagger \\ \mathbf{W}_3^\dagger \end{bmatrix}. \quad (25)$$

Define the $M\times L$ matrix $\overline{\mathbf{W}} = \begin{bmatrix} \mathbf{W}_1 & \mathbf{W}_3 \end{bmatrix}$. Using (25) and applying the matrix inversion lemma given in the appendix to the matrix $(\mathbf{D} - \mathbf{W}\mathbf{\Delta}\mathbf{W}^\dagger)^{-1}$, we get

$$\mathbf{S}'_{zf} = \mathbf{D}^{-1}\left(\mathbf{I} + \overline{\mathbf{W}}\mathbf{\Theta}_1\overline{\mathbf{W}}^\dagger\mathbf{D}^{-1}\right)\mathbf{W}, \quad (26)$$

where $\mathbf{\Theta}_1$ is an $L\times L$ matrix given by

$$\mathbf{\Theta}_1 = \left(\begin{bmatrix} \mathbf{0} & \mathbf{L}_1^{-1} \\ \mathbf{U}_1^{-1} & \mathbf{0} \end{bmatrix} - \overline{\mathbf{W}}^\dagger\mathbf{D}^{-1}\overline{\mathbf{W}}\right)^{-1}. \quad (27)$$

Though the expression of $\mathbf{S}'_{zf}$ in (26) is more complicated than that in (15), it actually has a much lower implementation cost. Note that $\overline{\mathbf{W}}$ is an $M\times L$ submatrix of the DFT matrix $\mathbf{W}$. In many applications, M is usually much larger than L. The matrices $\overline{\mathbf{W}}$ and $\overline{\mathbf{W}}^\dagger$ can be implemented directly with a complexity of ML or using an FFT/IFFT algorithm. The matrix $\mathbf{\Theta}_1$ is an $L\times L$ matrix whose implementation needs only L^2 multiplications.

4.2 MMSE Receiver $\mathbf{S}'_{mmse}$

For the MMSE case, consider the receiver given in (18). We will derive the result for the general case of $K \geq L/2$. Note that when $K > L/2$, we can partition $\mathbf{B}$ as (19), where $\mathbf{B}_2$ is an $(2K-L)\times M$ Toeplitz matrix given by

$$\begin{aligned} \mathbf{B}_2 &= \begin{bmatrix} 0 & \cdots & 0 & c_L & c_{L-1} & \cdots & c_{L-K+2} & c_{L-K+1} \\ \vdots & & & & \ddots & & & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & c_L & \cdots & c_K \end{bmatrix} \\ &\triangleq \begin{bmatrix} \mathbf{0} & \mathbf{U}_2 \end{bmatrix}, \end{aligned} \quad (28)$$

where $\mathbf{U}_2$ is an $(2K-L)\times K$ upper triangular Toeplitz matrix. Using (19) and (22), we can write

$$\mathbf{B}^\dagger\mathbf{B}+\rho\mathbf{I}=\mathbf{B}_1^\dagger\mathbf{B}_1+\mathbf{B}_2^\dagger\mathbf{B}_2+\rho\mathbf{I} \quad (29)$$

$$=\mathbf{W}^\dagger\left(\underbrace{\mathbf{D}^\dagger\mathbf{D}+\rho\mathbf{I}+[\widehat{\mathbf{W}}\ \ \mathbf{D}^\dagger\widetilde{\mathbf{W}}]\mathbf{\Psi}\left[\begin{array}{c}\widehat{\mathbf{W}}^\dagger\\ \widetilde{\mathbf{W}}^\dagger\mathbf{D}\end{array}\right]}_{\mathbf{\Pi}}\right)\mathbf{W},$$

where $\widehat{\mathbf{W}}$ is an $M\times L$ matrix consisting of the first $(L-K)$ and the last K columns of $\mathbf{W}$, and $\widetilde{\mathbf{W}}$ is an $M\times L$ matrix consisting of the first K and the last $(L-K)$ columns of $\mathbf{W}$. The matrix $\mathbf{\Psi}$ is an $2L\times 2L$ matrix given by

$$\mathbf{\Psi}=\begin{pmatrix}\mathbf{L}_1^\dagger\mathbf{L}_1 & \mathbf{0} & \mathbf{0} & -\mathbf{L}_1^\dagger\\ \mathbf{0} & \mathbf{U}_1^\dagger\mathbf{U}_1+\mathbf{U}_2^\dagger\mathbf{U}_2 & -\mathbf{U}_1^\dagger & \mathbf{0}\\ \mathbf{0} & -\mathbf{U}_1 & \mathbf{0} & \mathbf{0}\\ -\mathbf{L}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0}\end{pmatrix}.$$

Applying the matrix inversion lemma to the matrix $\mathbf{\Pi}^{-1}$ and defining the diagonal matrix $\mathbf{D}_{mmse}=(\mathbf{D}^\dagger\mathbf{D}+\rho\mathbf{I})^{-1}$, we can write $(\mathbf{B}^\dagger\mathbf{B}+\rho\mathbf{I})^{-1}$ as

$$\mathbf{W}^\dagger\mathbf{D}_{mmse}\left(\mathbf{I}-[\widehat{\mathbf{W}}\ \ \mathbf{D}^\dagger\widetilde{\mathbf{W}}]\mathbf{\Theta}_2\left[\begin{array}{c}\widehat{\mathbf{W}}^\dagger\\ \widetilde{\mathbf{W}}^\dagger\mathbf{D}\end{array}\right]\mathbf{D}_{mmse}\right)\mathbf{W}, \quad (30)$$

where $\mathbf{\Theta}_2$ is an $2L\times 2L$ matrix given by

$$\mathbf{\Theta}_2=\left(\mathbf{\Psi}^{-1}+\left[\begin{array}{c}\widehat{\mathbf{W}}^\dagger\\ \widetilde{\mathbf{W}}^\dagger\mathbf{D}\end{array}\right]\mathbf{D}_{mmse}[\widehat{\mathbf{W}}\ \ \mathbf{D}^\dagger\widetilde{\mathbf{W}}]\right)^{-1}.$$

Using (20) and (22), we can write $\mathbf{B}^\dagger=[\mathbf{B}_1^\dagger\ \ \mathbf{B}_2^\dagger]$ where

$$[\mathbf{B}_1^\dagger\ \ \mathbf{B}_2^\dagger]=[\mathbf{W}^\dagger\mathbf{D}^\dagger\mathbf{W}-\mathbf{\Delta}^\dagger\ \ \ \mathbf{B}_2^\dagger]. \quad (31)$$

Combining the results in (30) and (31), we can rewrite the MMSE receiver given in (18) as

$$\mathbf{S}'_{mmse}=\ \ \mathbf{D}_{mmse}\left(\mathbf{I}-[\widehat{\mathbf{W}}\ \ \mathbf{D}^\dagger\widetilde{\mathbf{W}}]\mathbf{\Theta}_2\left[\begin{array}{c}\widehat{\mathbf{W}}^\dagger\\ \widetilde{\mathbf{W}}^\dagger\mathbf{D}\end{array}\right]\mathbf{D}_{mmse}\right)$$
$$\mathbf{W}\left[\ \mathbf{W}^\dagger\mathbf{D}^\dagger\mathbf{W}-\mathbf{\Delta}^\dagger\ \ \ \mathbf{B}_2^\dagger\ \right] \quad (32)$$

For the case of minimum redundancy, $K=L/2$. In this case, the matrices $\widetilde{\mathbf{W}}=\widehat{\mathbf{W}}=\overline{\mathbf{W}}$ and $\mathbf{B}=\mathbf{B}_1$. One can obtain the expression of $\mathbf{S}'_{mmse}$ for the minimum redundancy case by appropriately modifying (32).

4.3 Complexity Comparison

We will compare the number of complex multiplications (CMUL) required for the low complexity receivers in (26) and (32) with that needed for the direct implementation of $\mathbf{S}'_{zf}$ and $\mathbf{S}'_{mmse}$ respectively. The implementation cost for the least-squares receiver $\mathbf{S}'_{ls}$ is the same as the MMSE receiver $\mathbf{S}'_{mmse}$. We will evaluate the efficiency of the low complexity implementation by computing the following ratio:

$$\eta=\frac{\text{no. of CMUL in low cost implementation}}{\text{no. of CMUL in direct implementation}}. \quad (33)$$

The direct implementation of an $(M+2K-L)\times M$ receiver in general needs $M(M+2K-L)$ CMULs. For the low cost implementations in (26) and (32), we need to implement the $M\times M$ DFT and IDFT matrices. In practice, M is often a power of 2 so that these matrices can be efficiently implemented using the FFT/IFFT algorithms. In this paper, we will use the split-radix FFT/IFFT algorithms in [6]. The number of CMULs of an 2^n-point FFT/IFFT is listed in Table VI of [6] for different n. In addition to the DFT and IDFT matrices, we have to realize the $M\times L$ matrices $\overline{\mathbf{W}}$, $\widehat{\mathbf{W}}$, $\widetilde{\mathbf{W}}$ and their transpose conjugates. Note that these matrices are submatrices of DFT/IDFT matrices. They can either be implemented directly with ML CMULs or be implemented using FFT/IFFT algorithms. For most values of M and L, it is found that the FFT/IFFT algorithms in [6] have a smaller number of CMULs than ML. So the matrices $\overline{\mathbf{W}}$, $\widehat{\mathbf{W}}$, $\widetilde{\mathbf{W}}$ and their transpose conjugates are implemented using the split-radix FFT/IFFT algorithms. The $L\times L$ matrix $\mathbf{\Theta}_1$ and $2L\times 2L$ matrix $\mathbf{\Theta}_2$ in (26) and (32) are implemented directly and they need L^2 and $4L^2$ CMULs respectively.

We compute the ratio η for different values of M and L. In practice, L is usually much smaller than M so that the bandwidth efficiency of the system is large. The ratio L/M is seldom larger than 0.2. Some typical values of M and L are given in Table 1. The values of η for $\mathbf{S}'_{zf}$ are shown in Table 1. For the MMSE case, the ratio η varies slightly with respect to the redundancy K. Table 2 shows the results for $K=0.75L$. From these two tables, it is seen that a lot of computational saving can be attained by implementing the receivers using the expressions in (26) and (32). For $M=512$ and $L=32$, we have a saving of around 98% and 94% for $\mathbf{S}'_{zf}$ and $\mathbf{S}'_{mmse}$ respectively.

For completeness, we also compare the number of CMULs of the low cost implementation of $\mathbf{S}'_{zf}$ in (26) with that of ZP DMT receiver in (7) (The CP DMT receiver in (10) has the same number of CMULs). The ratio is listed in Table 3. From the table, we see that the ZP-ZJ DMT receiver is about 3 to 8 times more costly than the ZP DMT receiver.

5. SIMULATION RESULTS

In this section, we carry out the Monte Carlo simulation to verify the performance. The transmitted symbols are QPSK with signal power E_s. The size of the DFT matrix is $M=64$. The channel order is $L=16$. The channel taps $c(l)$ are independent zero-mean complex Gaussian random variables with variance $\sigma_l^2=1/17$ for $0\le l\le 16$. A total of 20000 random channels are used in the numerical simulation. The channel noise is AWGN with noise power N_0. We plot the bit error rate (BER) versus the signal to noise ratio E_s/N_0.

Fig. 1(a) shows the performance of ZP-ZJ DMT systems with least-squares receivers for different values of K. For the purpose of comparison, we also plot the BER of the conventional CP DMT system with 16 redundant samples. When the redundancy K is 10, the performance of the ZP-ZJ DMT system is comparable to the CP DMT system. When $K>10$, the performance of the proposed system is better than CP DMT system. A ZP-ZJ DMT system can achieve a better performance in both BER and bandwidth efficiency.

Fig. 1(b) shows the performance for the MMSE case. From the figure, we see that the ZP-ZJ DMT system with a minimum of 8 redundant samples is only slightly inferior to

L\M	64	128	256	512	1024
8	0.0996	0.0536	0.0297	0.0165	0.0092
16	0.1465	0.0653	0.0326	0.0173	0.0094
32	x	0.1122	0.0443	0.0202	0.0101
64	x	x	0.0912	0.0319	0.0130
128	x	x	x	0.0788	0.0247

Table 1: η for $\mathbf{S}'_{zf}$.

L\ M	64	128	256	512	1024
8	0.2569	0.1323	0.0716	0.0395	0.0218
16	0.4312	0.1789	0.0835	0.0425	0.0225
32	x	0.3593	0.1314	0.0548	0.0257
64	x	x	0.3143	0.1035	0.0382
128	x	x	x	0.2879	0.0875

Table 2: η for the MMSE receiver with $K = 0.75L$.

L\M	64	128	256	512	1024
8	3	2.7962	2.7303	2.7189	2.7279
16	4.4118	3.4076	3	2.8394	2.7828
32	x	5.8535	4.0787	3.3212	3
64	x	x	8.3933	5.2484	3.8707
128	x	x	x	x	7.3537

Table 3: Ratio of number of CMULs of $\mathbf{S}'_{zf}$ to that of the ZP receiver in (7).

the conventional CP DMT system with 16 redundant samples. When $K \geq 9$, the proposed ZP-ZJ DMT system outperforms the CP DMT system. For a BER of 10^{-4} or less, the gain can be substantial.

6. CONCLUDING REMARKS

We propose a ZP-ZJ DMT system that can eliminate IBI with fewer redundant samples. Both the zero-forcing and MMSE receivers are studied. Moreover their low cost implementations are derived. Compared with the direct implementation, these low cost implementations have a significantly smaller number of complex multiplications. Simulations show that the proposed system with fewer redundant samples can outperform the CP DMT system.

7. APPENDIX. MATRIX INVERSE LEMMA

Let $\mathbf{A}$ and $\mathbf{R}$ be respectively $m \times m$ and $n \times n$ invertible matrices. Then

$$(\mathbf{A}+\mathbf{XRY})^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{X}(\mathbf{R}^{-1}+\mathbf{YA}^{-1}\mathbf{X})^{-1}\mathbf{YA}^{-1}.$$

The matrices $\mathbf{X}$ and $\mathbf{Y}$ need not be square.

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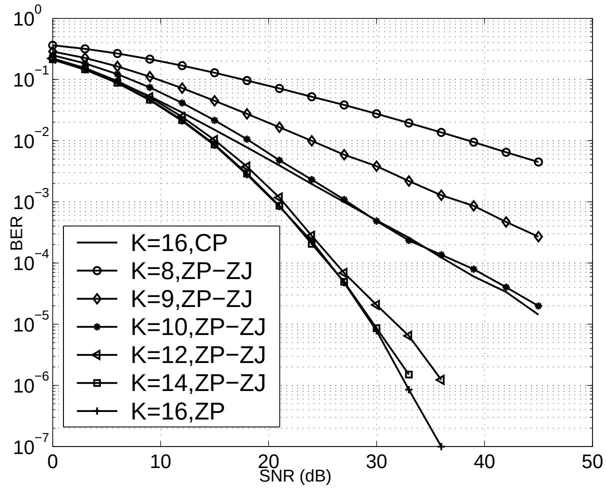

Figure 1(a): Performance of the least-squares receivers

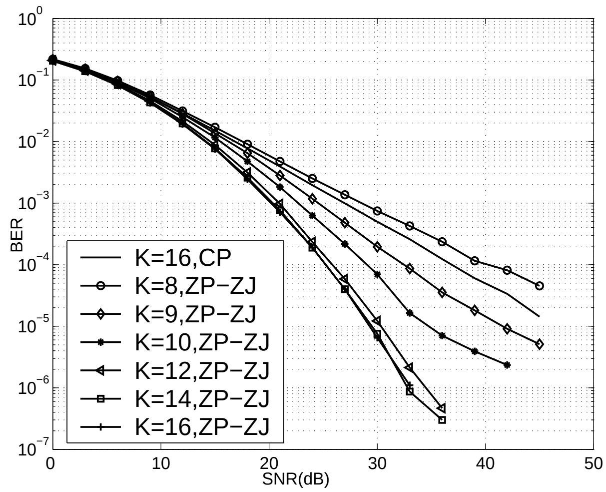

Figure 1(b): Performance of the MMSE receivers

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