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LOCALIZATION OF TACTILE INTERACTIONS THROUGH TDOA ANALYSIS: GEOMETRIC VS. INVERSION-BASED METHOD

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ABSTRACT

In this paper we propose an comparison of three different methods for the localization of tactile interactions with a planar surface, all based on the analysis of differences between time of arrivals from a single source to a set of contact sensors, located around the area of interest. We tested these methods with both synthetic and real data, taken recording a finger touch with four sensors placed over a MDF (Medium Density Fiberboard) board, to prove their accuracy and robustness against time delay estimation errors. The aim is to create tangible interfaces without intrusive sensors or devices placed inside the interaction area.

1. INTRODUCTION

The vast majority of Human Computer Interfaces (HCI) are tangible (e.g. keyboards, mouses, touch pads and touch screens) and different methods for the detection of interactions have been developed. These methods can be divided into two groups: active and passive. Active techniques, like resistive or capacitive touch screens, are based on the measure of the acoustic energy that is absorbed at the points of contacts: two common disadvantages with these peripherals are the presence of mechanical or electronic devices, such as switches, sensitive layers, force resistive sensors, near or under the interaction area, and the fact that they restrict the mobility of users, constraining them to be in certain locations during interaction with the computer. Passive techniques rely on the analysis of the acoustic vibrations generated at the points of contact: these methods are more promising if you want to develop new touch-based interfaces, that have to be scalable in dimensions, cheap, and built with materials and devices that allow them to be suitable for any condition and environment. Within passive approach many algorithms for source detecting and localization can be developed: in this work we focus our attention on three Time Delay Of Arrival (TDOA) related methods. This paper is organized as follows. In section 2 we briefly describe the three methods. Time Delay Estimation problem in solid materials is presented in section 3. In section 4 we test the algorithms first with synthetic data and then with an experimental set-up consisting in four contact sensors positioned near the corners of a rectangular MDF board.

2. LOCALIZATION METHODS

2.1 Tobias method

The first method was proposed in [1]. Consider the case depicted in figure 1: the difference in distance covered by the wave to a pair of sensors (Δd) generates an infinite number of possible solutions, which lie on an hyperbola. These curves, drawn for different pairs of sensors, are then intersected to identify the source location. In general, N receivers yields $N-1$ relatives distances and coordinates: thus, for localization in the 2-D case, at least three sensors are required. However, an extra sensor is needed to resolve ambiguities that arise when the source is positioned closed to or behind a sensor.

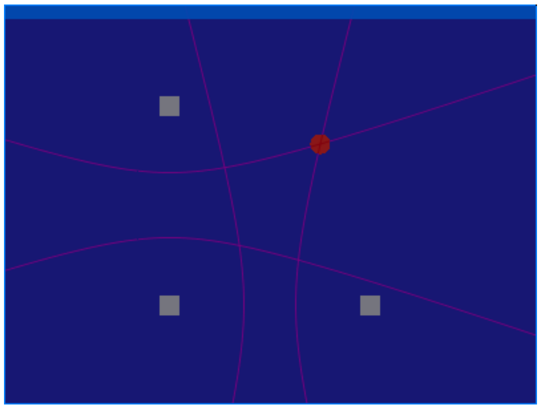

Figure 1: Source position (circle) estimation with three sensors (squares) using Tobias method.

Let Δd_1 be the relative distance between sensor R_1, positioned in (x_1, y_1), and sensor R_0, positioned in $(0,0)$, and Δd_2 the relative distance between R_2, positioned in (x_2, y_2), and R_0.

The source location (x,y) is given by

$$\begin{aligned}
A_1 &= (x_1^2 + y_1^2) - \Delta d_1^2 \\
A_2 &= (x_2^2 + y_2^2) - \Delta d_2^2 \\
B &= \begin{cases} tan^{-1}\frac{A_1 y_2 - A_2 y_1}{A_1 x_2 - A_2 x_1} & \text{if } A_1 x_2 - A_2 x_1 \geq 0 \\ tan^{-1}\frac{A_1 y_2 - A_2 y_1}{A_1 x_2 - A_2 x_1} + \pi & \text{otherwise} \end{cases} \\
C &= cos^{-1}\frac{A_2 \Delta d_1 - A_1 \Delta d_2}{\sqrt{(A_1 x_2 - A_2 x_1)^2 + (A_1 y_2 - A_2 y_1)^2}}
\end{aligned}$$

From these relations two solutions can be computed, in polar coordinates :

$$r_1 = \frac{A_1}{2(x_1 cos\theta_1 + y_1 sin(\theta_1) + \Delta d_1)} \quad \text{where} \quad \theta_1 = B - C$$

$$r_2 = \frac{A_2}{2(x_1 cos\theta_2 + y_1 sin(\theta_2) + \Delta d_2)} \quad \text{where} \quad \theta_2 = B + C$$

If r_1 and r_2 are both negative, then the solution is invalid. If r_1 and r_2 are both positive, there is ambiguity, and we need a fourth sensor R_3, positioned in (x_3, y_3). For both solutions calculated above we have to compute

$$D = \frac{\sqrt{(x-x_3)^2 + (y-y_3)^2} - \sqrt{x^2 - y^2} - \Delta d_3}{\sqrt{(x-x_3)^2 + (y-y_3)^2}}$$

and choose the solution with the smaller D value.

2.2 Mahajan-Walworth method

The second method was proposed in [2]. Referring to figure 2, if the transmitter, located in position (x,y), sends a signal at time $T = 0$, receivers will be reached at times T_1, T_2, T_3, T_4. Let d be the distance between the source and the first receiver that senses the signal and

$$\begin{aligned} \Delta T_{12} &= T_2 - T_1 \\ \Delta T_{13} &= T_3 - T_1 \\ \Delta T_{14} &= T_4 - T_1. \end{aligned}$$

The next receiver reached by the signal is at distance $d + c\Delta T_{12}$, where c is the signal velocity. The third and fourth receivers are then at distances $d + c\Delta T_{13}$ and $d + c\Delta T_{14}$ from the source. Since signal travels in circular waves, four concentric circles, all centered in source position, can be drawn.

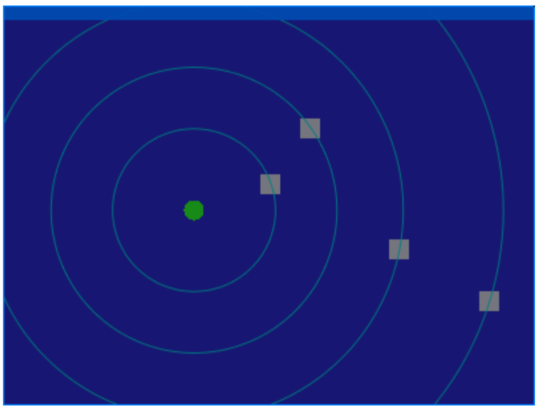

Figure 2: Source position (circle) estimation with four sensors (squares) using Mahajan-Walworth method.

Writing circles equations

$$\begin{cases} (x_1 - x)^2 + (y_1 - y)^2 = d^2 \\ (x_2 - x)^2 + (y_2 - y)^2 = (d + c\Delta T_{12})^2 \\ (x_3 - x)^2 + (y_3 - y)^2 = (d + c\Delta T_{13})^2 \\ (x_4 - x)^2 + (y_4 - y)^2 = (d + c\Delta T_{14})^2 \end{cases}$$

solving the first equation for d^2

$$d^2 = x_1^2 - 2x_1 x + x^2 + y_1^2 - 2y_1 y + y^2$$

and substituting d^2 into the remaining three equations we obtain :

$$\begin{bmatrix} 2(x_1 - x_2) & 2(y_1 - y_2) & -2c\Delta T_{12} \\ 2(x_1 - x_3) & 2(y_1 - y_3) & -2c\Delta T_{13} \\ 2(x_1 - x_4) & 2(y_1 - y_4) & -2c\Delta T_{14} \end{bmatrix} * \begin{bmatrix} u \\ v \\ d \end{bmatrix} = \qquad (1)$$

$$\begin{bmatrix} c^2\Delta T_{12}^2 + x_1^2 + y_1^2 - x_2^2 - y_2^2 \\ c^2\Delta T_{13}^2 + x_1^2 + y_1^2 - x_3^2 - y_3^2 \\ c^2\Delta T_{14}^2 + x_1^2 + y_1^2 - x_4^2 - y_4^2 \end{bmatrix}$$

2.3 The inverse problem formulation method

Consider again the situation depicted in figure 2: the source localization problem can be formulated as follows :

- Parameters (experiment configuration): sensors position, $S_i(x_i, y_i)$, $i = 0, 1, 2, 3$;
- Unknown quantities (model): source position $m = [x, y]^T$;
- Observed data: let d_i be the distance between source and i-th sensor and consider R_0 as reference sensor; the observed data vector is $d_{obs} = [\Delta d_1, \Delta d_2, \Delta d_3]^T$, where $\Delta d_i = d_i - d_0$.

The link between model and observed data leads to a Jacobian matrix G, linearized around reference model $m_0 = [(x_0, y_0)]^T$

$$G = \begin{bmatrix} \frac{x-x_0}{d_0} & \frac{x-x_1}{d_1} & \frac{y-y_0}{d_0} & \frac{y-y_1}{d_1} \\ \frac{x-x_0}{d_0} & \frac{x-x_2}{d_2} & \frac{y-y_0}{d_0} & \frac{y-y_2}{d_2} \\ \frac{x-x_0}{d_0} & \frac{x-x_3}{d_3} & \frac{y-y_0}{d_0} & \frac{y-y_3}{d_3} \end{bmatrix}$$

and to the linear system

$$\begin{bmatrix} \Delta d_1 \\ \Delta d_2 \\ \Delta d_3 \end{bmatrix} = G \begin{bmatrix} x \\ y \end{bmatrix}$$

The following iterative fix-point algorithm, based on the Tarantola technique for non-linear inverse problems, has been proposed [3]:

$$m_{k+1} = m_{pr} - \left[G_k^T C_d^{-1} G_k + C_m^{-1}\right]^{-1}$$
$$G_k^T C_d^{-1} \left[(g(m_k) - d_{obs}) - G_k(m_k) - m_{pr}\right],$$

where m_{pr} and C_m are respectively the mean and the covariance matrix of the a priori model, d_{obs} is the observed data vector, C_d is the covariance matrix of the measured uncertainties and of the modelling errors, G_k is the linearized Jacobian matrix at iteration k, m_k and m_{k+1} are the model vector at k_{th} and $(k+1)_{th}$ iteration.

3. TIME DELAY ESTIMATION IN SOLID MATERIALS

TDOA-based locators are based on a simple geometrical approach and their reliability strongly depends on the accuracy of the Time Delay Estimation (TDE), because the relative distances are calculated as $\Delta d_i = v\Delta t_i$, where v is the wave speed in the medium. Two techniques are normally employed : cross-correlation, that estimates Δt as the peak of the cross-correlation function between the two signals acquired by the receivers, and first arrival detection, that measures the

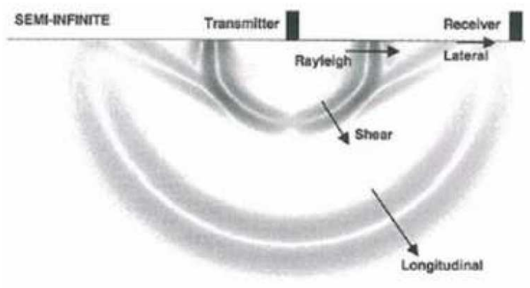

Figure 3: Propagation of different elastic wave modes inside a solid object.

instant when the signal reaches a sensor by defining a threshold value, slightly higher than noise level but not too low in order to avoid spurious events detection: so Δt is the time elapsed between signal arrivals at different sensors.

In solid materials several modes of wave propagation are present (see figure 3): Bulk waves, that propagate deep inside the material and that can be further divided into P waves (longitudinal) and S waves (transversal), and surface waves, that propagates only close to the surface of the objects as a circular wave from the contact point (Rayleigh waves, Love waves, lateral waves); inside thin plates the most important modes are Rayleigh waves, in this case known as Lamb waves.

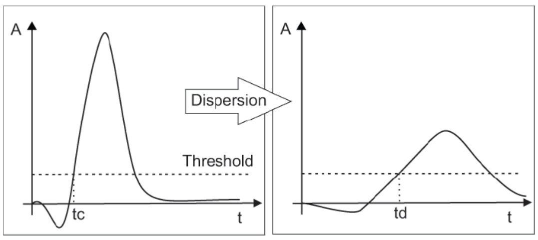

Figure 4: Error introduced by dispersion in TDE with threshold.

Physical phenomena involved in in-solid wave propagation, like reflections and, above all, dispersion, make TDE more complicated compared with the in-air case. Dispersion is defined as any phenomenon in which the velocity of a wave is wavelength dependent and its effect is substantially the modification of the signal wavefront, as depicted in figure 4: this spread introduces errors in time arrival estimation. Better TDE performances can be achieved with techniques that, through the modeling of the elastic wave propagation in plates, are able to estimate this modification.

4. TESTING AND RESULTS

In order to assess the effectiveness of the three algorithms, we first conducted some tests based on synthetic data. We developed a Graphical User Interface (see figure 4) that allows to position source and sensors in arbitrary positions. TDE errors are simulated adding a random white gaussian noise to the geometric distances between the source and the receivers.

We conducted our simulations by placing four sensors at the corners of a 1 x 1 meter board. We immediately found

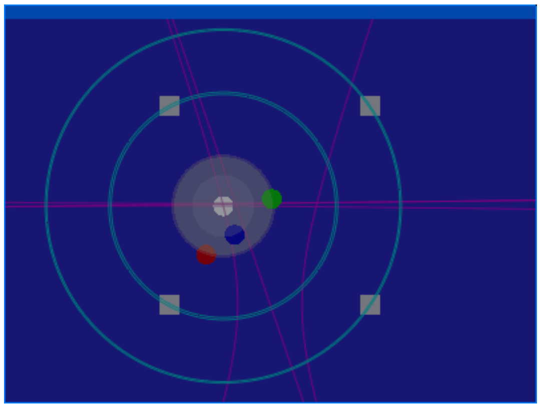

Figure 5: The Graphical User Interface : real source position (white cicle) and estimated source positions (Tobias-red, Mahajan-green, Tarantola-blue).

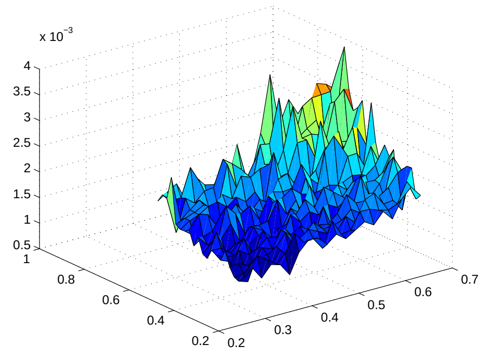

Figure 6: Variance of the source estimation error using Tobias method in the central zone.

Tobias method to perform well only in the central region. Figure 6 shows the variance of the estimation error in the 20*cm* per 20*cm* central square, calculated over 50 tests. This method turns out to have two drawbacks: the former is that it can manage information coming only from three sensor at a time (the fourth is used only in a partial way to select between double solutions). The latter, more crucial, is the poor localization accuracy when the source is close to one of the sensors (in particular the reference sensor R_0): an example of this situation is presented in figure 7.

The method proposed by Mahajan is affected by significant errors when the source is placed in specific positions. This is due to the fact that a circle can have more than a receiver on it: when this happens (see the example of figure 8) the algorithm returns clearly wrong estimates. Another drawback of this approach is associated to the dispersed propagation, i.e. with in-solid source localization. The problem formulation presented in 1, in fact, treats wave speed c as constant, therefore it does not consider its frequency dependence. A formulation with c treated as unknown is proposed in [2], but it requires at least five sensors.

Inverse problem formulation method proved by far the best solution in terms of both accuracy and robustness. We test it with real data using an experimental equipment (see figure 9) made of a 152*x*106*cm* MDF panel of thickness

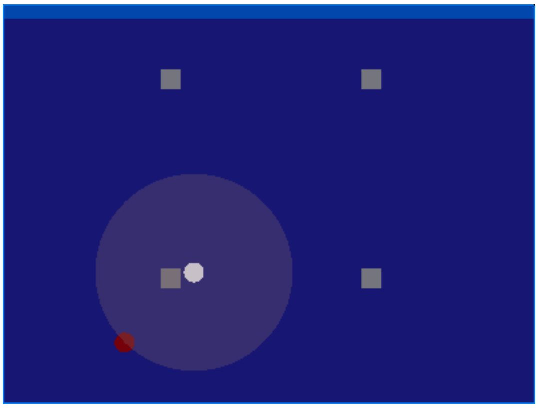

Figure 7: Source estimation error using Tobias method with the source placed near the reference sensor.

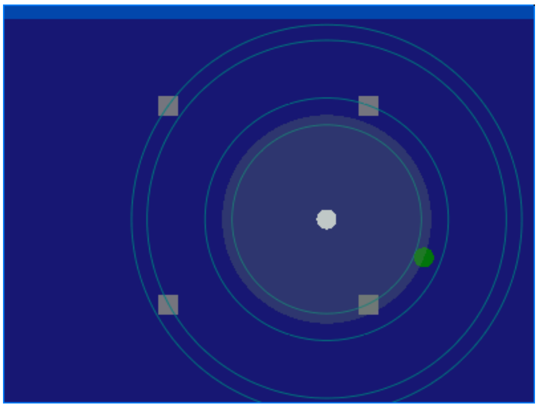

Figure 8: Source estimation error using Mahajan method with two receivers on the same circle.

$t = 0.5cm$. The contact sensors that we used for the acquisition are Knowles BU-1771 piezo-sensors, characterized by a bandwidth of $10kHz$. Such sensors were stuck to the board using double sided adhesive tape and connected to a Presonus Firepod unit, a semi-professional 24bit/96K firewire audio acquisition rack. The PC that we used was a Intel Pentium4 3GHz, with 1Gb RAM. The experiment consisted of five repeated taps in coordinates $(x = 10cm, y = 10cm)$, $(x = 50cm, y = 10cm)$, $(x = 30cm, y = 30cm)$, $(x = 50cm, y = 10cm)$, $(x = 50cm, y = 50cm)$; sensors are placed in $(x = 0cm, y = 0cm)$, $(x = 60cm, y = 0cm)$, $(x = 0cm, y = 45cm)$, $(x = 45cm, y = 45cm)$. The iteration starting point m_0 has been set as $(x_0 = 30cm, y_0 = 30cm)$ when the tap occurred in the board center, and as the center of the quadrant otherwise, for example $(x = 15cm, y = 12.75cm)$ where the tap was in $(x = 10cm, y = 10cm)$. Results are shown in figure 10.

Compared with the other two methods, the accuracy that can be achieved with this method is slightly better in terms of mean square localization error in central area, and far better at the corner. The greatest improvement, however, is in the extremely reduced number of outliers that we obtain with the inverse theory method with respect to the other two solutions.

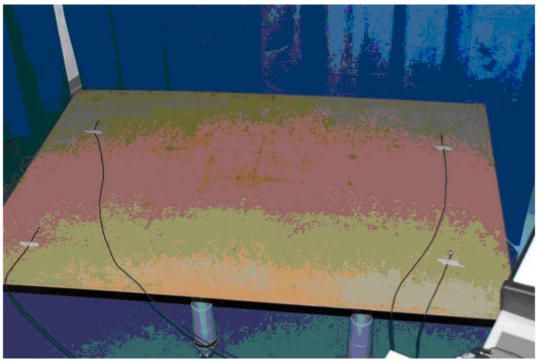

Figure 9: The experimental equipment.

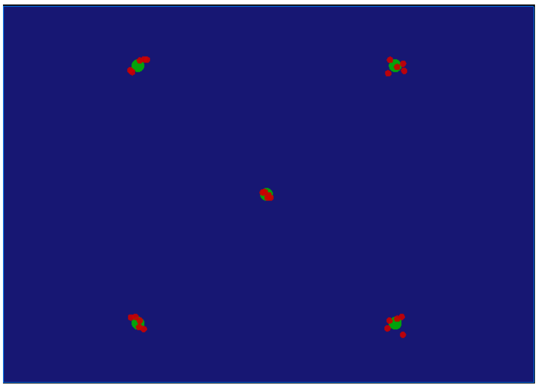

Figure 10: Experiment results.

5. CONCLUSIONS

In this paper we presented a comparison between three TDOA-based methods for the localizations of tactile interactions. The inverse problem formulation exhibit the best performances in terms of precision and robustness against TDE errors mainly due to the dispersion phenomenon.

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