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REDUCING POWER CONSUMPTION IN A SENSOR NETWORK BY INFORMATION FEEDBACK

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ABSTRACT

We study the role of information feedback for the problem of distributed signal tracking/estimation using a sensor network with a fusion center. Assuming that the fusion center has sufficient energy to reliably feed back its intermediate estimates, we show that the sensors can substantially reduce their power consumption by using the feedback information in a manner similar to the stochastic approximation scheme of Robbins-Monro. For the problem of tracking an autoregressive source or estimating an unknown parameter, we quantify the total achievable power saving (as compared to the distributed schemes with no feedback), and provide numerical simulations to confirm the theoretical analysis.

1. PROBLEM FORMULATION

Consider the problem of tracking a signal source or estimating an unknown parameter by a wireless sensor network with a fusion center (FC). Given that the energy supply to each sensor is limited, it is important that sensor operations are energy-efficient so as to ensure a maximum network lifetime. In practice, a major part of sensor energy is used for communication with the FC. As a result, some recent work has focussed on reducing power consumption either by optimally adjusting quantization and transmission power levels at each sensor in an inhomogeneous sensing environment [3], or by exploiting the spatial correlation in sensor observations [4]. In both studies, communication between the sensors and the FC is assumed to be one-way, and no information feedback from the FC is allowed. In some practical situations, the FC (e.g., an unmanned aerial vehicle) may have substantially more energy supply than the sensors, and therefore can feed back information to the sensors reliably if needed. An interesting question is: what benefits can information feedback bring to a sensor network system?

From a signal processing standpoint, allowing information feedback cannot increase the estimation performance of a sensor network. This is because the information provided by the FC can only be a suitable summary of the information collected by the sensors; there cannot be any "fresh information" from the FC. Thus, even with feedback, the sensor network performance is still upper bounded by that of a centralized counterpart which is further limited by the same Cramer-Rao bound as in the case without feedback. However, from a power consumption point of view, allowing information feedback does bring a major benefit. As we show in this paper, by suitably exploiting feedback information from the FC, a sensor network can significantly reduce its power consumption required to achieve a given mean squared error (MSE) performance. Thus, in some sense, information feedback effectively lets the FC tradeoff its own power with that of the sensors as both sides collaborate to achieve a given signal processing performance. The center piece of our proposed scheme is a low-power communication strategy based on the stochastic approximation procedure of Robbins-Monro. For the problem of tracking an autoregressive (AR) source, our analysis shows that information feedback can result in a constant factor power reduction, the size of which is dependent on final MSE requirement as well as the power spectral density of the source. For the problem of estimating an unknown parameter, it is shown that the power reduction factor grows unbounded as the final MSE requirement tends to zero.

Our work is inspired by the information theoretic study [2] of receiver feedback for a memoryless point-to-point communication channel. In that context, even though information feedback cannot increase channel capacity, it can significantly reduce the probability of error at the receiver [2]. Our current work can be viewed as exploiting information feedback in the context of distributed signal processing with multiple sensors, as opposed to the single point-to-point communication considered by [2].

A. Source model

Consider a p-th order autoregressive (AR) source modeled as

$$s[n] = -\sum_{k=1}^{p} a[k]s[n-k] + u[n], \;\; u[n] \sim \mathcal{N}(0, \sigma_u^2). \qquad (1)$$

The source is initialized with $s[n] = 0$ for $n \le 0$. The model coefficients $a[k]$, $k = 1,\ldots,p$ as well as the model order p are assumed to be known. The AR model can also be expressed in a vector form

$$\mathbf{s}[n] = \mathbf{A}\mathbf{s}[n-1] + \mathbf{b}u[n], \qquad (2)$$

where the vector source state $\mathbf{s}[n] \in \mathbb{R}^p$ is defined by

$$\mathbf{s}[n] = \begin{bmatrix} s[n-p+1] \\ \vdots \\ s[n] \end{bmatrix}$$

and matrix $\mathbf{A} \in \mathbb{R}^{p\times p}$ and vector $\mathbf{b} \in \mathbb{R}^{p\times 1}$ are defined by

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0\ldots 0 \\ 0 & 0 & 1 & 0\ldots 0 \\ \vdots & & & \ddots \\ 0 & 0 & 0\ldots & 0 \;\; 1 \\ -a[p] & \ldots & & -a[1] \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}. \qquad (3)$$

B. Sensor observation model

Suppose source $s[n]$ is observed by a sensor network with K sensors. The source observations are corrupted by i.i.d. (spatially and temporally) Gaussian noise

$$x_i[n] = s[n] + w_i[n], \;\; w_i[n] \sim \mathcal{N}(0, \sigma_w^2), \quad i = 1,\ldots,K.$$

An equivalent vector model is

$$x_i[n] = \mathbf{h}^T \mathbf{s}_i[n] + w_i[n], \quad \mathbf{h} = [0,\ldots,0,1]^T, \quad i = 1,\ldots,K. \qquad (4)$$

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We assume that, at each time n, the i-th sensor transmits an analog message $f_i(x_i[n])$ of the observed signal $x_i[n]$ to the FC. No local interaction among sensors is considered in this paper.

C. Communication channel to the fusion center

Different designs of MAC layer are possible in practical implementation of sensor networks. We consider two specific MAC strategies: i) uncoded orthogonal transmissions and ii) uncoded coherent combining. The former can be implemented using CDMA/TDMA/FDMA schemes, while the latter can be realized in systems with synchronized transmission over static channels.

- **Uncoded orthogonal transmissions:** the FC receives separate signals from all transmitting sensors

$$y_i^{or}[n] = f_i(x_i[n]) + z_i[n], \;\; z_i[n] \sim \mathcal{N}(0, \sigma_z^2), \tag{5}$$

where $z_i[n]$ is the the channel noise from the i-th sensor to the FC at the time n. The communication noise is assumed to be spatially and temporally uncorrelated.

- **Uncoded coherent combining:** in this case the FC receives

$$y^{cc}[n] = \sum_{i=1}^{K} f_i(x_i[n]) + z[n], \; z[n] \sim \mathcal{N}(0, \sigma_z^2), \tag{6}$$

where noise $z[n]$ is assumed to be temporally uncorrelated.

These two multi-access models have also been considered in [6].

2. SOURCE TRACKING AT THE FUSION CENTER

Let sensor nodes adopt the following message functions

$$f_i(x_i[n]) = \beta(x_i[n] - \mu[n]), \; \forall\, i, \tag{7}$$

where $\mu[n]$ is a reference signal, and β is an amplification factor common to all sensors. The value of β affects the transmission power and the final estimation quality at the FC; it will be chosen later. To reduce transmission power, $\mu[n]$ should be chosen to closely approximate $x_i[n]$ or $s[n]$. Thus, if feedback channel is available, the FC should broadcast to all the sensors its predicted value of $s[n]$ so that the sensors can use it as their reference signal $\mu[n]$. In the absence of information feedback, the sensors should set $\mu[n] = E\{x_i[n]\} = E\{s[n]\}$ instead.

A. An equivalent observation model at fusion center

- When orthogonal multi-access is used, the FC can first compute the estimate (c.f. (5), (7))

$$\begin{aligned} y_{fc}^{or}[n] &= \frac{1}{K\beta}\sum_{i=1}^{K} y_i^{or}[n] + \mu[n] \\ &= \frac{1}{K}\sum_{i=1}^{K} x_i[n] + \frac{1}{K\beta}\sum_{i=1}^{K} z_i[n] \\ &= s[n] + \frac{1}{K}\sum_{i=1}^{K} w_i[n] + \frac{1}{K\beta}\sum_{i=1}^{K} z_i[n]. \end{aligned}$$

Thus, we obtain an equivalent observation model at the FC

$$y_{fc}[n] = s[n] + z_{fc}[n], \tag{8}$$

where the equivalent noise at the FC is

$$z_{fc}[n] = z_{fc}^{or}[n] = \frac{1}{K}\sum_{i=1}^{K} w_i[n] + \frac{1}{K\beta}\sum_{i=1}^{K} z_i[n] \sim \mathcal{N}\left(0, \frac{\sigma_w^2}{K} + \frac{\sigma_z^2}{K\beta^2}\right).$$

- With the coherent combining (6) channel, the FC has

$$\begin{aligned} y_{fc}^{cc}[n] &= \frac{1}{K\beta}\, y^{cc}[n] + \mu[n] \\ &= \frac{1}{K}\sum_{i=1}^{K} x_i[n] + \frac{1}{K\beta}\, z[n] \\ &= s[n] + \frac{1}{K}\sum_{i=1}^{K} w_i[n] + \frac{1}{K\beta}\, z[n], \end{aligned}$$

which also can be expressed as (8) with Gaussian noise

$$z_{fc}[n] = z_{fc}^{cc}[n] \sim \mathcal{N}\left(0, \frac{\sigma_w^2}{K} + \frac{\sigma_z^2}{K^2\beta^2}\right).$$

B. Kalman filter at the fusion center

Given the equivalent observation model (8), the FC can apply vector Kalman filter to track the source $s[n]$. Let us define two error covariance matrices

$$\mathbf{M}[n|n-1] = E\left\{(\mathbf{s}[n] - \hat{\mathbf{s}}[n|n-1])(\mathbf{s}[n] - \hat{\mathbf{s}}[n|n-1])^T\right\}$$
$$\mathbf{M}[n|n] = E\left\{(\mathbf{s}[n] - \hat{\mathbf{s}}[n|n])(\mathbf{s}[n] - \hat{\mathbf{s}}[n|n])^T\right\}.$$

The Kalman Filtering operation at the FC is summarized below [1]:

- *Prediction:*
$$\hat{\mathbf{s}}[n|n-1] = \mathbf{A}\hat{\mathbf{s}}[n-1|n-1]. \tag{9}$$
- *Update the Prediction Error Covariance Matrix:*
$$\mathbf{M}[n|n-1] = \mathbf{A}\mathbf{M}[n-1|n-1]\mathbf{A}^T + \sigma_u^2\mathbf{b}\mathbf{b}^T. \tag{10}$$
- *Update Kalman Gain Vector:*
$$\mathbf{g}[n] = \frac{\mathbf{M}[n|n-1]\mathbf{h}}{\sigma_{fc}^2 + \mathbf{h}^T\mathbf{M}[n|n-1]\mathbf{h}}. \tag{11}$$
- *Correction:*
$$\hat{\mathbf{s}}[n|n] = \hat{\mathbf{s}}[n|n-1] + \mathbf{g}[n](y_{fc}[n] - \mathbf{h}^T\hat{\mathbf{s}}[n|n-1]). \tag{12}$$
- *Update Error Covariance Matrix:*
$$\mathbf{M}[n|n] = (\mathbf{I} - \mathbf{g}[n]\mathbf{h}^T)\mathbf{M}[n|n-1]. \tag{13}$$

Here, σ_{fc}^2 is the equivalent noise variance in the FC observation model (8) with

$$\sigma_{fc}^2 = \begin{cases} \sigma_w^2/K + \sigma_z^2/(K\beta^2), & \text{for orthogonal multi-access,} \\ \sigma_w^2/K + \sigma_z^2/(K^2\beta^2), & \text{for coherent multi-access.} \end{cases} \tag{14}$$

The source estimate $\hat{s}_p[n|n]$ corresponds to the p-th entry of the vector estimate $\hat{\mathbf{s}}[n|n]$. Our goal is to minimize the steady state MSE $\gamma = \lim_{n\to\infty} E\{(s[n] - \hat{s}_p[n|n])^2\}$. Note that at time $n-1$, the FC should broadcast $\hat{s}_p[n|n-1]$ (the predicted value of $s[n]$) so that it can be used as the reference signal $\mu[n]$ by all sensors at time n.

3. ANALYSIS OF SENSOR TRANSMISSION POWER

We now analyze the total network transmission power required to achieve a *fixed* MSE target γ at the FC, and quantify the extent of power reduction resulted from information feedback under both the orthogonal and coherent combining channel models. Notice that, given the AR source model (2) and the channel model (5)-(6), the steady state MSE γ achievable by the Kalman filter (13) is uniquely determined by the noise variance σ_{fc}^2 for the equivalent FC observation model (8). Thus, the consequence of fixing γ is equivalent to fixing the noise variance σ_{fc}^2. The latter is in turn uniquely specified by (14) through the choice of channel model and amplification factor β, *regardless* of whether information feedback from the FC is allowed or not. Consequently, in our ensuing power consumption analysis, the value of β will be the same, with or without feedback. Let $P_f[n]$ and $P_{nf}[n]$ denote the (instantaneous) power consumption of the sensor network at the time interval n with and without feedback from the FC respectively. When we refer to the power consumption in the steady state, we will drop the reference to the specific time interval. Similarly, let $\mu_f[n]$ and $\mu_{nf}[n]$ respectively denote the reference signal at time n with and without feedback. Then, we have from (7)

$$\begin{aligned} P_{nf}[n] &= K\beta^2 E\left\{(x_i[n]-\mu_{nf}[n])^2\right\} \\ P_f[n] &= K\beta^2 E\left\{(x_i[n]-\mu_f[n])^2\right\} \end{aligned}$$

Selecting β according to (14), we obtain the following total steady state network power consumption for the two multi-access channel models (5)–(6):

$$P_{nf}^{or} = \frac{K\sigma_z^2 \lim_{n\to\infty} E\left\{(x_i[n]-\mu_{nf}[n])^2\right\}}{K\sigma_{fc}^2-\sigma_w^2}, \quad P_{nf}^{cc} = \frac{P_{nf}^{or}}{K}, \qquad (15)$$

where P_{nf}^{or} (respectively, P_{nf}^{cc}) stands for the the steady-state total network power consumption with orthogonal channel (respectively, coherent combining channel) at the FC. Similarly, we can define P_f^{cc}, P_f^{or}. The difference of a factor K in P_{nf}^{or} and P_{nf}^{cc} is due to the fact that there are K independent noise samples accumulated in the orthogonal channels, whereas only one noise sample is present in the coherent combining case.

Theorem 1. *Given a steady-state target MSE γ, information feedback from the FC reduces power consumption of the wireless sensor network by a factor of*

$$\frac{P_{nf}^{cc}}{P_f^{cc}} = \frac{P_{nf}^{or}}{P_f^{or}} = \frac{\sigma_w^2+\sigma_s^2}{\sigma_w^2+\sigma_u^2+\gamma\,\|\mathbf{a}\|^2}, \qquad (16)$$

where $\mathbf{a}$ *is the vector of AR coefficients $a[k]$ and source variance σ_s^2 is given by*

$$\sigma_s^2 = \frac{\sigma_u^2}{2\pi}\int_{-\pi}^{\pi}\frac{d\omega}{\left|1+\sum_{k=1}^{p} a[k]\exp(-j\omega k)\right|^2}. \qquad (17)$$

In the special case of the AR(1) source given by the scalar Gauss-Markov process $s[n] = as[n-1]+u[n]$, $a=-a[1]$, the expression for the steady state source variance (17) can be simplified. Substituting $p=1$ in (17) we arrive at the well-known [1] steady state variance of the scalar Gauss-Markov process:

$$\sigma_s^2 = \frac{\sigma_u^2}{1-a^2}, \quad a^2<1.$$

Notice that σ_s^2 blows up when $a\to 1$. In this case, the power reduction factor (16) also becomes unbounded. For a general AR(p) source, the steady state variance depends on its spectral property. If the frequency response has a pole approaching the unit circle, the integral (17) diverges and the power reduction due to feedback from the FC goes to infinity.

To prove Theorem 1 we will need the following lemma.

Lemma 1. *In the steady state the minimum MSE matrix of Kalman filter (13) is a factor γ of the identity matrix. The factor γ is defined by the following equation*

$$\sigma_{fc}^2 = \frac{\gamma\,(\gamma\,\|\mathbf{a}\|^2+\sigma_u^2)}{\gamma\,(\|\mathbf{a}\|^2-1)+\sigma_u^2}. \qquad (18)$$

The expression (18) identifies the maximum noise variance σ_{fc}^2 at the FC that achieves the target minimum MSE γ. According to (14), the knowledge of noise variance σ_{fc}^2 in the equivalent FC observation model (8) allows the optimal amplification factors β for both channels to be precomputed in advance.

Proof of Lemma 1. The proof consists of three steps. First, we derive the expression for $\mathbf{M}[n|n]$ from Kalman filter operations (9)-(13). Then, we prove that in the steady state matrix $\mathbf{M}[n|n]$ is a constant factor γ of the identity matrix. Finally, we show that γ satisfies (18).

Substituting Kalman gain vector (11) into the minimum MSE matrix (13) we obtain

$$\mathbf{M}[n|n] = \mathbf{M}[n|n-1] - \frac{\mathbf{M}[n|n-1]\mathbf{h}\mathbf{h}^T\mathbf{M}[n|n-1]}{\sigma_{fc}^2+\mathbf{h}^T\mathbf{M}[n|n-1]\mathbf{h}}, \qquad (19)$$

where the minimum prediction MSE matrix $\mathbf{M}[n|n-1]$ is given by (10). In the steady state, matrices $\mathbf{M}[n|n]$ and $\mathbf{M}[n|n-1]$ do not depend on the time interval n, so we will drop the dependence on n to simplify the notations and write $\mathbf{M}$ for steady state $\mathbf{M}[n|n]$ and $\mathbf{M}_p$ for steady state $\mathbf{M}[n|n-1]$. With these new notations we can rewrite (19) in the steady state as follows

$$\mathbf{M} = \mathbf{M}_p - \frac{\mathbf{M}_p\mathbf{h}\mathbf{h}^T\mathbf{M}_p}{\sigma_{fc}^2+\mathbf{h}^T\mathbf{M}_p\mathbf{h}}, \;\; \mathbf{M}_p = \mathbf{A}\mathbf{M}\mathbf{A}^T+\sigma_u^2\mathbf{b}\mathbf{b}^T. \qquad (20)$$

The expression in the denominator can be simplified by taking into account the structure of matrix $\mathbf{A}$ and vector $\mathbf{b}$ introduced in (2) as well as the structure of vector $\mathbf{h}$ in (4)

$$\mathbf{h}^T\mathbf{M}_p\mathbf{h} = \mathbf{h}^T\mathbf{A}\mathbf{M}\mathbf{A}^T\mathbf{h}+\sigma_u^2\;\mathbf{h}^T\mathbf{b}\mathbf{b}^T\mathbf{h} = \mathbf{a}^T\mathbf{M}\mathbf{a}+\sigma_u^2. \qquad (21)$$

Let $M(i,k)$ denote the (i,k)-th entry of the minimum MSE matrix $\mathbf{M}$, and let vector $\mathbf{a}^T$ denote the last row of matrix $\mathbf{A}$ or, equivalently, the vector of source coefficients $a[k]$. Then, using the structure of matrix $\mathbf{A}$ and vector $\mathbf{b}$ for every entry $M(i,k)$ such that $i\neq p$ and $k\neq p$ we obtain from (20) and (21)

$$M(i,k) = M(i+1,k+1) - \frac{f(i+1)f(k+1)}{\sigma_{fc}^2+\sigma_u^2+\mathbf{a}^T\mathbf{M}\mathbf{a}}, \qquad (22)$$

where $f(n+1) = \sum_{m=1}^{p} M(n+1,m)A(p,m),\; 0\le n<p$.

From the definition of the MSE achieved at the FC, all the diagonal entries of $\mathbf{M}$ are equal to γ in the steady state. This implies that in the steady state $M(i,i) = M(i+1,i+1),\;\forall\, i,\; 0\le i<p$. According to (22) we must have $f(i+1)=0,\;\forall\, i,\; i\le 0<p$, to ensure that all diagonal entries of $\mathbf{M}$ are equal. Therefore, the steady state MSE matrix $\mathbf{M}$ must be Toeplitz. By analogy, for the entries (p,k) of the last row of matrix $\mathbf{M}$ such that $0\le k<p$, we have from (20) and (21)

$$M(p,k) = \frac{\sigma_{fc}^2}{\sigma_{fc}^2+\sigma_u^2+\mathbf{a}^T\mathbf{M}\mathbf{a}}\; f(k+1),$$

which are all equal to 0 because $f(k+1)=0,\;\forall\, k,\; 0\le k<p$. The Toeplitz structure of matrix $\mathbf{M}$ implies that all entries below the diagonal are zeros. Since the Minimum MSE matrix $\mathbf{M}$ is symmetric, it must be diagonal and its diagonal entries are steady state MSE of the source γ at the FC.

Now, we derive the factor γ as a function of σ_{fc}^2 and source parameters $\mathbf{a}$ and σ_u^2. Multiplying the two sides of (20) by $\mathbf{h}^T$ and $\mathbf{h}$ respectively and noticing that $\mathbf{h}^T\mathbf{M}\mathbf{h} = M(p,p) = \gamma$ due to the structure of vector $\mathbf{h}$, we obtain

$$\gamma = \mathbf{h}^T\mathbf{M}_p\mathbf{h} - \frac{(\mathbf{h}^T\mathbf{M}_p\mathbf{h})(\mathbf{h}^T\mathbf{M}_p\mathbf{h})}{\sigma_{fc}^2+\mathbf{h}^T\mathbf{M}_p\mathbf{h}} = \frac{\sigma_{fc}^2\mathbf{h}^T\mathbf{M}_p\mathbf{h}}{\sigma_{fc}^2+\mathbf{h}^T\mathbf{M}_p\mathbf{h}}. \qquad (23)$$

The quadratic form $\mathbf{h}^T\mathbf{M}_p\mathbf{h}$ can be calculated from (21) by taking into account the fact that $\mathbf{M} = \gamma\mathbf{I}$, so that $\mathbf{h}^T\mathbf{M}_p\mathbf{h} = \gamma\|\mathbf{a}\|^2 + \sigma_u^2$. Substituting the last expression into (23) and using simple algebraic manipulations, we obtain the desired equation (18). □

Proof of Theorem 1. Consider first the case without feedback. As was discussed in the beginning of Section 3, we have in this case $\mu_{nf} = E\{s[n]\}$. Therefore, the power consumed by a sensor in the network is given by

$$P_{nf} = E\left\{\beta^2\left(x_i[n] - E\{s[n]\}\right)^2\right\} = \beta^2\left(\sigma_s^2 + \sigma_w^2\right), \qquad (24)$$

where the source variance is defined as $\sigma_s^2 = E\left\{(s[n] - E\{s[n]\})^2\right\}$. Selecting β according to (14) we can specify $P_{nf} = P_{nf}^{or}$ or P_{nf}^{cc}. The power spectral density $f_s(\omega)$ of the source (1) is given by

$$f_s(\omega) = \frac{\sigma_u^2}{2\pi\left|1 + \sum_{k=1}^p a[k]\exp(-j\omega k)\right|^2}.$$

Hence, the source variance σ_s^2 can be easily calculated

$$\sigma_s^2 = \int_{-\pi}^{\pi} f_s(\omega)d\omega,$$

which leads to the expression stated in (17).

Let us now consider the case with feedback. In that case sensors use the source prediction $\mu_f[n] = \hat{s}_p[n|n-1]$ as the reference signal for transmission. Thus, the consumed power per sensor is

$$P_f = E\{\beta^2\left(x_i[n] - \hat{s}_p[n|n-1]\}\right)^2\} = \beta^2\left(M[n|n-1](p,p) + \sigma_w^2\right),$$

where $M[n|n-1](p,p)$ is the (p,p)-th entry of minimum prediction MSE matrix $\mathbf{M}[n|n-1]$. In the steady state, the (p,p)-th entry of minimum prediction MSE matrix $\mathbf{M}[n|n-1]$ can be calculated from the Kalman filter minimum prediction MSE matrix (10)

$$M[n|n-1](p,p) = \mathbf{aMa}^T + \sigma_u^2 = \gamma\|\mathbf{a}\|^2 + \sigma_u^2,$$

where $\mathbf{M}$ is the steady state minimum MSE matrix, and the last equality follows from Lemma 1. Thus, the power consumption of each sensor is given by

$$P_f = \beta^2\left(\gamma\|\mathbf{a}\|^2 + \sigma_u^2 + \sigma_w^2\right). \qquad (25)$$

Selecting a β according to (14) allows us to specify power consumption for either orthogonal channel $P_f = P_f^{or}$ or coherent combining channel $P_f = P_f^{cc}$. Finally, the power reduction factor stated in Theorem 1 is obtained as a ratio of (24) and (25). □

4. UNKNOWN PARAMETER ESTIMATION PROBLEM

Now we consider a constant signal model with

$$s[n] = \theta,$$

which corresponds to $a = 1$ and $\sigma_u^2 = 0$ in the AR source model (2). In this case, the source tracking problem reduces to the problem of estimating an unknown static parameter θ. The goal is to construct a sequence of parameter estimates at the FC with increasing quality until a specified target MSE γ is achieved. As in the AR case, we are interested in the effect of information feedback on the sensor network's power consumption. Notice that the power reduction factor in (16) is undefined for $a = 1$ and $\sigma_u^2 = 0$. We show in this section that the power reduction factor in this case is in fact unbounded.

Let each sensor collect observations $x_i[m] = \theta + w_i[m]$, $m = 1,\ldots,n$, and calculate its local average $\bar{x}_i[n] = \sum_{m=1}^n x_i[m]/n$. Given a sequence of reference signals $\mu[n]$ known at the FC, each sensor transmits the difference

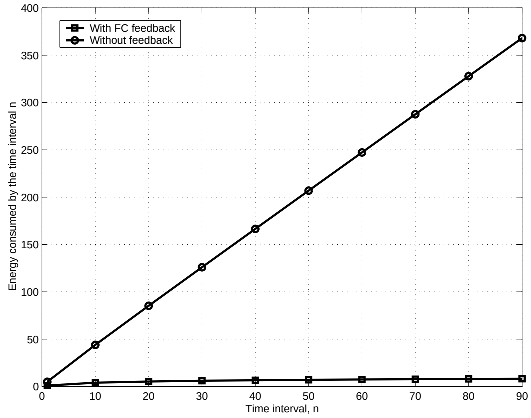

Figure 1: The comparison of sensor energy consumption in sensor networks with and without FC feedback, $K = 100, \sigma_w^2 = 2, \sigma_z^2 = 10$.

$$f_i(\bar{x}_i[n]) = \mu[n] - \bar{x}_i[n],\ i = 1,\ldots,K. \qquad (26)$$

Depending on the type of the communication channel, the received signal at time interval n can be expressed as

$$y^{cc}[n] = \tfrac{1}{K}\left(\sum_{i=1}^K f_i(\bar{x}_i[n]) + z[n]\right), \quad \text{for coherent multi-access,}$$
$$y^{or}[n] = \tfrac{1}{K}\sum_{i=1}^K\left(f_i(\bar{x}_i[n]) + z_i[n]\right), \quad \text{for orthogonal multi-access.}$$

Subtracting the contribution due to the known sequence $\mu[m]$, $m = 1,\ldots,n$, the FC extracts the updated estimate

$$\hat{\theta}[n] = \frac{1}{n}\sum_{m=1}^n\left(\mu[m] - y[m]\right), \qquad (27)$$

where $y[m]$ can be specialized to either $y^{cc}[m]$ or $y^{or}[m]$ for the considered communication channel. In the presence of feedback, the FC broadcasts its estimate $\hat{\theta}[n]$ for sensors to use as a reference signal: $\mu_f[n+1] = \hat{\theta}[n]$, $n = 1,\ldots,N-1$, and $\mu_f[1] = \hat{\theta}[0]$ is the initial guess of θ at the FC. Then, simple algebraic transformations of (27) lead to the following sequence of estimates at the FC

$$\hat{\theta}[n] = \theta + \frac{1}{nK}\sum_{m=1}^n a_n[m]\sum_{i=1}^K w_i[m] - \frac{1}{nK}\sum_{m=1}^n z_{fc}[m], \qquad (28)$$

where $a_n[m] = 1/m + 1/(m+1) + \ldots + 1/n$, and the effective channel noise is given by:

$$z_{fc}[m] = z[m], \quad \text{for coherent multi-access,}$$
$$z_{fc}[m] = \sum_{i=1}^K z_i[m], \quad \text{for orthogonal multi-access.}$$

Suppose that the estimation takes N time intervals to achieve a target MSE at the FC. Combining (26) and (28) we calculate the the energy consumed by a sensor to perform N transmissions to the FC, which can be upper bounded by the function

$$E_f[N] \le E_1 + \sum_{n=1}^{N-1}\left(\frac{(K-2)\sigma_w^2}{K(n+1)} + \frac{\sigma_z^2}{K^\alpha n} + \frac{2\sigma_w^2}{Kn}\right). \qquad (29)$$

where $E_1 = E\{\left(\theta - \hat{\theta}[0]\right)^2\} + \sigma_w^2$ is the energy consumed at the first time interval with the initial parameter guess $\hat{\theta}[0]$; and α is a parameter of the communication channel: $\alpha = 1$ for orthogonal channels and $\alpha = 2$ for the coherent combining at the FC. It can be

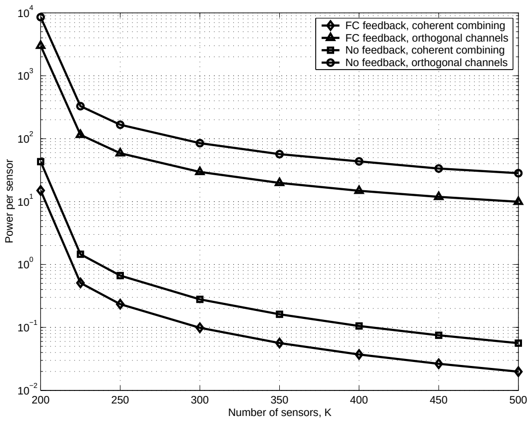

Figure 2: Power consumption per sensor as a function of the network size. The target MSE at the Fusion Center is $\gamma = 0.01$.

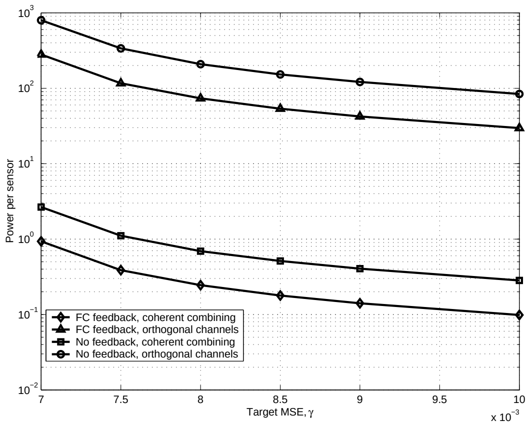

Figure 3: Power consumption per sensor as a function of the target MSE at the Fusion Center. The network size is $K = 300$ sensors.

shown that bound (29) is asymptotically tight. Thus, with information feedback, the consumed energy per sensor scales as $\log N$ with number of transmissions N. In the absence of feedback from the FC, the sensors must use a constant reference signal known at the FC, $\mu_{nf}[n] = \hat{\theta}[0],\ n = 1,\ldots,N$. The FC calculates the parameter estimates according to $\hat{\theta}[n] = (\hat{\theta}[0] - y[n])/n$. In this case, for both types of communication channel, the transmission of a sequence of local observations results in a sensor energy consumption that grows linearly with N:

$$E_{nf}[N] = E_1 N + \sigma_w^2 \sum_{n=1}^{N} \frac{1}{n}. \qquad (30)$$

Therefore, the energy (or power) reduction factor $E_{nf}[N]/E_f[N]$ asymptotically scales like $N/\log N \to \infty$ as $N \to \infty$. Note, in both cases the total energy spent on estimation goes to infinity with increasing N, but the scaling factor dramatically favors the network with feedback. Figure 1 demonstrates that information feedback allows the sensors to make efficient use of their energy budget while delivering the same MSE performance at the FC.

5. SIMULATIONS

In simulations we track a source specified by AR(3) model:

$$s[n] = 0.5s[n-1] + 0.3s[n-2] + 0.15s[n-3] + u[n].$$

The source is initialized with zeros: $s[n] = 0$, for all $n \leq 0$. The variance of $u[n]$ is taken to be $\sigma_u^2 = 1$. The observation noise variance is $\sigma_w^2 = 2$ and the communication noise variance is $\sigma_z^2 = 10$. The network starts tracking the source at time interval $n = 10$ and proceeds until $n = 10000$. All tracking/estimation algorithms are tested with 100 runs.

Figure 2 demonstrates the power consumption per sensor as a function of the network size. The four curves correspond to two communication channel models, orthogonal channels to the FC and coherent combining at the FC, and two tracking models, with and without the feedback from the FC. The target MSE of $\gamma = 0.01$ has been chosen for the simulations. According to the Theorem 1 the power reduction due to the feedback from the FC is constant for a fixed target MSE. Evaluating the expression (16) of the power reduction with the specified parameters we obtain

$$\frac{P_{nf}^{cc}}{P_f^{cc}} = \frac{P_{nf}^{or}}{P_f^{or}} = 2.86. \qquad (31)$$

The maximum deviation from this factor in Figure 2 is within 6%.

Figure 3 shows the dependence of power consumption of each sensor as a function of the target MSE at the FC. These simulations are performed with the network of 300 sensors. The difference in communication model results in the power factor of $K = 300$ which was justified by (15). For the target MSE ranging from 0.007 to 0.01 the power reduction decreases slightly from 2.7423 to 2.7414, which is consistent with (16). In Figure 3, the maximum deviation from the theoretical bound (16) is again within 6%.

6. CONCLUSIONS

This paper considers the role of information feedback in the context of distributed signal tracking by a sensor network with a FC. It is shown that if the FC can feed back its intermediate estimates reliably to the sensors, there can be a constant factor power reduction in the sensor transmission power required to achieve a target MSE at the FC. This constant factor depends on the power spectral density of the source signal being tracked, and can grow unbounded if the source is static. Notice that the sensor power required to receive the feedback information has not been accounted for in our analysis. In [5], it was noted that the energy cost for packet reception is only slightly more than that of listening to an idle channel, while transmitting energy is 1.4 times listening. Thus, the receive power required for implementing the information feedback scheme studied herein is about 70% of the sensors' transmit power. This effectively reduces the total power saving (16) by a factor of 1.7.

REFERENCES

[1] S. M. Kay, *Fundamentals of Statistical Signal Processing, Volume I: Estimation Theory,* Prentice Hall, 1993.

[2] J.P. Schalkwijk and T. Kailath, "A coding scheme for additive noise channels with feedback, Part. I: no bandwidth constraint," *IEEE Trans. Information Theory,* vol. 12, pp. 172–182, Apr. 1966.

[3] J.-J. Xiao, S. Cui, Z.-Q. Luo and A.J. Goldsmith, "Power scheduling of universal decentralized estimation in sensor networks," *IEEE Transactions on Signal Processing,* vol. 54, no. 2, pp. 413–422, Feb. 2006.

[4] A. Krasnopeev, J.-J. Xiao and Z.-Q. Luo, "Minimum energy decentralized estimation in a wireless sensor network with correlated sensor noise," *EURASIP Journal on Wireless Communication and Networking*, no. 4, pp. 473–482, Nov. 2005.

[5] M. Stemm and R. Katz, "Measuring and reducing energy consumption in hand-held devices," *E80-B*, vol. 8, pp. 1125–1131, Aug. 1997.

[6] I.D. Schizas, G.B. Giannakis and Z.-Q. Luo, "Distributed estimation using reduced dimensionality sensor observations," *Proc. of 39th Asilomar Conf. on Signals, Systems, and Computers,* Pacific Grove, Oct. 30 – Nov. 2, 2005.