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ITERATIVE EQUALIZATION FOR SEVERE TIME DISPERSIVE MIMO CHANNELS

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ABSTRACT

In this work, a minimum mean-squared error (MMSE) iterative equalization method for a severe time dispersive MIMO channel is proposed. To mitigate the severe time dispersiveness of the channel single carrier with cyclic prefix (SCCP) is employed and the equalization is performed in the frequency domain. The use of cyclic prefix (CP) and equalization in the frequency domain simplifies the challenging problem of equalization in MIMO channels due to both the inter-symbol-interference (ISI) and co-antenna interference (CAI). The proposed iterative algorithm works in two stages. The first stage estimates the transmitted frequency domain symbols using a low complexity MMSE equalizer. The second stage finds the *a posteriori* probabilities of the estimated symbols to find their means and variances to use in the MMSE equalizer in the following iteration. Simulation results show the superior performance of the iterative algorithm when compared with the conventional MMSE equalizer.

1. INTRODUCTION

Multimedia services in mobile communication require very high data rates [1]. Communication theory suggests that high data rates can be achieved by using multiple antenna at the transmitter and receiver, so called MIMO systems [2, 3, 4]. When MIMO systems are used to increase the data rates, to design an equalizer is a challenging task due to co-antenna and inter-symbol interference. Most of the recent research on MIMO systems focused on flat fading channels to decrease the computational complexity of the receiver using space time block codes [5, 6, 7] that increase the diversity of the signal. But, these schemes do not increase the data rates. In order to increase the data rates using MIMO systems in the presence of co-antenna and inter-symbol interference in [8, 9] time domain MIMO decision feedback equalizers (DFE) have been proposed. In these algorithms, hard decisions are input in the feedback filter due to which, in most cases, the phenomenon of error propagation may occur that may degrade the bit-error-rate (BER) performance of these equalizers [10]. Moreover, the computational complexity of these equalizers is very high. To improve the performance of DFE Shoumin *et. al* [11] proposed several DFE iterative algorithms that input soft decisions in the feedback filter that improve the performance but the computational complexity of these algorithm is prohibitively high and increases with the channel support.

Orthogonal frequency division multiplexing (OFDM) with frequency domain equalization (FDE) methods are robust to severe time dispersive channels [12]. Dinis *et. al* [13] proposed an iterative layered space time receiver that is based on a frequency domain DFE in which soft decisions are found for the feedback filter. Here, the filter coefficients for feedback and feed forward filters are required that are computationally very demanding. OFDM with FDE is robust to severe time dispersive channels, however, it has a peak-to-average power ratio (PAPR) problem. An SCCP is a closely related scheme that has most of the benefits of OFDM but does not have PAPR problem [14].

In this work, an iterative equalization scheme is proposed that exploits SCCP and to mitigate the severe time dispersiveness of the channel the equalization is performed in the frequency domain. Working in the frequency domain and the use of SCCP simplifies the detection in both CAI and ISI to the detection in only inter-carrier-interference (ICI) as explained in Section 2. In the proposed algorithm, the equalization is split into two stages. The first stage estimates the transmitted samples in the frequency domain with a low complexity MMSE equalizer and then converts them into time domain by applying an inverse fast Fourier transform (IFFT) operation. While, the second stage determines the *a posteriori* probabilities of the estimated time domain symbols to find the mean and variance, which are used in the MMSE equalizer in the first stage in the next iteration.

This paper is organized as follows. In the following section the signal model is presented. Then, in Section III, the iterative symbol estimation algorithm is presented. Simulation results are given in section IV, followed by our conclusions in Section V.

Notations: Bold upper case letters, $\mathbf{X}(k)$, and lower case letters, $\mathbf{x}(k)$, with indices respectively denote the matrices and vectors having all the elements at frequency k, while without indices denote the general matrices and vectors. Conjugate transposition of a matrix is denoted by $(.)^H$; $\text{diag}(\mathbf{x})$ is a diagonal matrix with diagonal elements taken from the vector $\mathbf{x}$. $\mathbf{I}_N$ is an identity matrix of dimension $N \times N$ and $\mathbf{i}_n$ denotes its nth column. $\mathbf{F}$ denotes the FFT matrix and $\mathbf{f}_n$ denotes its nth column. $E\{.\}$ and $\langle . \rangle_N$ denote respectively the statistical expectation and the modulo-N operation. $\mathbf{X}(n,l)$ denotes the nth row and lth column element of matrix $\mathbf{X}$. Finally, $\bar{s}(k)$ and $\text{Cov}[s(k), s(k)]$ respectively represent the mean (expected value) and co-variance of $s(k)$.

2. SYSTEM MODEL

The MIMO-SCCP transmission and reception model with n_T transmit and n_R receive antennas used in this paper is given in Figure 1. On each transmit antenna, data are converted into blocks each of symbol N and a CP is appended at the beginning of each block. Then, the data are transmitted serially with the help of parallel to serial (PTS) converter through the L multi-path channel. At each receive antenna, the received samples are collected and converted into blocks each of $N+L-1$ symbols with the help of serial to parallel (STP) converter and the CP part is removed. To understand the signal model of a MIMO-SCCP system, consider

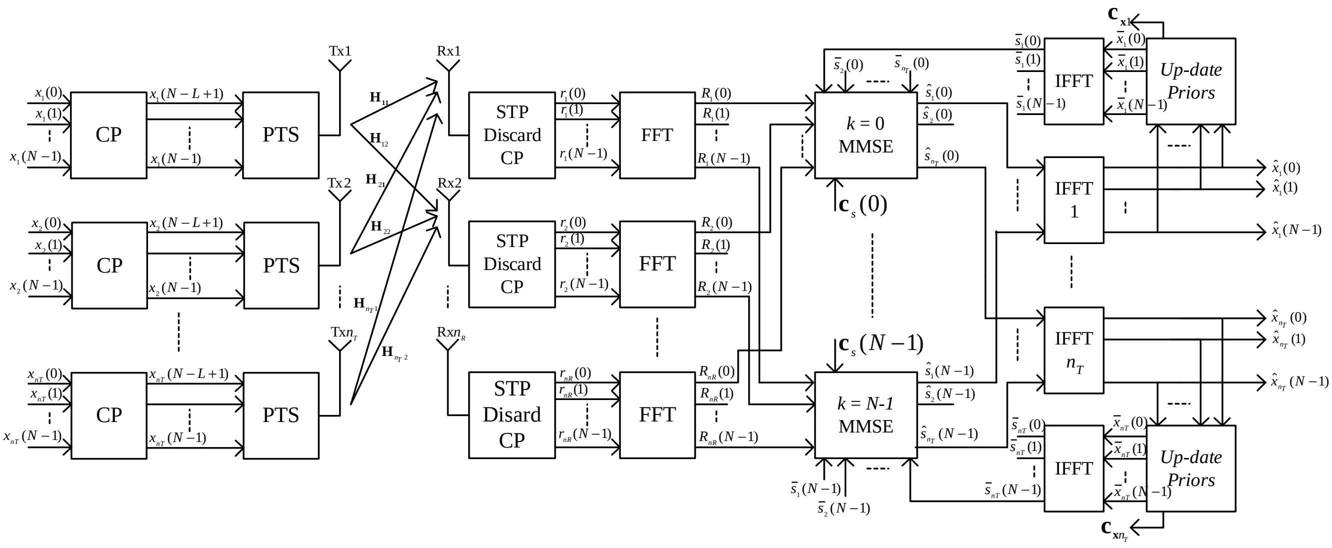

Figure 1: Basic baseband model of the transmitter, channel and the iterative receiver.

an isolated transmit and receive antenna and suppose that the signal between the transmit and receive antenna has propagated through L different paths. If the sampling rate at the receiver is equal to the symbol transmission rate then the received baseband signal at sample time n after removing the CP can be written as

$$r(n) = \sum_{l=0}^{L-1} h(l)x(\langle n-l\rangle_N) + v(n), \tag{1}$$

where $h(l)$ is the unknown complex channel gain between the transmit and receive antenna for the lth multi-path and $v(n)$ is the complex white Gaussian noise at the receive antenna. The N received samples in vector form can be written as

$$\mathbf{r} = \mathbf{H}\mathbf{x} + \mathbf{v} = \mathbf{H}\mathbf{F}^H\mathbf{s} + \mathbf{v}, \tag{2}$$

where $\mathbf{F}$ is the unitary discrete Fourier transform (DFT) matrix, $\mathbf{H}$ is the circulant channel convolution matrix (CCM) of dimension $N \times N$ and $\mathbf{H}(n,l) = h(n, \langle n-l\rangle_N)$. Moreover, $\mathbf{x} = [\ x(0)\ \ x(1)\ \ \cdots\ \ x(N-1)\ ]^T$ and $\mathbf{s} = [\ s(0)\ \ s(1)\ \ \cdots\ \ s(N-1)\ ]^T$. Where, $\{x(n)\}$ are the time domain symbols to be transmitted and $\{s(k)\}$ are the corresponding frequency domain samples after the FFT operation. The relationship between them can be described by the following operation

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} s(k) e^{j\frac{2\pi}{N}kn}. \tag{3}$$

In (1), it can be noted that the receiver experiences ISI. However, working in the frequency domain can make this problem ISI free. Therefore, applying the FFT on (2) yields the frequency domain received sample vector

$$\mathbf{y} = \mathbf{F}\mathbf{H}\mathbf{F}^H\mathbf{s} + \mathbf{F}\mathbf{v} = \mathbf{H}_{IC}\mathbf{s} + \mathbf{w}, \tag{4}$$

where $\mathbf{y} = [\ y(0)\ \ y(1)\ \ \cdots\ \ y(N-1)\ ]^T$, $y(k)$ is the received frequency domain sample at frequency k, $\mathbf{w} = [w(0)\ \ w(1)\ \ \cdots\ \ w(N-1)]^T$, $w(k) = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} v(n)e^{-j\frac{2\pi nk}{N}}$ and $\mathbf{H}_{IC}$ is the ICI matrix. If in (4), the channel is linear time invariant (LTI) then the ICI matrix, $\mathbf{H}_{IC}$, will be diagonal. Therefore, in order to estimate the symbols $\{s(k)\}$, the L-MMSE equalization requires the inversion of a diagonal matrix, which is computationally inexpensive.

However, contrasting to single-input single-output (SISO) system in MIMO system model the signals are transmitted from n_T and received by n_R antennas. Therefore, each receive antenna not only experiences ISI due to multi-paths but co-antenna interference too. The presence of CAI and ISI makes the equalization computationally very expensive. But, working in the frequency domain can simplify this problem into an only ICI problem. Therefore, as shown in Figure 1 the collection of received samples at frequency k, from received antenna $l = 1, 2, \ldots, n_R$, in vector form can be written as

$$\mathbf{y}(k) = \mathbf{H}(k)\mathbf{s}(k) + \mathbf{w}(k), \tag{5}$$

where

$$\begin{aligned}
\mathbf{y}(k) &= [\ y_1(k)\ \ y_2(k)\ \ \cdots\ \ y_{n_R}(k)\ ]^T, \\
\mathbf{H}(k) &= \begin{bmatrix} H_{11}(k) & H_{21}(k) & \cdots & H_{n_T1}(k) \\ H_{12}(k) & H_{22}(k) & \cdots & H_{n_T2}(k) \\ \vdots & \vdots & \ddots & \vdots \\ H_{1n_R}(k) & H_{2n_R}(k) & \cdots & H_{n_Tn_R}(k) \end{bmatrix}, \\
\mathbf{s}(k) &= [\ s_1(k)\ \ s_2(k)\ \ \cdots\ \ s_{n_T}(k)\ ]^T, \\
\text{and } \mathbf{w}(k) &= [\ w_1(k)\ \ w_2(k)\ \ \cdots\ \ w_{n_R}(k)\ ]^T.
\end{aligned}$$

Here, $H_{tr}(k)$ is the frequency response of the channel between the transmit antenna t and the receive antenna l at frequency k and $H_{tr}(k) = \sqrt{N}\sum_{l=0}^{L-1} h_{tr}(l)e^{-j\frac{2\pi lk}{N}}$.

3. MMSE-ITERATIVE EQUALIZATION

To estimate the transmitted symbols $\{x_t(n)\}$ from the antenna $t = 1, 2, ..., n_T$, we propose a relatively low complexity iterative algorithm. The iterative algorithm works in two stages. In the first stage an MMSE equalizer is designed to estimate the transmitted frequency domain symbols. The first stage accounts for already estimated variances on the equalizer design and means in estimator to cancel the interference from all other symbols.

In order to design the equalizer, the noise is assumed uncorrelated and zero mean, i.e., $E\{\mathbf{w}(k)\} = \mathbf{0}$, $E\{\mathbf{w}(k)\mathbf{w}(k)^H\} = \sigma_w^2 \mathbf{I}_{n_R}$ and $E\{s_t(k)\mathbf{w}(k)\} = \mathbf{0}$. Moreover, we define $\bar{s}(k) = E\{s(k)\}$, $\bar{\mathbf{s}} = E\{\mathbf{s}\}$, $c_t(k) = \text{var}[s_t(k), s_t(k)]$ and $\mathbf{c}_s(k) = [\ c_1(k) \quad c_2(k) \quad \cdots \quad c_{n_T}(k)\ ]^T$. The MMSE equalizer $\mathbf{q}_t(k)$ of length n_R for the soft estimates of $s_t(k)$ can be derived by minimizing the cost function

$$J(\mathbf{q}_t(k)) = E\{|\mathbf{q}_t^H(k)\mathbf{y}(k) - s_t(k)|^2\},$$

which yields the MMSE equalizer coefficient vector given in [15, 16] by

$$\mathbf{q}_t(k) = \left(\mathbf{H}(k)\text{diag}(\mathbf{c}_s(k))\mathbf{H}^H(k) + \sigma_w^2 \mathbf{I}_{n_R}\right)^{-1} \mathbf{h}_t(k) c_t(k) \tag{6}$$

and the estimate

$$\hat{s}_t(k) = \bar{s}_t(k) + \mathbf{q}_t^H(k)\left(\ \mathbf{y}(k) - \mathbf{H}(k)\bar{\mathbf{s}}\ \right) \tag{7}$$

with the assumption that $\{\bar{s}_t(k) \neq 0\}$, the mean values of the estimates of the individual symbols can not be equal to zero, in (6) $\mathbf{h}_t(k)$ is the tth column of the matrix $\mathbf{H}(k)$.

Now, we are interested in finding the *a posteriori* values of mean and variance of the frequency domain symbols to use in the next iteration, which requires the log-likelihood-ratios (LLR)s [17]. The frequency domain transmitted symbols have not finite constellations due to the FFT operation, therefore, it is very difficult to determine the LLRs. However, the LLRs can be found easily by converting the estimated frequency domain symbols into the time domain. The relationship between the time and frequency domain samples can be written as

$$\begin{aligned} s_t(k) &= \mathbf{i}_k^T \mathbf{F} \mathbf{x}_t &(8)\\ x_t(n) &= \mathbf{i}_n^T \mathbf{F}^H \mathbf{s}_t. &(9) \end{aligned}$$

Therefore, we have

$$\begin{aligned} \hat{x}_t(n) &= \mathbf{i}_n^T \mathbf{F}^H \sum_{k=0}^{N-1} \mathbf{i}_k \hat{s}_t(k) \\ &= \mathbf{i}_n^T \mathbf{F}^H \sum_{k=0}^{N-1} \mathbf{i}_k \left[\bar{s}_t(k) + \mathbf{q}_t^H(k)\left(\mathbf{y}(k) - \mathbf{H}(k)\bar{\mathbf{s}}\right)\right] \\ &= \bar{x}_t(k) + \mathbf{i}_n^T \mathbf{F}^H \sum_{k=0}^{N-1} \mathbf{i}_k \left[\mathbf{q}_t^H(k)\left(\mathbf{y}(k) - \mathbf{H}(k)\bar{\mathbf{s}}\right)\right]. \quad (10) \end{aligned}$$

In (5) the individual terms can be written as

$$\begin{aligned} \mathbf{s}(k) &= \begin{bmatrix} s_1(k) \\ s_2(k) \\ \vdots \\ s_{n_T}(k) \end{bmatrix} = \begin{bmatrix} \mathbf{i}_k^T \mathbf{F} \mathbf{x}_1 \\ \mathbf{i}_k^T \mathbf{F} \mathbf{x}_2 \\ \vdots \\ \mathbf{i}_k^T \mathbf{F} \mathbf{x}_{n_T} \end{bmatrix} \\ &= \Re_{s(k)} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \ddots \\ \mathbf{x}_{n_T} \end{bmatrix} = \begin{bmatrix} \mathbf{x}_1^T \\ \mathbf{x}_2^T \\ \vdots \\ \mathbf{x}_{n_T}^T \end{bmatrix} \mathbf{f}_k \quad (11) \end{aligned}$$

$$\begin{aligned} \mathbf{w}(k) &= \begin{bmatrix} w_1(k) \\ w_2(k) \\ \vdots \\ w_{n_R}(k) \end{bmatrix} = \begin{bmatrix} \mathbf{i}_k^T \mathbf{F} \mathbf{v}_1 \\ \mathbf{i}_k^T \mathbf{F} \mathbf{v}_2 \\ \vdots \\ \mathbf{i}_k^T \mathbf{F} \mathbf{v}_{n_R} \end{bmatrix} \\ &= \Re_w(k) \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \\ \vdots \\ \mathbf{v}_{n_R} \end{bmatrix} = \begin{bmatrix} \mathbf{v}_1^T \\ \mathbf{v}_2^T \\ \vdots \\ \mathbf{v}_{n_R}^T \end{bmatrix} \mathbf{f}_k \quad (12) \end{aligned}$$

where

$$\Re_s(k) = \underbrace{\begin{bmatrix} \mathbf{f}_k^T & 0 & 0 & 0 \\ 0 & \mathbf{f}_k^T & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \mathbf{f}_k^T \end{bmatrix}}_{n_T \times n_T N}$$

and

$$\Re_w(k) = \underbrace{\begin{bmatrix} \mathbf{f}_k^T & 0 & 0 & 0 \\ 0 & \mathbf{f}_k^T & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \mathbf{f}_k^T \end{bmatrix}}_{n_R \times n_R N}.$$

If we suppose $\mathbf{x} = [\mathbf{x}_1^T \quad \mathbf{x}_2^T \quad \cdots \quad \mathbf{x}_{n_T}^T]^T$, $\mathbf{X} = [\mathbf{x}_1 \quad \mathbf{x}_2 \quad \cdots \quad \mathbf{x}_{n_T}]^T$, $\mathbf{v} = [\mathbf{v}_1^T \quad \mathbf{v}_2^T \quad \cdots \quad \mathbf{v}_{n_R}^T]^T$ and $\mathbf{V} = [\mathbf{v}_1 \ \mathbf{v}_2 \ \cdots \ \mathbf{v}_{n_R}]^T$ then by exploiting (11) and (12), the equation (10) can be written as

$$\hat{x}_t(n) = \bar{x}_t(n) + \mathbf{i}_n^T \mathbf{F}^H \sum_{k=0}^{N-1} \mathbf{i}_k \mathbf{q}_t^H(k) \left[\mathbf{H}(k)(\mathbf{X} - \bar{\mathbf{X}}) + \mathbf{V}\right] \mathbf{f}_k \tag{13}$$

$$= \bar{x}_t(n) + \mathbf{f}_n^H \sum_{k=0}^{N-1} \mathbf{i}_k \mathbf{q}_t^H(k)[\mathbf{H}(k)\Re_s(k)(\mathbf{x} - \bar{\mathbf{x}}) + \Re_w(k)\mathbf{v}]. \tag{14}$$

Now, if we suppose

$$\begin{aligned} \mathbf{Q}_t &= \underbrace{\sum_{k=0}^{N-1} \mathbf{i}_k \mathbf{q}_t^H(k) \mathbf{H}(k) \Re_s(k)}_{N \times n_T N} \\ \mathbf{P}_t &= \underbrace{\sum_{k=0}^{N-1} \mathbf{i}_k \mathbf{q}_t^H(k) \Re_w(k),}_{N \times n_R N} \end{aligned}$$

then, (14) can be written as

$$\hat{x}_t(n) = \bar{x}_t(n) + \mathbf{f}_n^H \left\{\mathbf{Q}_t(\mathbf{x} - \bar{\mathbf{x}}) + \ \mathbf{P}_t \mathbf{v}\right\}. \tag{15}$$

Now, we wish to find the *a posteriori* values of $\{\bar{x}_t(n)\}$ and $\{c_t(n)\}$ to use in (6) and (7) in the next iteration. To find these values the following steps are required to form the proposed iterative algorithm.

Step 1: In the first iteration, we have no prior knowledge. Therefore, we initialize all the mean values $\{\bar{x}_t(n)\} = 0$ that corresponds to $\{\bar{s}_t(k) = 1\}$ and $\text{diag}(\mathbf{c}_s(k)) = \mathbf{I}_{n_T}$, the estimate $\hat{s}_t(k)$ is obtained using (6) and (7).

Step 2: The *a priori* and *a posteriori* LLR of $x_t(n)$ are defined in [17, 18] as

$$L[x_t(n)] = \ln\frac{\Pr\{x_t(n)=1\}}{\Pr\{x_t(n)=-1\}} \text{ and}$$
$$L[x_t(n)|_{\hat{x}_t(n)}] = \ln\frac{\Pr\{x_t(n)=1|_{\hat{x}_t(n)}\}}{\Pr\{x_t(n)=-1|_{\hat{x}_t(n)}\}}.$$

The difference between the *a posteriori* and *a priori* LLRs (which is the extrinsic information) of $x_t(n)$ is

$$\begin{aligned}\triangle L[x_t(n)] &= L[x_t(n)|_{\hat{x}_t(n)}] - L[x_t(n)] = L[\hat{x}_t(n)|_{x_t(n)}] \\ &= \ln\frac{\Pr\{\hat{x}_t(n)|_{x_t(n)=1}\}}{\Pr\{\hat{x}_t(n)|_{x_t(n)=-1}\}}. \end{aligned} \quad (16)$$

In order to find the extrinsic LLR, $L[\hat{x}(n)|_{x(n)}]$, in (16), it is assumed that the probability density function (PDF) of $\hat{x}_t(n)$ is Gaussian with variance $\sigma^2_{x_t}(n)$ and can be written as

$$\Pr\{\hat{x}_t(n)\} = \frac{1}{\sqrt{2\pi}\sigma_x(n)}\exp\left(-\frac{(\hat{x}_t(n)-E\{\hat{x}_t(n)\})^2}{2\sigma_x^2(n)}\right). \quad (17)$$

Therefore, the conditional PDF, when the transmitted signal $x_t(n) = b \in \{+1,-1\}$, of $\hat{x}_t(n)$ becomes

$$\Pr\{\hat{x}_t(n)|_{x_t(n)=b}\} = \frac{1}{\sqrt{2\pi}\sigma_x(n)}\exp\left(-\frac{(\hat{x}_t(n)-m_{tn}(b))}{2\sigma_x^2(n)|_{x_t(n)=b}}\right) \quad (18)$$

where $m_{tn}(b) = E\{\hat{x}_t(n)|_{x_t(n)=b}\}$ and $\sigma_x^2(n)|_{x_t(n)=b} = \mathrm{var}[\hat{x}_t(n),\hat{x}_t(n)|_{x_t(n)=b}]$, which are the conditional mean and variance of $\hat{x}_t(n)$.

Throughout the iterative receiver process, we exchange only extrinsic information. That is, when estimating $x_t(n)$, we use only the *a priori* information from $\{x_p(n), p \neq t\}$. Therefore, it is assumed that the *a priori* information $\bar{x}_t(n) = 0$ and $c_t(n) = 1$ in (6) and (7). Hence, the conditional mean can be determined by using (13) as

$$\begin{aligned} E\{\hat{x}_t(n)|_{x_t(n)=b}\} &= \mathbf{f}_n^H \sum_{k=0}^{N-1} \mathbf{i}_k \mathbf{q}_t^H(k)\mathbf{H}(k)\left[\; E(\mathbf{X}-\bar{\mathbf{X}}) \;\right]\mathbf{f}_k \\ &= b\mathbf{f}_n^H \sum_{k=0}^{N-1} \mathbf{i}_k \mathbf{q}_t^H(k)\mathbf{H}(k)\underbrace{\begin{pmatrix} 0 & \cdots & 0 & 0 \\ \vdots & b_{t,n} & \vdots & 0 \\ 0 & \cdots & 0 & 0 \end{pmatrix}}_{n_T \times N}\mathbf{f}_k \\ &= b\,\mathbf{f}_n^H \sum_{k=0}^{N-1} \mathbf{i}_k \frac{\mathbf{q}_t^H(k)\mathbf{h}_t(k)e^{-\frac{j2\pi}{N}kn}}{\sqrt{N}} \\ &= b\sum_{k=0}^{N-1}\frac{\mathbf{q}_t^H(k)\mathbf{h}_t(k)}{N} = b\,\mathbf{q}_t^H\mathbf{h}_t \end{aligned}$$

where $\mathbf{q}_t = \frac{1}{N}[\mathbf{q}_t^T(0) \;\; \mathbf{q}_t^T(1) \;\; \cdots \;\; \mathbf{q}_t^T(N-1)]^T$ and $\mathbf{h}_t = [\mathbf{h}_t^T(0) \;\; \mathbf{h}_t^T(1) \;\; \cdots \;\; \mathbf{h}_t^T(N-1)]^T$. It should be noted that $m_{tn}(b)$ depends on the particular value of b. Similarly, the conditional variance $\sigma_x^2(n)|_{x_t(n)=b}$, can be written as

$$\begin{aligned}\sigma_x^2|_{x_t(n)=b} &= E\left\{(\hat{x}_t(n)-m_{tn}(b))(\hat{x}_t(n)-m_{tn}(b))^H\right\} \\ &= E\{\hat{x}_t(n)\hat{x}_t^H(n)\}|_{x_t(n)=b} - m_{tn}(b)m_{tn}^H(b).\end{aligned}$$

From (15), we have

$$\begin{aligned} E\{\hat{x}_t(n)\hat{x}_t^H(n)\}|_{x_t(n)=b} &= \mathbf{f}_n^H[\mathbf{Q}_t E\{(\mathbf{x}-\bar{\mathbf{x}})(\mathbf{x}-\bar{\mathbf{x}})^H\}\mathbf{Q}_t^H \\ &+ \sigma_v^2\mathbf{P}_t\mathbf{P}_t^H]\mathbf{f}_n = \mathbf{f}_n^H[\mathbf{Q}_t\mathrm{Cov}(\mathbf{x},\mathbf{x})\mathbf{Q}_t^H + \sigma_v^2\mathbf{P}_t\mathbf{P}_t^H]\mathbf{f}_n \end{aligned}$$

and

$$\sigma_x^2|_{x_t(n)=b} = \mathbf{f}_n^H[\mathbf{Q}_t\mathrm{Cov}(\mathbf{x},\mathbf{x})\mathbf{Q}_t^H + \sigma_v^2\mathbf{P}_t\mathbf{P}_t^H]\mathbf{f}_n - \mathbf{q}_t^H\mathbf{h}_t\mathbf{h}_t^H\mathbf{q}_t.$$

Note that unlike the mean the variance of the estimator is independent of b. Now, the difference between the *a posteriori* and the *a priori* LLR of $x_t(n)$ becomes

$$\begin{aligned}\triangle L\left[x_t(n)\right] &= \ln\frac{\left[\exp\left(-\frac{(\hat{x}_t(n)-m_{tn}(+1))^2}{\sigma_x^2(n)|_{x_t(n)=+1}}\right)\right]}{\left[\exp\left(-\frac{(\hat{x}_t(n)-m_{tn}(-1))^2}{\sigma_x^2(n)|_{x_t(n)=-1}}\right)\right]} \\ &= \frac{\mathrm{Re}\{2\hat{x}_t(n)\mathbf{q}_t^H\mathbf{h}_t\mathbf{h}_t^H\mathbf{q}_t\}}{\mathbf{f}_n^H[\mathbf{Q}_t\mathrm{Cov}(\mathbf{x},\mathbf{x})\mathbf{Q}_t^H + \sigma_v^2\mathbf{P}_t\mathbf{P}_t^H]\mathbf{f}_n - \mathbf{q}_t^H\mathbf{h}_t\mathbf{h}_t^H\mathbf{q}_t} \end{aligned} \quad (19)$$

and the *a posteriori* LLR of $s_t(k)$

$$L\left[s_t(k)|_{\hat{s}_t(k)}\right] = L\left[s_t(k)\right] + \triangle L\left[s_t(k)\right]. \quad (20)$$

Step 3: Exploiting (20) and using the property $\Pr\{x_t(n)=1|_{\hat{x}_t(n)}\} + \Pr\{x_t(n)=-1|_{\hat{x}_t(n)}\} = 1$ the updated values for $\bar{x}_t(n)$ and $c_t(n)$ are obtained as

$$\begin{aligned}\bar{x}_t(n)_{\text{new}} &= \Pr\{x_t(n)=+1|_{\hat{x}_t(n)}\} - \Pr\{x_t(n)=-1|_{\hat{x}_t(n)}\} \\ &= \tanh\left(\frac{L\left[x_t(n)|_{\hat{x}_t(n)}\right]}{2}\right)\end{aligned} \quad (21)$$

and

$$\begin{aligned} c_t(k)_{\text{new}} &= \sum_{b\in\{+1,-1\}}(b-\bar{x}_t(n)_{\text{new}})^2\Pr(x_t(n)=b|_{\hat{x}_t(n)}) \\ &= 1-\bar{x}_t(n)^2_{\text{new}}. \end{aligned} \quad (22)$$

The equations (21) and (22) are used to update the values of $\bar{s}_t(k)$ and $c_t(k)$ in (6) and (7) in Step 1.

Step 4: We repeat steps 1 through to 3 until the specified number of iterations has elapsed.

4. SIMULATION

For simulations, it is assumed that the MIMO channel is frequency selective and slowly time varying, i.e., it is time invariant within each frame of $N+L-1$ symbol periods but changes independently and slowly from one frame to other. We suppose perfect knowledge of channel and noise variance at the receiver. For simulations a 2 transmit and 3 receive antenna MIMO channel model is used. The number of sub-carriers is chosen to be $N = 32$ and the length of the CP is kept equal to the length of the channel. We use an 8-tap wireless fading channel model in which each channel tap is represented by a complex Gaussian random process independently generated. Here, we assume $\sum_{l=0}^{L-1}\sigma_{ltr}^2 = 1$, where σ_{ltr}^2 is the variance of the lth path between the transmit antenna t and receive antenna r. The symbols $\{x_t(n)\}$ are BPSK. All simulations are performed for 5000 monte-carlo simulations. In the first stage of the algorithm, all frequency domain symbols are estimated using the low complexity MMSE equalizer. In the second stage estimated symbols are converted back into time domain and means and variances of the estimated time domain symbols are found, which are used in the following iteration in the first stage. Figure 2 shows the BER performance of iterative estimation technique up to three iterations. In the figure BER performance is compared with the MMSE equalizer and matched filter bound (MFB). The MFB is obtained from the model given in 1 by assuming the symbols $\{x_p(m)|_{m\neq n \text{ when } p=t}\}$ are known when estimating $x_t(n)$. It can be seen from the figure that the performance of the iterative algorithm outperforms the MMSE equalizer and almost converges after first iteration.

5. CONCLUSIONS

We have considered the design of an iterative receiver for the severe time dispersive slowly time variant MIMO channel. Due to CP and equalization in the frequency domain the algorithm can work for severe time dispersive channels. The simulation results support the expected superiority of the proposed iterative scheme over the L-MMSE equalization that has poor performance.

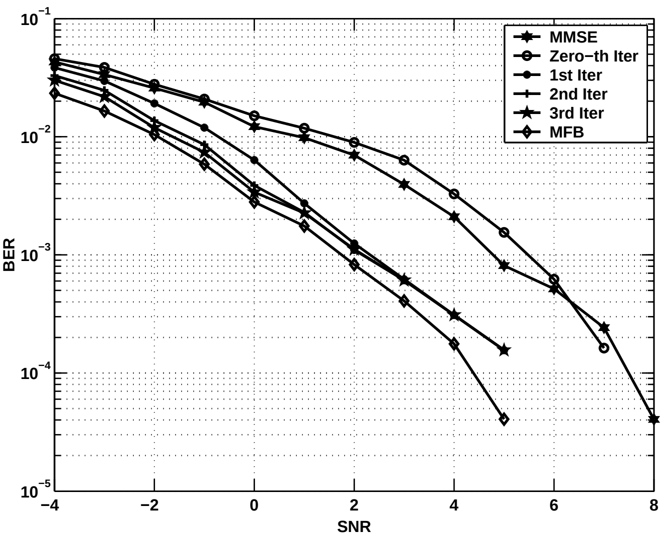

Figure 2: BER performance comparison of the proposed iterative algorithm with the MMSE equalizer and MFB at different number of iterations for the SCCP block length of 32.

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