14th European Signal Processing Conference (EUSIPCO 2006), Florence, Italy, September 4-8, 2006, copyright by EURASIP

FINGER SELECTION FOR UWB RAKE RECEIVERS

Sinan Gezici, Mung Chiang†, Hisashi Kobayashi† and H. Vincent Poor†*

∗ Mitsubishi Electric Research Laboratories
201 Broadway, Cambridge, MA 02139
phone:+(1)-617-621-7500
fax:+(1)-617-621-7550
gezici@ieee.org

† Department of Electrical Engineering
Princeton University, Princeton, NJ 08544
phone:+(1)-609-258-5071
fax:+(1)-609-258-2158
{chiangm,hisashi,poor}@princeton.edu

ABSTRACT

The problem of choosing the multipath components to be employed at a selective Rake receiver, the finger selection problem, is considered for an impulse radio ultra-wideband system. First, the finger selection problem for MRC-Rake receivers is considered and the suboptimality of the conventional scheme is shown by formulating the optimal solution according to the SINR maximization criterion. Due to the complexity of the optimal solution, a convex formulation is obtained by means of integer relaxation techniques. Then, the finger selection for MMSE-Rake receivers is studied and optimal and suboptimal schemes are presented. Finally, a genetic algorithm based solution is proposed for the finger selection problem, which works for various multipath combining schemes. Simulation studies are presented to compare the performance of different algorithms.

Index Terms— Ultra-wideband (UWB), impulse radio (IR), Rake receiver, convex optimization, integer programming, genetic algorithm (GA).

1. INTRODUCTION

Recently impulse radio (IR) ultra wideband (UWB) systems ([1]-[4]) have drawn considerable attention due to their suitability for short-range high-speed data transmission and precise location estimation. In an IR-UWB system, very short pulses with a low duty cycle are transmitted, and each information symbol is represented by positions or polarities of a number of pulses. Each pulse resides in an interval called "frame", and the positions of pulses within frames are determined by time-hopping (TH) sequences specific to each user, which prevents catastrophic collisions among pulses of different users [1].

Commonly, users in an IR-UWB system employ Rake receivers to collect energy from different multipath components. A Rake receiver combining all the paths of the incoming signal is called an *all-Rake* (*ARake*) receiver. Since a UWB signal has a very wide bandwidth, the number of resolvable multipath components is usually very large. Hence, an ARake receiver is not implemented in practice due to its complexity. However, it serves as a benchmark for the performance of more practical Rake receivers. A feasible implementation of multipath diversity combining can be obtained by a *selective-Rake* (*SRake*) receiver, which combines the M best, out of L, multipath components [5]. Those M best components are determined by a finger selection algorithm.

For a maximal ratio combining (MRC) Rake receiver, the paths with highest signal-to-noise ratios (SNRs) are selected, which is an optimal scheme for a linear modulation, in the absence of interference, such as multi-access interference (MAI), inter-symbol interference (ISI) or narrowband interference (NBI). This finger selection method is called hybrid-selection/MRC (H-S/MRC) scheme in [6] and generalized selection combining (GSC) in [7], [8]. Although the strongest multipath components optimize the system performance for linearly modulated IR-UWB systems, the optimal finger selection can be more complicated for pulse-position modulated (PPM) IR-UWB systems, as investigated in [9] and [10].

Although finger selection techniques for MRC-Rake receivers have been considered, finger selection for minimum mean square error (MMSE)-Rake has not been studied thoroughly before. As a straightforward solution, one can consider a finger selection algorithm that chooses the paths with the highest signal-to-interference-plus-noise ratios (SINRs). However, this scheme is not necessarily optimal since it ignores the correlation of the noise terms at different multipath components. A finger selection algorithm for generalized Rake (GRake) receivers is proposed in [11]. However, this structure is usually not suitable for UWB systems, since it considers a system in which the number of fingers is larger than the number of multipath components, and the extra fingers are used to obtain samples from the noise process. In other words, the finger selection in [11] focuses on the selection of "noise fingers".

In this paper, we provide a complete picture of the finger selection problem for MRC- and MMSE-Rake receivers[1]. In both cases, we present optimal finger selection algorithms that provide upper bounds on the receiver performance. Then, we consider suboptimal algorithms with lower complexity, which are obtained by means of integer relaxation techniques and Taylor approximations. Moreover, we present a genetic algorithm (GA) based approach to the finger selection problem, which can be used independently from the combining scheme; i.e., it is valid for MRC-Rake, MMSE-Rake, etc.

The remainder of this paper is organized as follows. Section 2 describes the transmitted and received signal models in a multiuser frequency-selective environment. The finger selection problem for the MRC-Rake and the MMSE-Rake is investigated in Section 3 and Section 4, respectively. Section 5 presents the GA based finger selection algorithm, which can be applied to both the MRC-Rake and the MMSE-Rake receivers. Simulation results are presented in Section 6, and concluding remarks are made in the last section.

2. SIGNAL MODEL

We consider a synchronous, binary phase shift keyed IR-UWB system with K users, in which the transmitted signal

[0] This research is supported in part by the National Science Foundation under grants ANI-03-38807, CNS-0417603, and CCR-0440443, and in part by the New Jersey Center for Wireless Telecommunications.

[1] Note that the authors have considered the finger selection problem for MMSE-Rake receivers before in [20] and [17]; however, the results of those papers are briefly presented here again in order to provide a complete overview of the finger selection problem.

from user k is represented by:

$$s_{\rm tx}^{(k)}(t) = \sqrt{\frac{E_k}{N_f}} \sum_{j=-\infty}^{\infty} d_j^{(k)}\, b_{\lfloor j/N_f \rfloor}^{(k)} p_{\rm tx}(t - jT_f - c_j^{(k)} T_c), \quad (1)$$

where $p_{\rm tx}(t)$ is the transmitted UWB pulse, E_k is the bit energy of user k, T_f is the "frame" time, N_f is the number of pulses representing one information symbol, and $b_{\lfloor j/N_f \rfloor}^{(k)} \in \{+1, -1\}$ is the binary information symbol transmitted by user k. In order to allow the channel to be shared by many users and avoid catastrophic collisions, a TH sequence $\{c_j^{(k)}\}$, where $c_j^{(k)} \in \{0, 1, ..., N_c - 1\}$, is assigned to each user. This TH sequence provides an additional time shift of $c_j^{(k)} T_c$ seconds to the jth pulse of the kth user where T_c is the chip interval and is chosen to satisfy $T_c \leq T_f / N_c$ in order to prevent the pulses from overlapping. We assume $T_f = N_c T_c$ without loss of generality. The random polarity codes $d_j^{(k)}$ are binary random variables taking values ± 1 with equal probability [12]-[14].

Consider the discrete presentation of the channel, $\boldsymbol{\alpha}^{(k)} = [\alpha_1^{(k)} \cdots \alpha_L^{(k)}]$ for user k, where L is assumed to be the number of multipath components for each user, and T_c is the multipath resolution. Note that this channel model can model any channel of the form $\sum_{l=1}^{\hat{L}} \hat{\alpha}_l^{(k)} \delta(t - \hat{\tau}_l^{(k)})$ if the channel is bandlimited to $1/T_c$. Then, the received signal can be expressed as

$$r(t) = \sum_{k=1}^{K} \sqrt{\frac{E_k}{N_f}} \sum_{j=-\infty}^{\infty} \sum_{l=1}^{L} \alpha_l^{(k)} d_j^{(k)}\, b_{\lfloor j/N_f \rfloor}^{(k)} \times p_{\rm rx}(t - jT_f - c_j^{(k)} T_c - (l-1)T_c) + \sigma_n n(t), \quad (2)$$

where $p_{\rm rx}(t)$ is the received unit-energy UWB pulse, which is usually modelled as the derivative of $p_{\rm tx}(t)$ due to the effects of the receive antenna, and $n(t)$ is zero mean white Gaussian noise with unit spectral density.

For the simplicity of the analysis, we assume that the TH sequence is constrained to the set $\{0, 1, \ldots, N_T - 1\}$, where $N_T \leq N_c - L$, so that there is no inter-frame interference (IFI). However, the finger selection algorithms considered in this paper are valid for scenarios with IFI as well [15].

Due to the high resolution of UWB signals, chip-rate and frame-rate sampling are not very practical for such systems. In order to have a lower sampling rate, the received signal can be correlated with symbol-length template signals that enable symbol-rate sampling of the correlator output [16]. The template signal for the lth path of the incoming signal can be expressed as

$$s_{{\rm temp},l}^{(1)}(t) = \sum_{j=iN_f}^{(i+1)N_f - 1} d_j^{(1)}\, p_{\rm rx}(t - jT_f - c_j^{(1)} T_c - (l-1)T_c), \quad (3)$$

for the ith information symbol, where we consider user 1 as the desired user, without loss of generality. In other words, by using a correlator for each multipath component that we want to combine, we can have symbol-rate sampling at each branch, as shown in Figure 1.

Note that the use of such template signals results in equal gain combining (EGC) of different frame components. This

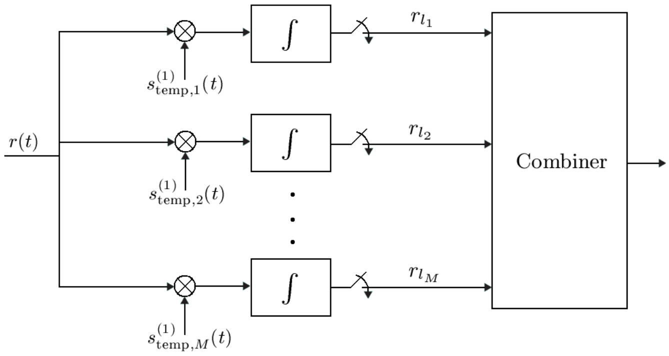

Fig. 1. The receiver structure. There are M multipath components that are used by the combiner to obtain a decision variable.

may not be optimal under some conditions (see [15] for (sub)optimal schemes). However, it is very practical since it facilitates symbol-rate sampling. Since we consider a system that employs template signals of the form (3), i.e. EGC of frame components, it is sufficient to consider the problem of selection of the optimal paths for just one frame. Hence, we assume $N_f = 1$ without loss of generality. Note, however, that we can directly apply the finger selection algorithms in this paper for systems with $N_f > 1$ as well.

Let $\mathcal{L} = \{l_1, \ldots, l_M\}$ denote the set of multipath components that the receiver collects (Figure 1). At each branch, the signal is effectively passed through a matched filter (MF) matched to the related template signal in (3) and sampled once for each symbol. Then, the discrete signal for the lth path can be expressed, for the ith information symbol, as[2]

$$r_l = \mathbf{s}_l^T \mathbf{A} \mathbf{b}_i + n_l, \quad (4)$$

for $l = l_1, \ldots, l_M$, where $\mathbf{A} = {\rm diag}\{\sqrt{E_1}, \ldots, \sqrt{E_K}\}$, $\mathbf{b}_i = [b_i^{(1)} \cdots b_i^{(K)}]^T$ and $n_l \sim \mathcal{N}(0, \sigma_n^2)$. $\mathbf{s}_l$ is a $K \times 1$ vector, which can be expressed as a sum of the desired signal part (SP) and multiple-access interference (MAI) terms:

$$\mathbf{s}_l = \mathbf{s}_l^{\rm (SP)} + \mathbf{s}_l^{\rm (MAI)}, \quad (5)$$

where the kth elements can be expressed as

$$\left[\mathbf{s}_l^{\rm (SP)}\right]_k = \begin{cases} \alpha_l^{(1)}, & k = 1 \\ 0, & k = 2, \ldots, K \end{cases} \quad (6)$$

and

$$\left[\mathbf{s}_l^{\rm (MAI)}\right]_k = \begin{cases} 0, & k = 1 \\ d_1^{(1)} d_1^{(k)} \sum_{m=1}^{L} \alpha_m^{(k)} I_{l,m}^{(k)}, & k = 2, \ldots, K \end{cases}, \quad (7)$$

with $I_{l,m}^{(k)}$ being the indicator function that is equal to 1 if the mth path of user k collides with the lth path of user 1, and 0 otherwise.

Out of L incoming multipath components, the received signals from M of them can be expressed using a selection matrix $\mathbf{X}$ as follows:

$$\mathbf{r} = \mathbf{X}\mathbf{S}\mathbf{A}\mathbf{b}_i + \mathbf{X}\mathbf{n}, \quad (8)$$

[2]Note that the dependence of r_l on the index of the information symbol, i, is not shown explicitly.

where $\mathbf{n}$ is the vector of background noise components $\mathbf{n} = [n_1 \cdots n_L]^T$, and $\mathbf{S}$ is the signature matrix given by $\mathbf{S} = [\mathbf{s}_1 \cdots \mathbf{s}_L]^T$, with $\mathbf{s}_l$ as in (5). The $M \times L$ selection matrix $\mathbf{X}$ is defined as follows: M of the columns of $\mathbf{X}$ are the unit vectors $\mathbf{e}_1, \ldots, \mathbf{e}_M$ ($\mathbf{e}_i$ having a 1 at its ith position and zero elements for all other entries), and the other columns are all zero vectors. The column indices of the unit vectors determine the subset of the multipath components that are selected. For example, for $L = 4$ and $M = 2$, $\mathbf{X} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ chooses the second and third multipath components.

Using (5)-(7), (8) can be expressed as

$$\mathbf{r} = b_i^{(1)} \sqrt{E_1} \mathbf{X} \boldsymbol{\alpha}^{(1)} + \mathbf{X}\mathbf{S}^{(\mathrm{MAI})}\mathbf{A}\mathbf{b}_i + \mathbf{X}\mathbf{n}, \qquad (9)$$

where $\mathbf{S}^{(\mathrm{MAI})}$ is the MAI part of the signature matrix $\mathbf{S}$.

Let $\tilde{\mathbf{n}} = \mathbf{S}^{(\mathrm{MAI})}\mathbf{A}\mathbf{b}_i + \mathbf{n}$. Then, the received signal can be expressed as the summation of the signal and the total noise terms:

$$\mathbf{r} = b_i^{(1)} \sqrt{E_1} \mathbf{X} \boldsymbol{\alpha}^{(1)} + \mathbf{X}\tilde{\mathbf{n}}. \qquad (10)$$

Note that this signal model can represent even more general scenarios assuming that all the noise components are collected in $\tilde{\mathbf{n}}$. For example, we can include the effects of the NBI as well. As long as the correlation matrix for $\tilde{\mathbf{n}}$ is known, the algorithms in this paper are valid in general.

We consider a linear receiver structure that combines the elements of $\mathbf{r}$ linearly,

$$y = \boldsymbol{\theta}^T \mathbf{r}, \qquad (11)$$

where $\boldsymbol{\theta}$ is the weighting vector, and obtain the bit estimate as the sign of the decision variable, i.e. $\hat{b}_i = \mathrm{sign}\{\boldsymbol{\theta}^T \mathbf{r}\}$.

For the linear receiver structure defined above, the SINR of the output y in (11) can be calculated as

$$\mathrm{SINR} = \frac{E_1 |\boldsymbol{\theta}^T \mathbf{X} \boldsymbol{\alpha}^{(1)}|^2}{\boldsymbol{\theta}^T \mathbf{X}\mathbf{R}\mathbf{X}^T \boldsymbol{\theta}}, \qquad (12)$$

where $\mathbf{R} = \mathrm{E}\{\tilde{\mathbf{n}}\tilde{\mathbf{n}}^T\}$ is the correlation matrix of the total noise vector.

3. FINGER SELECTION FOR MRC-RAKE

For the MRC scheme, the received signal samples selected by the finger selection algorithm are weighed in proportion to their channel gains such that the weight vector in (11) is given by[3]

$$\boldsymbol{\theta} = \mathbf{X}\boldsymbol{\alpha}^{(1)}. \qquad (13)$$

¿From (12), the SINR for the MRC-Rake can be obtained after some manipulation as

$$\mathrm{SINR}_{\mathrm{MRC}} = \frac{E_1 \left(\mathbf{x}^T \mathbf{q}\right)^2}{\mathbf{x}^T \tilde{\mathbf{R}} \mathbf{x}}, \qquad (14)$$

where $\mathbf{x} = \mathrm{diag}\{\mathbf{X}^T\mathbf{X}\}$,

$$\mathbf{q} = \left[\left(\alpha_1^{(1)}\right)^2 \cdots \left(\alpha_L^{(1)}\right)^2 \right]^T, \quad \text{and} \qquad (15)$$

$$\tilde{\mathbf{R}} = \mathrm{diag}\{\alpha_1^{(1)}, \ldots, \alpha_L^{(1)}\} \mathbf{R} \, \mathrm{diag}\{\alpha_1^{(1)}, \ldots, \alpha_L^{(1)}\}, \qquad (16)$$

[3]Note that we consider real channel coefficients assuming a carrierless system. In general, $\boldsymbol{\theta} = \left(\mathbf{X}\boldsymbol{\alpha}^{(1)}\right)^*$, where $*$ denotes the complex conjugate operation.

with $\mathrm{diag}\{a_1, \ldots, a_L\}$ denoting an $L \times L$ diagonal matrix (a_i representing the ith diagonal element).

Due to the structure of the selection matrix $\mathbf{X}$, the $L \times 1$ vector $\mathbf{x}$ in (14) has the structure that $[\mathbf{x}]_i = 1$ if the ith path is selected, and $[\mathbf{x}]_i = 0$ otherwise; and $\sum_{i=1}^{L} [\mathbf{x}]_i = M$. In other words, $\mathbf{x}$ can be considered as an "assignment vector", which selects the multipath components corresponding to indices of its non-zero elements [17].

3.1. Conventional Scheme

The conventional way of choosing the multipath components to be used at the MRC-Rake receiver is to select the signal paths with the largest channel gains [6]-[8]. This scheme is called H-S/MRC [6], or GSC [7] in the literature. In this case, the assignment vector $\mathbf{x}$ is given by

$$[\mathbf{x}]_i = \begin{cases} 1, & \text{if } \left|\alpha_i^{(1)}\right| \geq \left|\alpha_{(M)}^{(1)}\right| \\ 0, & \text{otherwise} \end{cases}, \qquad (17)$$

where $\alpha_{(1)}^{(1)}, \ldots, \alpha_{(L)}^{(1)}$ denotes the ordered set of channel coefficients such that $\left|\alpha_{(1)}^{(1)}\right| > \cdots > \left|\alpha_{(L)}^{(1)}\right|$. Then, the SINR expression in (14) can be expressed as

$$\mathrm{SINR}_{\mathrm{MRC}} = \frac{E_1 \left(\sum_{l \in \mathcal{S}} \left|\alpha_l^{(1)}\right|^2\right)^2}{\sum_{i,j \in \mathcal{S}} \alpha_i^{(1)} [\mathbf{R}]_{ij} \, \alpha_j^{(1)}}, \qquad (18)$$

where $\mathcal{S}$ is the set of indices of the strongest multipath components.

Note that the conventional scheme is optimal if $\mathbf{R} = \sigma^2 \mathbf{I}$; i.e., if the noise components are i.i.d[4]. In that case, the SINR is given by

$$\mathrm{SINR} = \frac{E_1}{\sigma^2} \sum_{l \in \mathcal{S}} \left|\alpha_l^{(1)}\right|^2. \qquad (19)$$

In some cases, the noise components are dependent and $\mathbf{R}$ is not diagonal. For example, the MAI, IFI or NBI can cause colored noise, hence dependent noise components. In such cases, the conventional algorithm is not optimal, since it does not the maximize the SINR expression in (18) in general. In the next subsection, we investigate the overall optimal scheme.

3.2. Optimal Solution

In order to evaluate the performance of the conventional finger selection algorithm for MRC-Rake receivers, we consider the maximization of the SINR expression in (14) assuming that the correlation matrix $\mathbf{R}$ is known. This solution not only provides an upper bound for the performance of any finger selection algorithm that operates in the presence of unknown colored noise, but also helps quantify the performance loss due to the conventional finger selection algorithm.

¿From (14), the optimal finger selection algorithm for MRC-Rake can be expressed as the solution of the follow-

[4]More generally, the MRC scheme weights the selected multipath components in proportion to their SNR values. In that case, the MRC scheme with conventional finger selection is optimal for independent noise components.

ing optimization problem:

$$\arg\max_{\mathbf{x}} \quad \frac{\mathbf{x}^T\mathbf{q}\mathbf{q}^T\mathbf{x}}{\mathbf{x}^T\tilde{\mathbf{R}}\mathbf{x}} \tag{20}$$

$$\text{subject to} \quad \mathbf{x}^T\mathbf{1} = M \tag{21}$$

$$\mathbf{x} \in \{0,1\}^L \tag{22}$$

Note that this is a combinatorial problem, and the optimal solution requires an exhaustive search. Therefore, we consider a suboptimal solution in the next subsection.

3.3. Suboptimal Solution

Since the optimization problem in (20) is NP-hard, we aim to simplify the problem in such a way to obtain a convex optimization problem. Because over the past decade, both powerful theory and efficient numerical algorithms have been developed for nonlinear convex optimization, it is now recognized that the watershed between "easy" and "difficult" optimization problems is not linearity but convexity. For example, the interior-point algorithms for nonlinear convex optimization are highly efficient, both in worst case complexity (provably polynomial time) and in practice (very fast even for a large number variables and constraints) [18].

As a first step to obtain a convex problem, we first relax the integer constraint in (22). In other words, instead of optimizing the objective function in (20) over all possible binary values, we relax the constraint to a convex region, such as a hypercube, and optimize the objective function considering this convex constraint. Assuming that we relax (22) to a hypercube, we can replace (22) by the following constraint:

$$\mathbf{0} \preceq \mathbf{x} \preceq \mathbf{1}, \tag{23}$$

where $\mathbf{u} \succeq \mathbf{v}$ means that $[\mathbf{u}]_1 \geq [\mathbf{v}]_1, \ldots, [\mathbf{u}]_L \geq [\mathbf{v}]_L$.

After the relaxation of the integer constraint, we still need some manipulation in order to formulate the problem as a convex optimization problem. To that end, we define new variables $\mathbf{w} = \mathbf{x}/(\mathbf{q}^T\mathbf{x})$ and $z = (\mathbf{q}^T\mathbf{x})^{-1}$. Then, the optimization problem in (20)-(22) can be expressed as

$$\arg\min_{\mathbf{w},z} \quad \mathbf{w}^T\tilde{\mathbf{R}}\mathbf{w} \tag{24}$$

$$\text{subject to} \quad \mathbf{w}^T\mathbf{1} - Mz = 0 \tag{25}$$

$$z \geq 0 \tag{26}$$

$$\mathbf{w} \succeq \mathbf{0} \tag{27}$$

$$\mathbf{w} - z\mathbf{1} \preceq \mathbf{0} \tag{28}$$

$$\mathbf{q}^T\mathbf{w} = 1 \tag{29}$$

Note that the optimization problem given by (24)-(29) is convex in the new variables, since the objective function is quadratic and the constraints are linear. In other words, this is a linearly constrained quadratic programming (LCQP), which can be solved by interior-point algorithms [18] for the optimal $[\mathbf{w}^T z]^T$. Then, the optimal $\mathbf{x}$ is obtained by the relation $\mathbf{x} = (z)^{-1}\mathbf{w}$, and the indices of the largest elements of the optimal $\mathbf{x}$ vector determines the multipath components to be used at the MRC-Rake receiver.

4. FINGER SELECTION FOR MMSE-RAKE

For an MMSE-Rake receiver, the received signal samples selected by the finger selection algorithm are combined in such a way to minimize the MSE between the information bit $b_i^{(1)}$ and the decision variable in (11). In this case, the weighting vector $\boldsymbol{\theta}$ in (11) is given by [19]

$$\boldsymbol{\theta} = \hat{\mathbf{R}}^{-1}\mathbf{X}\boldsymbol{\alpha}^{(1)}, \tag{30}$$

where $\hat{\mathbf{R}} = \mathbf{X}\mathrm{E}\{\tilde{\mathbf{n}}\tilde{\mathbf{n}}^T\}\mathbf{X}^T$ is the correlation matrix of noise, with $\tilde{\mathbf{n}} = \mathbf{S}^{(\mathrm{MAI})}\mathbf{A}\mathbf{b}_i + \mathbf{n}$. Note that, for equiprobable binary symbols, $\hat{\mathbf{R}}$ can be expressed as

$$\hat{\mathbf{R}} = \mathbf{X}\mathbf{S}^{(\mathrm{MAI})}\mathbf{A}^2(\mathbf{S}^{(\mathrm{MAI})})^T\mathbf{X}^T + \sigma_n^2\mathbf{I}. \tag{31}$$

Hence, the SINR of the system can be obtained from (10), (11), (30) and (31) as

$$\mathrm{SINR}_{\mathrm{MMSE}}(\mathbf{X}) = \frac{E_1}{\sigma_n^2}\left(\boldsymbol{\alpha}^{(1)}\right)^T\mathbf{X}^T \times \left(\mathbf{I} + \frac{1}{\sigma_n^2}\mathbf{X}\mathbf{S}^{(\mathrm{MAI})}\mathbf{A}^2(\mathbf{S}^{(\mathrm{MAI})})^T\mathbf{X}^T\right)^{-1}\mathbf{X}\boldsymbol{\alpha}^{(1)}. \tag{32}$$

4.1. Conventional Scheme

Instead of minimizing the SINR expression in (32), the "conventional" finger selection algorithm for MMSE-Rake chooses the M paths with largest individual SINRs, where the SINR for the lth path can be expressed as

$$\mathrm{SINR}_l = \frac{E_1(\alpha_l^{(1)})^2}{(\mathbf{s}_l^{(\mathrm{MAI})})^T\mathbf{A}^2\mathbf{s}_l^{(\mathrm{MAI})} + \sigma_n^2}, \tag{33}$$

for $l = 1, \ldots, L$.

This algorithm is not optimal because it ignores the correlation of the noise components of different paths. Therefore, it does not always maximize the overall SINR of the system given in (32). For example, the contribution of two highly correlated strong paths to the overall SINR might be less than the contribution of one strong and one relatively weaker, but uncorrelated, paths. The correlation between the multipath components is the result of the MAI from the other users in the system[5].

4.2. Optimal Scheme

The optimal finger selection problem can be formulated as

$$\arg\max_{\mathbf{X}} \mathrm{SINR}_{\mathrm{MMSE}}(\mathbf{X}), \tag{34}$$

where $\mathrm{SINR}_{\mathrm{MMSE}}(\mathbf{X})$ is given by (32) and $\mathbf{X}$ has the previously defined structure.

Note that the objective function to be maximized is not concave and the optimization variable $\mathbf{X}$ takes binary values, with the previously defined structure. In other words, two major difficulties arise in solving (34) globally: nonconvex optimization and integer constraints. Either makes the problem NP-hard. Therefore, it is an intractable optimization problem in this general form.

4.3. Suboptimal Schemes

Since the optimal solution in (34) is quite difficult, we first consider an approximation of the objective function in (12). When the eigenvalues of $\frac{1}{\sigma_n^2}\mathbf{X}\mathbf{S}^{(\mathrm{MAI})}\mathbf{A}^2(\mathbf{S}^{(\mathrm{MAI})})^T\mathbf{X}^T$ are

[5] More generally, the correlations can result from any colored noise process.

considerably smaller than 1, which occurs when the MAI is not very strong compared to the background noise, we can approximate the SINR expression in (12) as follows[6]:

$$\mathrm{SINR}_{\mathrm{MMSE}}(\mathbf{X}) \approx \frac{E_1}{\sigma_n^2}(\boldsymbol{\alpha}^{(1)})^T\mathbf{X}^T \times \left(\mathbf{I} - \frac{1}{\sigma_n^2}\mathbf{X}\mathbf{S}^{(\mathrm{MAI})}\mathbf{A}^2(\mathbf{S}^{(\mathrm{MAI})})^T\mathbf{X}^T\right)\mathbf{X}\boldsymbol{\alpha}^{(1)}. \quad (35)$$

Then, the SINR expression can be expressed, after some manipulation, as follows [20]:

$$\mathrm{SINR}_{\mathrm{MMSE}}(\mathbf{x}) = \frac{E_1}{\sigma_n^2}\left(\mathbf{q}^T\mathbf{x} - \frac{1}{\sigma_n^2}\mathbf{x}^T\mathbf{P}\mathbf{x}\right), \quad (36)$$

where $\mathbf{x}$ is the $L \times 1$ assignment vector, $\mathbf{q}$ is as in (15), and

$$\mathbf{P} = \mathrm{diag}\{\alpha_1^{(1)} \cdots \alpha_L^{(1)}\}\mathbf{S}^{(\mathrm{MAI})}\mathbf{A}^2(\mathbf{S}^{(\mathrm{MAI})})^T\mathrm{diag}\{\alpha_1^{(1)} \cdots \alpha_L^{(1)}\} \quad (37)$$

Then, we can formulate the finger selection problem as follows:

$$\begin{aligned} \text{minimize} \quad & \frac{1}{\sigma_n^2}\mathbf{x}^T\mathbf{P}\mathbf{x} - \mathbf{x}^T\mathbf{q} \\ \text{subject to} \quad & \mathbf{x}^T\mathbf{1} = M, \\ & \mathbf{x} \in \{0,1\}^L. \end{aligned} \quad (38)$$

Note that the objective function is convex since $\mathbf{P}$ is positive definite, and that the first constraint is linear. However, the integer constraint increases the complexity of the problem. Similar to the previous section, we consider integer relaxation techniques to obtain convex problems. In this case, we consider two different relaxation techniques.

4.3.1. Case-1: Relaxation to Sphere

Consider the relaxation of the integer constraint in (38) to a sphere that passes through all possible integer values. Then, the relaxed problem becomes

$$\begin{aligned} \text{minimize} \quad & \frac{1}{\sigma_n^2}\mathbf{x}^T\mathbf{P}\mathbf{x} - \mathbf{x}^T\mathbf{q} \\ \text{subject to} \quad & \mathbf{x}^T\mathbf{1} = M, \\ & (2\mathbf{x}-\mathbf{1})^T(2\mathbf{x}-\mathbf{1}) \le L. \end{aligned} \quad (39)$$

Note that the problem becomes a convex quadratically constrained quadratic programming (QCQP) [18]. Hence it can be solved for global optimality using interior-point algorithms in polynomial time.

4.3.2. Case-2: Relaxation to Hypercube

As an alternative approach, we can relax the integer constraint in (38) to a hypercube constraint and get

$$\begin{aligned} \text{minimize} \quad & \frac{1}{\sigma_n^2}\mathbf{x}^T\mathbf{P}\mathbf{x} - \mathbf{x}^T\mathbf{q} \\ \text{subject to} \quad & \mathbf{x}^T\mathbf{1} = M, \\ & \mathbf{0} \preceq \mathbf{x} \preceq \mathbf{1}. \end{aligned} \quad (40)$$

Note that the problem is now an LCQP, and can be solved by interior-point algorithms [18] for the optimal $\mathbf{x}$.

After solving the approximate problem (39) or (40), the finger locations are obtained by the indices of the M largest elements of the optimal $\mathbf{x}$ vector.

5. FINGER SELECTION VIA GENETIC ALGORITHMS

Note that the low-complexity finger selection schemes in Section 4.3 for the MMSE-Rake receiver are suboptimal due to both the integer relaxation and objective function approximation steps. Similarly, the scheme in Section 3.3 for the MRC-Rake is suboptimal due to the integer constraint relaxation.

Another approach to Rake finger selection is to employ binary GAs to solve the finger selection problem without using any integer relaxation or objective function approximation [20].

The GA is an iterative technique for searching for the global optimum of an objective function [21]. It starts with a set of binary vectors[7], and iteratively searches for the optimal value by updating the set of vectors in each iteration.

For the finger selection problem, we consider the exact SINR expression given by (14) for the MRC-Rake or by (32) for the MMSE-Rake. Then, we try to maximize the exact SINR expression over the assignment vector $\mathbf{x}$. The GA-based finger selection algorithm can be summarized as follows [20]:

- Generate N_{ipop} different assignment vectors randomly.
- Select N_{pop} of them with the largest SINR values.
- **Pairing:** Pair N_{good} of the finger assignments according to the weighted random scheme [21], in which each assignment vector is chosen with a probability that is proportional to its SINR value.
- **Mating:** Generate two new assignments from each pair (as described below).
- **Mutation:** Change the finger locations of some assignments randomly except for the best assignment (one 1 and one 0 are randomly chosen and flipped from a selected assignment vector).
- Choose the assignment vector with the highest SINR if the threshold criterion is met; go to the pairing step otherwise.

In the mating step, from each assignment pair, two new pairs are generated in the following manner: Let $\mathbf{x}_1$ and $\mathbf{x}_2$ denote two finger assignments, and let $\mathbf{p}_{\mathbf{x}_1}$ and $\mathbf{p}_{\mathbf{x}_2}$ consist of the indices of the multipath components chosen as the Rake fingers. Then, the indices of the new assignments are chosen randomly from the vector $\mathbf{p} = [\mathbf{p}_{\mathbf{x}_1}\ \mathbf{p}_{\mathbf{x}_2}]$. If the new assignment is the same as $\mathbf{x}_1$ or $\mathbf{x}_2$, then the procedure is repeated for that assignment.

Commonly, the algorithm is stopped after a certain number of iterations. In other words, the threshold criterion is that the number of iterations exceeds a given value. The parameters that determine the tradeoff between complexity and performance of the algorithm are the number of iterations, N_{ipop}, N_{pop}, N_{good}, and the number of mutations at each iteration.

[6]More accurate approximations can be obtained by using higher order series expansions for the matrix inverse in (12). However, the solution of the optimization problem does not lend itself to low complexity solutions in those cases.

[7]Although we consider only the binary GA, continuous parameter GAs are also available [21].

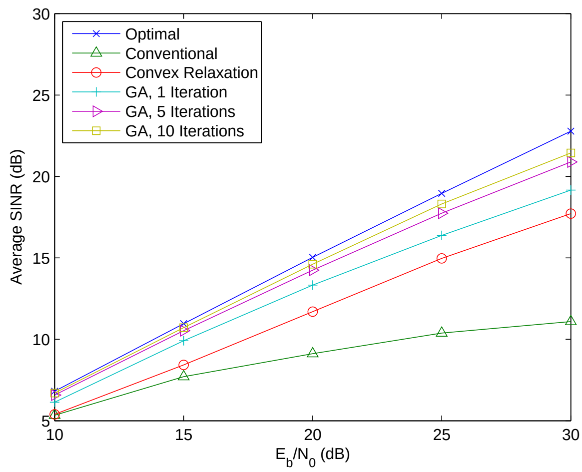

Fig. 2. Average SINR of an MRC-Rake receiver versus E_b/N_0 for $M = 5$ fingers, where E_b is the bit energy.

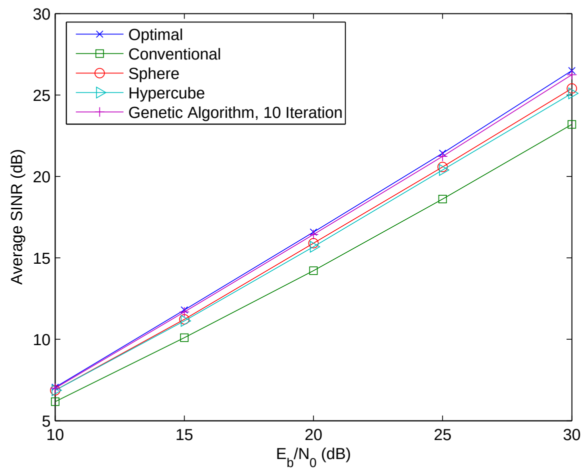

Fig. 3. Average SINR of an MMSE-Rake receiver versus E_b/N_0 for $M = 5$ fingers.

6. SIMULATION RESULTS

Simulations have been performed to evaluate the performance of various finger selection algorithms for an IR-UWB system with $N_c = 20$ and $N_f = 1$. In these simulations, there are five equal energy users in the system ($K = 5$) and the users' TH and polarity codes are randomly generated. We model the channel coefficients as $\alpha_l = \text{sign}(\alpha_l)|\alpha_l|$ for $l = 1, \ldots, L$, where $\text{sign}(\alpha_l)$ is ± 1 with equal probability and $|\alpha_l|$ is distributed lognormally as $\mathcal{LN}(\mu_l, \sigma^2)$. Also the energy of the taps is exponentially decaying as $\text{E}\{|\alpha_l|^2\} = \Omega_0 e^{-\lambda(l-1)}$, where λ is the decay factor and $\sum_{l=1}^{L} \text{E}\{|\alpha_l|^2\} = 1$ (so $\Omega_0 = (1 - e^{-\lambda})/(1 - e^{-\lambda L})$). For the channel parameters, we choose $\lambda = 0.1$, $\sigma^2 = 0.5$ and μ_l can be calculated from $\mu_l = 0.5 \left[\ln(\frac{1-e^{-\lambda}}{1-e^{-\lambda L}}) - \lambda(l-1) - 2\sigma^2 \right]$, for $l = 1, \ldots, L$. We average the overall SINR of the system over different realizations of channel coefficients, TH and polarity codes of the users.

In Figure 2, we plot the average SINR of an MRC-Rake receiver for different noise variances when $M = 5$ fingers are to be chosen out of $L = 15$ multipath components. For the GA, $N_{\text{ipop}} = 32$, $N_{\text{pop}} = 16$, and $N_{\text{good}} = 8$ are used, and 8 mutations are performed at each iteration. As is observed from the figure, the convex relaxation of the optimal finger selection and the GA based scheme perform considerably better than the conventional scheme, and the GA gets very close to the optimal exhaustive search scheme after 10 iterations. Note that the gain achieved by using the proposed algorithms over the conventional one increases as the background noise decreases, since the noise correlation increases in that case as the MAI becomes more dominant. Also note that the GA performs considerably better than the suboptimal solution based on the convex relaxation.

In Figure 3, the same performance evaluation is obtained for an MMSE-Rake receiver with various finger selection algorithms. Note the increase in the average SINR levels compared to Figure 2, which is due to the optimal linear MMSE combining after the selection step. Another significant observation is that the suboptimal solutions based on convex relaxations and the GA are very close to the optimal solution. Also the conventional scheme is within 3.3 dB of the optimal scheme in this case.

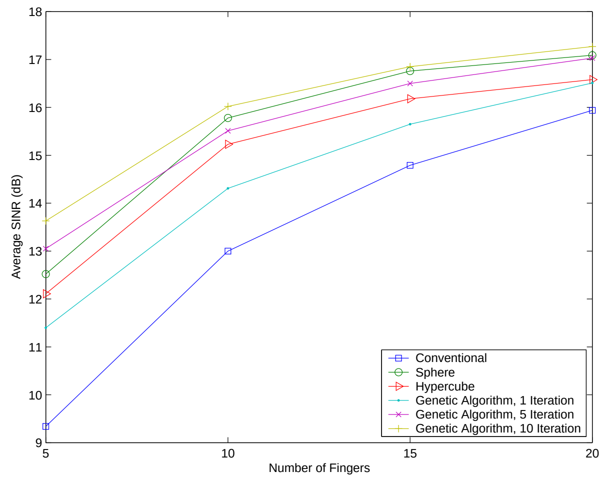

Fig. 4. Average SINR versus number of fingers M. There are 10 users with each interferer having 10dB more power than the desired user.

Finally, we plot, in Figure 4, the average SINR at an MMSE-Rake receiver using the proposed suboptimal and conventional techniques for different numbers of fingers, where there are 50 multipath components and $E_b/N_0 = 20$ dB. The number of chips per frame, N_c, is set to 75, and 10 users with $E_1 = 1$ and $E_k = 10\ \forall k \neq 1$ are considered (all other parameters are kept the same as before). In this case, the optimal algorithm takes a very long time to simulate since it needs to perform exhaustive search over many different finger combinations and therefore it was not implemented. The improvement using the convex relaxations of optimal finger selection or the GA based scheme over the conventional technique decreases as M increases since the channel is exponentially decaying and most of the significant multipath components are already combined by all the algorithms. Also, the GA based scheme performs very close to the suboptimal schemes using convex relaxations after 10 iterations with $N_{\text{ipop}} = 128$, $N_{\text{pop}} = 64$, $N_{\text{good}} = 32$, and 32 mutations.

7. CONCLUSIONS

We have investigated the finger selection problem for MRC- and MMSE-Rake receivers, and presented optimal and suboptimal finger selection schemes. Specifically, the conventional finger selection algorithms have very low complexity; however, performance loss can be considerable in interference-limited scenarios compared to the other suboptimal algorithms. Also, the GA based scheme provides a good performance-complexity trade-off due to its iterative nature with flexible parameters.

8. REFERENCES

[1] M. Z. Win and R. A. Scholtz, “Impulse radio: How it works,” *IEEE Communications Letters,* 2(2): pp. 36-38, Feb. 1998.

[2] M. Z. Win and R. A. Scholtz, “Ultra-wide bandwidth time-hopping spread-spectrum impulse radio for wireless multiple-access communications,” *IEEE Transactions on Communications*, vol. 48, pp. 679-691, April 2000.

[3] R. A. Scholtz, “Multiple access with time-hopping impulse modulation,” *Proc. IEEE Military Communications Conference (MILCOM 1993),* vol. 2, pp. 447-450, Boston, MA, Oct. 1993.

[4] D. Cassioli, M. Z. Win and A. F. Molisch, “The ultra-wide bandwidth indoor channel: From statistical model to simulations,” *IEEE Journal on Selected Areas in Communications*, vol. 20, pp. 1247-1257, Aug. 2002.

[5] D. Cassioli, M. Z. Win, F. Vatalaro and A. F. Molisch, “Performance of low-complexity RAKE reception in a realistic UWB channel,” *Proc. IEEE International Conference on Communications (ICC 2002),* vol. 2, pp. 763-767, New York, NY, April 28-May 2, 2002.

[6] M. Z. Win and J. H. Winters, “Analysis of hybrid selection/maximal-ratio combining of diversity branches with unequal SNR in Rayleigh fading,” *Proc. IEEE 49th Vehicular Technology Conference (VTC 1999-Spring),* vol. 1, pp. 215-220, Houston, TX, May 16-20, 1999.

[7] N. Kong and L. B. Milstein, “Combined average SNR of a generalized diversity selection combining scheme,” *Proc. IEEE International Conference on Communications (ICC 1998),* vol. 3, pp. 1556-1560, Atlanta, GA, June 7-11, 1998.

[8] L. Yue, “Analysis of generalized selection combining techniques,” *Proc. IEEE 51st Vehicular Technology Conference (VTC 2000-Spring),* vol. 2, pp. 1191-1195, Tokyo, Japan, May 15-18, 2000.

[9] J. H. Cho and Q. Zhang, “Design of RAKE receivers for ultra-wideband binary block-coded PPM in dense multipath channels,” *Proc. IEEE Military Communications Conference (MILCOM 2003),* vol. 2, pp. 874-879, Boston, MA, Oct. 13-16, 2003.

[10] F. E. Aranda, N. Brown and H. Arslan, “RAKE receiver finger assignment for ultra-wideband radio,” *Proc. 4th IEEE Workshop on Signal Processing Advances in Wireless Communications (SPAWC 2003),* pp. 239-243, Rome, Italy, June 15-18, 2003.

[11] A. S. Khayrallah and D. A. Cairns, “Fast finger selection for GRAKE,” *Proc. IEEE 61st Vehicular Technology Conference (VTC 2005-Spring),* vol. 2, pp. 728-732, May 30-June 1, 2005.

[12] E. Fishler and H. V. Poor, “On the tradeoff between two types of processing gain,” *IEEE Transactions on Communications,* vol. 53, issue 10, pp. 1744-1753, Oct. 2005.

[13] S. Gezici, H. Kobayashi, H. V. Poor and A. F. Molisch, “Performance evaluation of impulse radio UWB systems with pulse-based polarity randomization in asynchronous multiuser environments,” *Proc. IEEE Wireless Communications and Networking Conference (WCNC 2004),* vol. 2, pp. 908-913, Atlanta, GA, March 2004.

[14] Y.-P. Nakache and A. F. Molisch, “Spectral shape of UWB signals - Influence of modulation format, multiple access scheme and pulse shape,” *Proc. IEEE 57th Vehicular Technology Conference, (VTC 2003-Spring)*, vol. 4, pp. 2510-2514, Jeju, Korea, April 2003.

[15] S. Gezici, H. Kobayashi, H. V. Poor, and A. F. Molisch, “Optimal and suboptimal linear receivers for time-hopping impulse radio systems,” *Proc. IEEE Conference on Ultra Wideband Systems and Technologies (UWBST 2004),* Kyoto, Japan, May 18-21, 2004.

[16] A. F. Molisch, Y. P. Nakache, P. Orlik, J. Zhang, Y. Wu, S. Gezici, S. Y. Kung, H. Kobayashi, H. V. Poor, Y. G. Li, H. Sheng and A. Haimovich, “An efficient low-cost time-hopping impulse radio for high data rate transmission,” *Proc. IEEE 6th International Symposium on Wireless Personal Multimedia Communications (WPMC 2003),* Yokosuka, Kanagawa, Japan, Oct. 19-22, 2003.

[17] S. Gezici, M. Chiang, H. V. Poor and H. Kobayashi, “A genetic algorithm based finger selection scheme for UWB MMSE Rake receivers,” *Proc. IEEE International Conference on Ultra-Wideband (ICU 2005),* Zurich, Switzerland, Sept. 5-8, 2005.

[18] S. Boyd and L. Vandenberghe, *Convex Optimization,* Cambridge University Press, Cambridge, UK, 2004.

[19] S. Verdú. *Multiuser Detection,* Cambridge University Press, Cambridge, UK, 1998.

[20] S. Gezici, M. Chiang, H. V. Poor and H. Kobayashi, “Optimal and suboptimal finger selection algorithms for MMSE Rake receivers in impulse radio ultra-wideband systems,” *Proc. IEEE Wireless Communications and Networking Conference (WCNC 2005),* New Orleans, LA, March 13-17, 2005.

[21] R. L. Haupt and S. E. Haupt, *Practical Genetic Algorithms,* John Wiley & Sons Inc., New York, 1998.