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SPATIAL MULTIPLEXING WITH LINEAR PRECODING IN TIME-VARYING CHANNELS WITH LIMITED FEEDBACK

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ABSTRACT

Combining spatial multiplexing with linear unitary precoding allows for high data rates, but requires a feedback link from the receiver to the transmitter. We focus on quantizing and feeding back the precoder itself, since it outperforms quantized channel feedback. More specifically, we propose a modified precoder quantization approach that outperforms the conventional one. We investigate both the linear minimum mean square error (LMMSE) detector, which minimizes the mean square error (MSE) between the transmitted and estimated symbols, and the singular value decomposition (SVD) detector, which is a unitary detector that aims at diagonalizing the channel matrix. In this context, we illustrate that the LMMSE detector performs slightly better than the SVD detector. We also study precoder extrapolation, when the precoder is only fed back at a limited number of time instances, as well as a related detector extrapolation scheme for the LMMSE and SVD detector, when the channel is only known at some specific time instances. Simulation results illustrate the efficiency of the proposed extrapolation methods.

1. INTRODUCTION

Spatial multiplexing has emerged in the last years as an efficient technique to reach high data rates. To make it more resistant against rank deficient channels, it is advantageous to use linear precoding [1] on top of spatial multiplexing. Note that the precoder is generally restricted to be unitary. Moreover, since it depends on the channel, which in general is only available at the receiver, we require some feedback from the receiver to the transmitter. To reduce the bandwidth-requirements of the feedback channel, the information is quantized before it is transmitted to the transmitter. Note that there are basically two types of information we can feed back, the channel or the precoder. However, it has been shown recently that the precoder matrix has less degrees of freedom than the channel matrix. Hence, it is more opportune to quantize and feed back the precoder than the channel. The quantization of the precoding matrices requires a suitable codebook design and code selection procedure. This could for instance be based on average mean square error (MSE) [2]. However, this does not necessarily lead to a decoupling of the different spatial streams, which generally results in a better performance if only a few of the spatial streams are used. This is due to the fact that when the spatial streams are stronger decoupled, the average performance of the stronger modes is better. Hence, we will focus on a precoder that consists of a few right singular vectors of the channel matrix, and we will adopt the codebook design and code selection procedure that was proposed in [3]. But since this approach still has a phase ambiguity for every singular vector, we modify the codebook design and code selection accordingly, leading to a better performance.

To limit the amount of feedback, we will only feed back the right singular vectors at regular time instances. At other time instances, we can extrapolate the precoder by exploiting the coherence of the channel in the time domain. The extrapolation scheme we will adopt here is similar to the one proposed in [4]. Since the precoder is only fed back at regular time instances, it is only necessary to estimate the channel at those time instances, which means we can also reduce the amount of training overhead. At other time instances, we can then extrapolate the channel, again exploiting its coherence in the time domain. We will investigate two receiver methods: the linear minimum mean square error (LMMSE) detector, which minimizes the mean square error (MSE) between the transmitted and estimated symbols, and the singular value decomposition (SVD) detector, which is a unitary detector that aims at diagonalizing the channel matrix.

Notation: We designate vectors with lowercase boldface letters, and matrices with capital boldface letters. The notation $[\mathbf{A}]_{i,j}$ denotes the (i, j)th entry of the matrix $\mathbf{A}$, and $[\mathbf{A}]_{:,1:n}$ is the submatrix of $\mathbf{A}$ consisting of the columns 1 to n. $\mathbf{I}_n$ is the $n \times n$ identity matrix. Further, $\mathbf{A}^H$ denotes the conjugate transpose of the matrix $\mathbf{A}$, $\mathbf{A}^{-1}$ the inverse, and $\mathrm{expm}(\mathbf{A})$ the matrix exponential. Finally, $E(\cdot)$ represents expectation and $p(\cdot)$ probability.

2. SYSTEM MODEL

We assume a narrowband spatial multiplexing MIMO system with N_T transmit antennas, N_R receive antennas, and $N_S \leq \min\{N_T, N_R\}$ spatial streams. The system input-output relation at time instant n is given by

$$\mathbf{y}[n] = \mathbf{H}[n]\mathbf{F}[n]\mathbf{s}[n] + \nu[n], \tag{1}$$

where $\mathbf{y}[n] \in \mathbb{C}^{N_R \times 1}$ is the received vector, $\mathbf{s}[n] \in \mathbb{C}^{N_S \times 1}$ is the data symbol vector, $\mathbf{H}[n] \in \mathbb{C}^{N_R \times N_T}$ is the channel matrix, $\mathbf{F}[n] \in \mathbb{C}^{N_T \times N_S}$ is the linear precoder, and $\nu[n] \in \mathbb{C}^{N_R \times 1}$ is the additive noise vector.

We assume that the elements of $\mathbf{s}[n]$ are i.i.d. and uniformly distributed over a finite alphabet with zero mean and variance 1. We further assume that the elements of $\nu[n]$ are i.i.d. and complex Gaussian distributed with zero mean and variance N_0. We finally assume that the elements of $\mathbf{H}[n]$ are i.i.d. and distributed according to Jakes' model [5] with zero mean and variance 1:

$$[\mathbf{H}[n]]_{n_R,n_T} = \frac{1}{\sqrt{S}} \sum_{s=1}^{S} A_{s,n_R,n_T} \exp(j2\pi f_d nT \cos\phi_{s,n_R,n_T}), \tag{2}$$

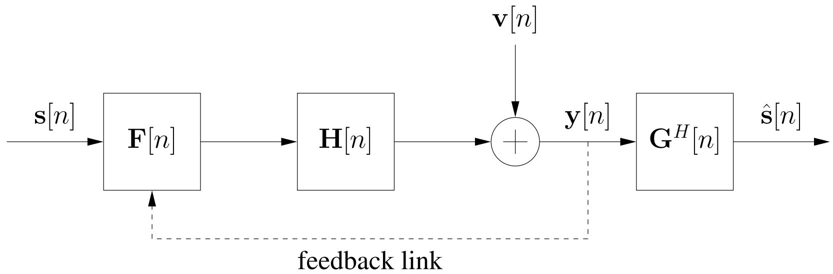

Figure 1: Spatial multiplexing MIMO system with linear precoding and a feedback link.

where S is the number of scatterers, A_{s,n_R,n_T} is complex Gaussian distributed with zero mean and variance 1, ϕ_{s,n_R,n_T} is uniformly distributed in $[0,2\pi]$, T is the symbol period, and f_d is the Doppler frequency. The singular value decomposition (SVD) of $\mathbf{H}[n]$ will be denoted as $\mathbf{H}[n] = \mathbf{U}[n]\Sigma[n]\mathbf{V}^H[n]$, where $\mathbf{U}[n]$ and $\mathbf{V}[n]$ belong to $\mathcal{U}_{N_R}$ and $\mathcal{U}_{N_T}$, respectively, with $\mathcal{U}_n$ denoting the set of unitary $n \times n$ matrices, and $\Sigma[n]$ is a diagonal $N_R \times N_T$ matrix with the diagonal starting in the top left corner.

Generally, the precoder is restricted to be unitary, i.e., $\mathbf{F}^H[n]\mathbf{F}[n] = \mathbf{I}_{N_S}$. Within that class, it can for instance be shown that $\mathbf{F}[n] = [\mathbf{V}[n]]_{:,1:N_S}\mathbf{Q}$ is optimal with respect to the average mean square error (MSE) [3], where $\mathbf{Q}$ is an arbitrary matrix belonging to $\mathcal{U}_{N_S}$.

Note that this $\mathbf{Q}$ matrix does not change the average MSE but can be used to enforce a certain MSE profile across the different spatial streams.

For instance, selecting a $\mathbf{Q}$ matrix with constant modulus entries enforces an even MSE profile across the different spatial streams, thereby minimizing the uncoded BER [6, 7]. However, selecting $\mathbf{Q} = \mathbf{I}_{N_S}$ links a specific spatial stream to a specific spatial mode of the channel, leading to a better seperation of the spatial streams. Hence, we consider $\mathbf{F}[n] = [\mathbf{V}[n]]_{:,1:N_S}$ to be the optimal precoder in this work.

To estimate the symbols, we make use of a linear detector $\mathbf{G}[n]$, i.e., $\hat{\mathbf{s}}[n] = \mathbf{G}^H[n]\mathbf{y}[n]$. We will consider two types of linear detectors in this work. The first detector is the LMMSE detector, which is given by

$$\mathbf{G}_{LMMSE}[n] = \mathbf{H}[n]\mathbf{F}[n] \\ (\mathbf{F}^H[n]\mathbf{H}^H[n]\mathbf{H}[n]\mathbf{F}[n] + N_0\mathbf{I}_{N_S})^{-1}. \quad (3)$$

The second detector, referred to as the SVD detector, relies on the left singular vectors $\mathbf{U}[n]$ of the channel matrix $\mathbf{H}[n]$, and is given by

$$\mathbf{G}_{SVD}[n] = [\mathbf{U}[n]]_{:,1:N_S}. \quad (4)$$

Note that the SVD detector only works for PSK modulation whereas the LMMSE detector works for any type of modulation.

3. PRECODER QUANTIZATION

It is clear that the use of the optimal precoder requires feedback from the receiver to the transmitter. However, since the feedback link only has a limited rate, we have to quantize the information. We can either feed back the channel or the precoder. But since the precoder has less degrees of freedom than the channel, it is better to send the quantized precoder than the quantized channel to the transmitter, as illustrated in the simulations section.

Hence, we have to select the precoder $\mathbf{F}[n]$ from a finite codebook $\mathcal{F} = \{\mathbf{F}_i\}$. The code selection and codebook design criteria can for instance be based on average MSE [2]. However, as we discussed before, this does not necessarily lead to a one-to-one link between the spatial streams and the spatial modes. Hence, in this work, we will quantize the right singular vectors $\mathbf{V}[n]$ and we will pick the quantized right singular vectors $\mathbf{V}_Q[n]$ from a finite codebook $\mathcal{V} = \{\mathbf{V}_i\}$. The precoder is then selected as $\mathbf{F}[n] = [\mathbf{V}_Q[n]]_{:,1:N_S}$. Note that if we follow this approach, the codebook design and code selection are independent of the number of spatial streams N_S that are selected. This allows for the use of code extrapolation when different spatial streams are selected at different time instances, a procedure known as multi-mode precoding. Code extrapolation will be discussed later on.

3.1 Codebook Design and Code Selection

The first codebook design we consider here is the same as in [3]. We review this design here shortly. First of all, assuming $\mathbf{H}[n]$ has i.i.d. Rayleigh fading taps, $\mathbf{V}[n]$ is isotropically distributed in $\mathcal{U}_{N_T}$ [8, 9]. Within that space we have to look for an optimal set of regions $\{\mathcal{R}_i\}$ and matrices $\mathcal{V} = \{\mathbf{V}_i\}$, such that $\mathbf{V}_Q[n] = \mathbf{V}_i$ if $\mathbf{V}[n] \in \mathcal{R}_i$. We can find such an optimal set of regions and matrices in $\mathcal{U}_{N_T}$, by minimizing the average quantization distortion, measured by the mean square error between $\mathbf{V}[n]$ and its quantized version. In other words, we try to solve

$$\{\mathcal{R}_i, \mathbf{V}_i\} = \underset{\{\mathcal{R}_i,\mathbf{V}_i\}|\mathcal{R}_i\subset\mathcal{U}_{N_T},\mathbf{V}_i\in\mathcal{U}_{N_T}}{\arg\min} \\ \sum_i E(\|\mathbf{V}[n] - \mathbf{V}_i\|_F^2 | \mathbf{V}[n] \in \mathcal{R}_i) p(\mathbf{V}[n] \in \mathcal{R}_i). \quad (5)$$

The solution is not known in closed form, but can be identified iteratively by the generalized Lloyd algorithm. Based on this codebook design, the optimal $\mathbf{V}_Q[n]$ is then found as

$$\mathbf{V}_Q[n] = \underset{\mathbf{V}_i\in\mathcal{V}}{\arg\min} \|\mathbf{V}[n] - \mathbf{V}_i\|_F^2. \quad (6)$$

3.2 Modified Codebook Design and Code Selection

Note that the right singular vectors $\mathbf{V}[n]$ are actually only known up to a phase shift of their columns. We refer to this ambiguity of $\mathbf{V}[n]$ as the orientation ambiguity of $\mathbf{V}[n]$, and it is characterized by a right multiplication of $\mathbf{V}[n]$ with an orientation matrix $\Theta[n] \in \mathcal{DU}_{N_T}$, where $\mathcal{DU}_{N_T}$ is the set of diagonal unitary $N_T \times N_T$ matrices. The previous codebook design and code selection, however, do not take this orientation ambiguity of $\mathbf{V}[n]$ into account. We can therefore improve the previous approach by not simply using the mean square error between $\mathbf{V}[n]$ and its quantized version, but between the optimally oriented $\mathbf{V}[n]$ and its quantized version. Hence, we have to solve a problem of the form

$$\min_{\Theta[n]\in\mathcal{DU}_{N_T}} \|\mathbf{V}[n]\Theta[n]-\mathbf{V}_i\|_F^2. \tag{7}$$

The solution can easily be computed in closed form and is given by [10, pp. 431-432]

$$[\Theta_{opt}[n]]_{p,q} = \begin{cases} \dfrac{[\mathbf{V}^H[n]\mathbf{V}_i]_{p,p}}{|[\mathbf{V}^H[n]\mathbf{V}_i]_{p,p}|}, & \text{if } p=q, \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

As a result, we modify the codebook design into

$$\{\mathcal{R}_i,\mathbf{V}_i\} = \underset{\{\mathcal{R}_i,\mathbf{V}_i\}|\mathcal{R}_i\subset\mathcal{U}_{N_T},\mathbf{V}_i\in\mathcal{U}_{N_T}}{\arg\min} \sum_i E(\|\mathbf{V}[n]\Theta_{opt}[n]-\mathbf{V}_i\|_F^2|\mathbf{V}[n]\in\mathcal{R}_i)p(\mathbf{V}[n]\in\mathcal{R}_i). \tag{9}$$

Again, the solution is not known in closed form, but can be identified iteratively by the generalized Lloyd algorithm. Based on this codebook design, the optimal $\mathbf{V}_Q[n]$ is then found as

$$\mathbf{V}_Q[n] = \underset{\mathbf{V}_i\in\mathcal{V}}{\arg\min}\|\mathbf{V}[n]\Theta_{opt}[n]-\mathbf{V}_i\|_F^2. \tag{10}$$

4. PRECODER EXTRAPOLATION

By exploiting the coherence of the channel in the time domain, we can avoid feeding back the quantized right singular vectors $\mathbf{V}_Q[n]$ at every time instant n. More specifically, we will feed back the quantized right singular vectors every N time instances, i.e., $\mathbf{V}_Q[kN]$ is fed back for all k. We can then extrapolate the quantized right singular vectors at some time instant $kN+n$, for $n=1,2,\ldots,N-1$, using the last K known quantized right singular vectors $\{\mathbf{V}_Q[(k+l)N]\}_{l=-K+1}^0$. The method we use is the same as proposed in [4]. To estimate $\mathbf{V}_Q[kN+n]$, for $n=1,2,\ldots,N-1$, we first transform the set $\{\mathbf{V}_Q[(k+l)N]\}_{l=-K+1}^0$ to a new set $\{\mathbf{V}_{k,l}\}_{l=-K+1}^0$, correcting the orientation of all matrices such that they are as close as possible to $\mathbf{V}_Q[kN]$ in Frobenius norm, and rotating all matrices such that $\mathbf{V}_Q[kN]$ becomes the identity matrix:

$$\begin{aligned} \mathbf{V}_Q[kN] &\rightarrow \mathbf{V}_{k,0} = \mathbf{I} \\ \mathbf{V}_Q[(k-1)N] &\rightarrow \mathbf{V}_{k,-1} \\ &= \mathbf{V}_Q^H[kN]\mathbf{V}_Q[(k-1)N]\Theta_{k,-1} \\ &\vdots \\ \mathbf{V}_Q[(k-K+1)N] &\rightarrow \mathbf{V}_{k,-K+1} \\ &= \mathbf{V}_Q^H[kN]\mathbf{V}_Q[(k-K+1)N]\Theta_{k,-K+1}, \end{aligned}$$

where $\Theta_{k,l}$ is the orientation matrix that minimizes the Frobenius norm between $\mathbf{V}_Q[(k+l)N]\Theta_{k,l}$ and $\mathbf{V}_Q[kN]$. The solution is similar to (8) and is given by

$$[\Theta_{k,l}]_{p,q} = \begin{cases} \dfrac{[\mathbf{V}_Q^H[(k+l)N]\mathbf{V}_Q[kN]]_{p,p}}{\left|[\mathbf{V}_Q^H[(k+l)N]\mathbf{V}_Q[kN]]_{p,p}\right|}, & \text{if } p=q, \\ 0, & \text{otherwise} \end{cases}. \tag{11}$$

In a next step we transform the set of unitary matrices $\{\mathbf{V}_{k,l}\}_{l=-K+1}^0$ to a set of related skew-Hermitian matrices $\{\mathbf{S}_{k,l}\}_{l=-K+1}^0$, such that $\mathbf{V}_{k,l}=\text{expm}(\mathbf{S}_{k,l})$. Then we try to fit a Pth order polynomial through $\{\mathbf{S}_{k,l}\}_{l=-K+1}^0$, i.e., we solve

$$\min_{\{\mathbf{C}_{k,p}\}} \sum_{l=-K+1}^{0} \|\mathbf{S}_{k,l} - \sum_{p=0}^{P}\mathbf{C}_{k,p}((k+l)N)^p\|_F^2. \tag{12}$$

Hence, extrapolating $\{\mathbf{S}_{k,l}\}_{l=-K+1}^0$ to time instant $kN+n$, for $n=1,2,\ldots,N-1$, we get $\sum_{p=0}^P \mathbf{C}_{k,p}(kN+n)^p$, and thus extrapolating $\{\mathbf{V}_{k,l}\}_{l=-K+1}^0$ to time instant $kN+n$, for $n=1,2,\ldots,N-1$, we get $\text{expm}(\sum_{p=0}^P \mathbf{C}_{k,p}(kN+n)^p)$. Note that this extrapolated $\mathbf{S}_{k,l}$ is still skew-Hermitian, and thus the extrapolated $\mathbf{V}_{k,l}$ is still unitary. Finally, correcting for the fact that all matrices $\{\mathbf{V}_Q[(k+l)N]\}_{l=-K+1}^0$ were rotated such that $\mathbf{V}_Q[kN]$ becomes the identity matrix, an estimate for $\mathbf{V}_Q[kN+n]$ is obtained as

$$\hat{\mathbf{V}}_Q[kN+n] = \mathbf{V}_Q[kN]\text{expm}\Big(\sum_{p=0}^{P}\mathbf{C}_{k,p}(kN+n)^p\Big). \tag{13}$$

5. RECEIVER DESIGN

Note that the above precoder extrapolation was intended to reduce the amount of feedback that would be required. However, it generally coincides with a reduction of the training overhead, since we can also exploit the coherence of the channel in the time domain to reduce the amount of training required to estimate the channel. We could for instance send some pilot symbols and estimate the channel at regular time instances, after which we can extrapolate the channel for future time instances. For simplicity, we assume that the feedback and training frequencies are the same. Hence, we may assume that the channel is known at the receiver every N time instances, i.e., $\mathbf{H}[kN]$ is known for all k. We can then extrapolate the channel at some time instant $kN+n$, for $n=1,2,\ldots,N-1$, using the last K known channels $\{\mathbf{H}[(k+l)N]\}_{l=-K+1}^0$. Although Wiener channel extrapolation would be possible if the channel statistics are known, we opt for a simple polynomial channel extrapolation. More specifically, in order to estimate $\mathbf{H}[kN+n]$, for $n=1,2,\ldots,N-1$, we try to fit a Pth order polynomial through $\{\mathbf{H}[(k+l)N]\}_{l=-K+1}^0$, i.e., we solve

$$\min_{\{\mathbf{D}_{k,p}\}} \sum_{l=-K+1}^{0} \|\mathbf{H}[(k+l)N] - \sum_{p=0}^{P}\mathbf{D}_{k,p}((k+l)N)^p\|_F^2. \tag{14}$$

The channel $\mathbf{H}[kN+n]$, for $n=1,2,\ldots,N-1$, is then estimated as

$$\hat{\mathbf{H}}[kN+n] = \sum_{p=0}^{P}\mathbf{D}_{k,p}(kN+n)^p. \tag{15}$$

5.1 LMMSE Detector

If we want to adopt the LMMSE detector (3), we carry out the above channel extrapolation approach and mimic the precoder extrapolation at the receiver, in order to find an estimate of the LMMSE receiver at every time instant.

5.2 SVD Detector

If we want to adopt the SVD detector [11], we can actually choose between two approaches. In the first approach, we carry out the above channel extrapolation approach and compute the left singular vectors $\hat{\mathbf{U}}[n]$ for every channel estimate $\hat{\mathbf{H}}[n]$ (note that for $n = kN$ these estimates are assumed to be exact). We then use $[\hat{\mathbf{U}}[n]]_{:,1:N_S}$ or $[\hat{\mathbf{U}}[n]\Theta_{opt}[\lfloor n/N \rfloor N]]_{:,1:N_S}$ as detector, for the conventional or modified feedback approach, respectively. In the second approach, we compute $\mathbf{U}[kN]$ for all k at the receiver, and we carry out an extrapolation approach that is similar to the one for the precoder. We extrapolate between the $\mathbf{U}[kN]$s or the $\mathbf{U}[kN]\Theta_{opt}[kN]$s, for the conventional or modified feedback approach, respectively, and use the first N_S columns of those matrices as detectors. Note that the latter approach is less complex than the first approach, because the amount of SVDs that has to be computed is reduced by a factor of N.

6. SIMULATION RESULTS

In this section, we study the performance of the proposed methods on a 2×2 MIMO system ($N_R = N_T = 2$). The system is modeled as in Section 2 with a Doppler frequency of $f_d = 30$ Hz. For simplicity, we assume the number of spatial streams is fixed and equal to the number of spatial modes, i.e., $N_S = \min\{N_T, N_R\} = 2$, and we assume that QPSK modulation is used on every spatial stream. To illustrate the decoupling between the two spatial streams, we will plot the symbol-error-rate (SER) of the two spatial streams separately. The larger the distance between the two SER curves, the larger the decoupling. Note that all performances are computed based on 10^4 channel realizations.

First, we consider no extrapolation, and a feedback link that is instantaneous, error-free, and limited to 8 bits per symbol period. Figs. 2 and 3 show the performance of the LMMSE and SVD detector, respectively, assuming perfect channel knowledge at the receiver. In both figures, we compare the conventional precoder quantization approach of Section 3.1 with the modified precoder quantization approach of Section 3.2. Also shown is the performance of channel quantization, where the sign of the real and imaginary part of every channel tap is fed back (optimal for the considered channel model). Clearly, precoder quantization realizes a larger decoupling between the spatial streams. In addition, the modified precoder quantization approach outperforms the conventional one.

Next, we include extrapolation in our simulations. We assume a feedback and training frequency of once every $NT = 10^{-3}$ s. Hence, the channel is assumed perfectly known once every $NT = 10^{-3}$ s, at which point 8 bits of information are fed back to the transmitter. Note that we do not give specific values for N and T, since the performance is only determined by their product NT and its relationship to the Doppler frequency f_d. We only consider the modified precoder quantization approach of Section 3.2 and compare the extrapolated LMMSE detector with the two extrapolation schemes for the SVD detector (see Section 5 for more details). In all extrapolation schemes, we consider a memory depth of $K = 3$ and a polynomial degree of $P = 2$. Clearly, the LMMSE detector performs the best. In addition, the SVD detector based on channel extrapolation performs worse than the SVD detector based on left singular vector extrapolation.

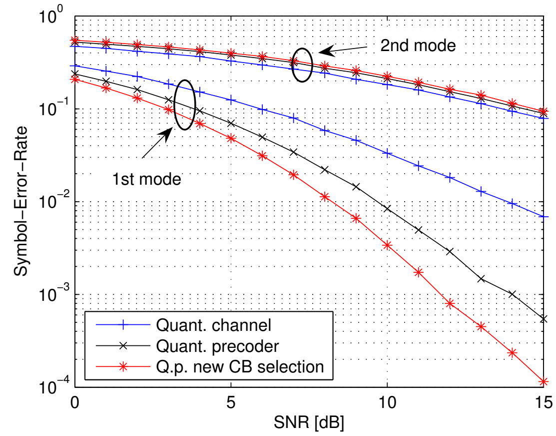

Figure 2: LMMSE detector, 8 bit feedback every time instant, $f_d = 30$ Hz.

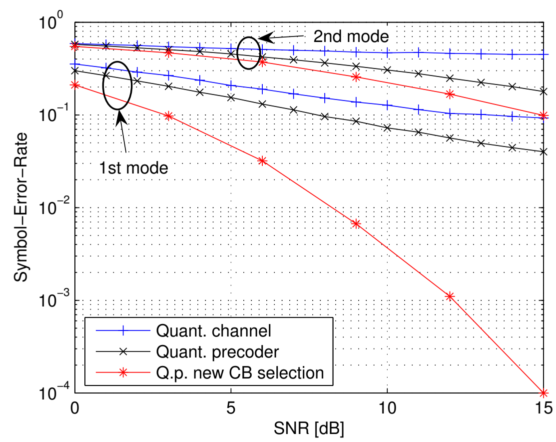

Figure 3: $M_T = 2$, $M_R = 2$, SVD detector, 8 bit feedback every time instant, $f_d = 30$ Hz.

7. CONCLUSIONS

This paper evaluates the performance of spatial multiplexing with linear precoding, exploiting a low-rate feedback link. As a performance metric we have considered the ability of the system to link the different spatial streams as tight as possible to the different spatial modes, leading to an increased gap between the SER curves of the different spatial streams, and as a result, to an increased average performance if only a few of the spatial streams are used. We have observed that quantizing and feeding back the precoder outperforms quantized channel feedback. Furthermore, we have proposed a modified precoder quantization approach that outperforms the conventional one. Both the LMMSE and SVD detector are investigated, assuming perfect channel knowledge at

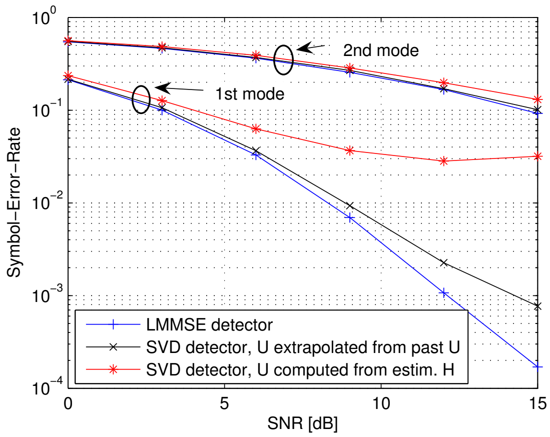

Figure 4: Results for extrapolation, 8 bit feedback every $NT = 10^{-3}$ s, $f_d = 30$ Hz, $K = 3$, and $P = 2$

every time instant. The LMMSE detector is shown to perform slightly better than the SVD detector. We have also discussed precoder extrapolation, when the precoder is only fed back at a limited number of time instances, as well as a related detector extrapolation scheme for the LMMSE and SVD detector, when the channel is only known at some specific time instances. Simulation results using these extrapolation ideas have revealed that the LMMSE detector works better or slightly better than the SVD detector, depending on the extrapolation method that has been chosen for the SVD detector.

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