RESOLVING MANIFOLD AMBIGUITIES IN DIRECTION FINDING SYSTEMS

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ABSTRACT

The issue of resolving manifold ambiguities in subspace–based Direction Finding (DF) systems is investigated in this paper. The cause of manifold ambiguities is due to linear dependence amongst manifold vectors. When an ambiguous situation occurs, subspace–based DF techniques fail to correctly identify a unique set of Directions–of–Arrival (DOA). This causes a performance degradation due to unreliable estimates of the parameters. In spite of the fact that there are infinitely many ambiguous scenarios in an array system, the problem in resolving manifold ambiguities has received very little attention. In this paper, two novel techniques are proposed, aiming at improving the DOA identification capability, while maintaining a minimum computational complexity. The proposed techniques adopt a beamforming class based on the Minimum Variance Distortionless Response (MVDR) criterion to estimate signal power, which is a measure for identifying the presence of a signal. Simulation results show the improved performance in terms of the identification capability over the previously proposed model fitting method.

1. INTRODUCTION

When estimating Directions–of–Arrival (DOA) in subspace–based Direction Finding (DF) systems, it is important that a unique and consistent set of solutions is obtained. In general, signals from different sets of DOA give different responses to a sensor array (i.e. different manifold vectors.) However, when this mapping is not one–to–one, a confusion is caused to decide which directions the signals were actually impinged from. This undesirable behavior is known as "*manifold ambiguity*."

By definition, manifold ambiguity is the inability of an array to distinguish a set of directions from at least one of its subsets (or one subset from another) due to linear dependence amongst manifold vectors [1]. Ambiguity is often categorized based on the rank of the manifold matrix. Rank–1 ambiguity (also known as *trivial,* ambiguity) occurs when there exists a manifold vector that is a scalar multiple of another [2]. Meanwhile, rank–k ambiguity (or *non–trivial* ambiguity) occurs when there exist l manifold vectors that are linearly dependent and no lower rank ambiguity existed amongst them.

Previously most researches in the area mainly focus on the performance analysis of specific arrays, or on the design of an array free of ambiguities up to a certain rank. In [3], Baygun and Tanik showed that the probability of having trivial ambiguities in a circular array is increased when enlarging the array's aperture. In [4], Godara and Cantoni derived a condition for an array to be free of rank–1 ambiguity. Tan *et al.* [5] extended the work of [4] and proposed a condition for an array to be free of rank–2 ambiguity. In [6], Tan *et al.* constructed a class of *cross* arrays that are free up to a certain rank.

On the other hand, Proukakis and Manikas analyzed the ambiguity problem from a different perspective based on the differential geometry of the array manifold. A crucial finding was that, in practice, there are *infinitely* many ambiguous scenarios in array systems, which can be represented in the form of an Ambiguous Generator Set (AGS). This concept was extended to planar arrays in [7].

Although it has been shown that manifold ambiguities are unavoidable in practice, the issue of *resolving* ambiguities has received very little attention.

In this paper, two novel techniques are proposed aiming to improve the DOA identification capability. The organization for the rest of the paper is as follows. In Section 2, the problem formulation is introduced, followed by a review of existing ambiguity resolving techniques. In Section 3, two novel techniques based on a beamforming approach are proposed. Simulation results are presented in Section 4 to examine the identification capability. Finally, the paper is concluded in Section 5.

2. PROBLEM FORMULATION

Consider an array system of N elements, located at $[\underline{r}_x, \underline{r}_y, \underline{r}_z] \in \mathfrak{R}^{N\times 3}$ (where $\underline{r}_x, \underline{r}_y,$ and $\underline{r}_z$ are column vectors containing the position of the elements respectively) receiving M co–channel signals from narrowband point sources. The basedband received signal vector $\underline{x}(t) = [x_1(t), \ldots, x_N(t)]^T \in \mathfrak{C}^{N\times 1}$ can be expressed as

$$\underline{x}(t) = \mathbb{S}\underline{m}(t) + \underline{n}(t) \tag{1}$$

where $\underline{m}(t)$ is the vector of M message signals, $\underline{n}(t)$ is the additive noise with covariance matrix $\sigma^2\mathbb{I}_N$, and $\mathbb{S}$ is the $(N\times M)$ manifold matrix (also known as the array response matrix) with columns the manifold vectors, i.e.

$$\mathbb{S} \triangleq [\ \underline{S}(\theta_1,\phi_1), \underline{S}(\theta_2,\phi_2), \ldots, \underline{S}(\theta_M,\phi_M)\ ] \tag{2}$$

Note that each $(N\times 1)$ manifold vector $\underline{S}(\theta_i,\phi_i)$ represents the complex array response to a unit amplitude plane wave impinging from the direction (θ_i,ϕ_i), expressed as

$$\underline{S}(\theta_i,\phi_i) = \exp(-j\ [\underline{r}_x, \underline{r}_y, \underline{r}_z]\ \underline{k}(\theta_i,\phi_i)) \tag{3}$$

where $\underline{k}(\theta_i,\phi_i) = \frac{2\pi}{\lambda}[\cos\theta_i\cos\phi_i, \sin\theta_i\cos\phi_i, \sin\phi_i]^T \in \mathfrak{R}^{3\times 1}$ is the wavenumber vector pointing toward the emitter at azimuth θ_i and elevation ϕ_i, with θ_i measured anticlockwise from the positive x–axis, and ϕ_i measured anticlockwise from the x–y plane.

Based on the model in (1), the second order statistics of the received signal vector $\underline{x}(t)$ can be expressed as

$$\mathbb{R}_{xx} \triangleq \mathscr{E}\{\underline{x}(t)\underline{x}^H(t)\} = \mathbb{S}\mathbb{R}_{mm}\mathbb{S}^H + \sigma^2\mathbb{I}_N \tag{4}$$

where $\mathbb{R}_{mm} \triangleq \mathscr{E}\{\underline{m}(t)\underline{m}^H(t)\}$ represents the source covariance matrix, $\mathbb{I}_N$ is the identity matrix, $(\cdot)^H$ is the Hermitian (complex conjugate transpose), and $\mathscr{E}\{\cdot\}$ is the expectation operator. In this work, it is assumed that $N > M$ and the sensors are isotropic of unity gain.

When Equation (4) is employed the main assumption is that the manifold vectors forming the matrix $\mathbb{S}$ are linearly independent. Otherwise an ambiguous situation is said to occur.

Majority of the work regarding *resolving* manifold ambiguities was extensively studied by Abramovich, Spencer, and Gorokhov through a list of their publications in [8],[9],[10]. Two techniques were proposed based on a matrix–valued transform of the intersensor covariance matrix $\mathbb{R}_{xx}$, namely by *association* and by *model fitting* techniques.

In the first technique, ambiguity is resolved through an association of the two sets of MUSIC estimates. The first set corresponds to the MUSIC spectrum based on the covariance matrix $\mathbb{R}_{xx}$, that is ambiguous but highly accurate. The second set is obtained from the MUSIC spectrum using another matrix $\mathbb{T}_{xx}$, which is a transformed matrix of $\mathbb{R}_{xx}$. It was proven that the DOA estimates from this set are asymptotically Gaussian unbiased, but not very accurate. By associating these two sets of estimates, the identification capability is improved [9], [10]. A major drawback of this approach is due to the computational complexity in obtaining the matrix $\mathbb{T}_{xx}$. For instance for a non–integer array the process may involve a non–linear programming routine. The complexity is increased even further for planar and 3–dimensional arrays.

The second technique, called resolving manifold ambiguity by *model fitting*, is more effective. The approach essentially estimates source powers associated with each of the estimated directions, including the ambiguous ones. A linear programming routine is adopted to find the best fit amongst the set of estimated spatial covariance lags and the source powers. The powers corresponding to the ambiguous DOA's tend to be equal to zero; meanwhile the DOA's associated with the M largest powers converge to the true DOA's. The disadvantage of this technique is based on the use of linear programming routines which can be computationally complex for a number of applications.

The objective for this work is to develop novel ambiguity resolving techniques that satisfy three preconditions. First, the new techniques should improve the ambiguity resolving ability in order to increase the correct identification rate. Second, the techniques should require a minimum computational complexity, so that they are feasible to be implemented in a practical array system. Third, the techniques should be suitable for any arbitrary array configuration including planar and 3–dimensional arrays. The proposed techniques are presented in the following Section.

3. RESOLVING MANIFOLD AMBIGUITIES BY MVDR BEAMFORMING TECHNIQUE

Similar to the model fitting approach [8], a key parameter to identify the presence of signals is the signal power. A beamformer class based on the Minimum Variance Distortionless Response (MVDR) criterion is adopted to estimate the signal powers in this work.

Beamforming is an array processing technique that uses an array to control the directionality of the radiation pattern. It aims to separate a desired signal from the co–channel interference and noise. The output of a beamformer is a linear combination of the received signal $\underline{x}(t)$, which can be expressed as

$$y(t) = \underline{w}^H\underline{x}(t) \tag{5}$$

where $\underline{w} = [w_1, w_2, \ldots, w_N]^T$ is a complex weight vector.

MVDR is a class of adaptive beamforming technique widely used in array processing, where the criterion is to choose $\underline{w}$ such that the total output power is minimized. However, this minimization should be achieved under the condition that a distortionless array response along the desired direction (θ_d, ϕ_d) is maintained. That is,

$$\min_{\underline{w}} \mathscr{E}\{|y(t)|^2\} \quad \text{subject to} \quad |\underline{w}^H\underline{S}(\theta_d, \phi_d)| = 1 \tag{6}$$

The solution of this constraint optimization is found [11] (using the Lagrange multiplier) and can be expressed as follows

$$\underline{w}_{MVDR} \triangleq \underline{w}_{MVDR}(\theta_d, \phi_d) = \frac{\mathbb{R}_{xx}^{-1}\underline{S}(\theta_d, \phi_d)}{\underline{S}^H(\theta_d, \phi_d)\mathbb{R}_{xx}^{-1}\underline{S}(\theta_d, \phi_d)} \tag{7}$$

Using the weight vector in the Equation (7), the array response corresponding to a single signal from the direction (θ_i, ϕ_i) is given as

$$g_{MVDR}(\theta_i, \phi_i) = \underline{w}_{MVDR}^H\underline{S}(\theta_i, \phi_i) \tag{8}$$

It is important to point out that Equation (8) evaluated for every (θ_i, ϕ_i) is known as the *array pattern*.

Two approaches are proposed to resolve ambiguities:

- by observing the array pattern, and
- by estimating the signal powers.

3.1 Array Pattern Observation

The directions of pseudo sources can be identified intuitively by observing the array pattern. Due to the nature of the MVDR beamformer, which attempts to minimize the total output power, nulls are placed to all the directions outside the "look–direction" that are associated with non–zero energy. However, the directions corresponding to the ambiguous signals do not have any energy attached, therefore the MVDR does not suppress any energy from these directions. In a manifold ambiguous scenario where a set of $(M + M_{amb})$ directions are ambiguously identified, the technique intuitively searches the gain responses amongst the $(M + M_{amb})$ points to identify the pseudo sources. The procedure is summarized as follows.

1. Form a weight–vector and the corresponding array pattern with mainlobe steered toward direction (θ_d, ϕ_d), where $d \in \{1, \ldots, M + M_{amb}\}$, and M_{amb} denotes the total number of the pseudo peaks.
2. Record the absolute values of the beam responses $g_{MVDR}(\theta_i, \phi_i)$ for $i = 1, 2, \ldots, M + M_{amb}$.
3. Select M_{amb} directions, which have maximum gain responses as pseudo source candidates. Note that this set must exclude the response from the direction (θ_d, ϕ_d).
4. Repeat steps 1–3 for all $(M + M_{amb})$ directions.
5. Based on the sets of candidates selected in each iteration, do a majority vote to decide the M_{amb} pseudo sources.

3.2 Power Estimation

Let us now compute a signal power, which is defined as

$$P = \mathscr{E}[\,|y(t)|^2] = \underline{w}^H \mathscr{E}[\underline{x}(t)\underline{x}^H(t)]\underline{w} = \underline{w}^H \mathbb{R}_{xx}\underline{w} \qquad (9)$$

Substituting the weight vector given by Equation (7) into Equation (9) gives,

$$P_{MVDR}(\theta_d, \phi_d) = \frac{1}{\underline{S}^H(\theta_d, \phi_d)\mathbb{R}_{xx}^{-1}\underline{S}(\theta_d, \phi_d)} \qquad (10)$$

The second proposed approach is to use Equation (10) for estimating the signal power for each of the $(M + M_{amb})$ directions. If all the power–estimates outside the look–directions are completely suppressed, then the M largest "peaks" will represent the true sources, while the remaining M_{amb} lowest powers will be identified as pseudo sources.

It is important to note that the proposed techniques should not cause any increase in terms of complexity due to the fact that beamforming process is already well established in an array system. Even in the case where a different class of beamformer is employed, the proposed techniques can still be implemented with a low computational cost. Furthermore, these techniques are applicable to an arbitrary array configuration.

4. SIMULATION

In this Section, the concept of resolving manifold ambiguities by MVDR beamforming techniques is demonstrated using a planar array of 5 elements located at

$$[\underline{r}_x, \underline{r}_y, \underline{r}_z] = \begin{pmatrix} -0.88 & -0.59 & 0.18 & 0.59 & 0.69 \\ -0.97 & 1.29 & -1.25 & -0.69 & 1.62 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}^T \qquad (11)$$

measured in half wavelengths. Suppose that the array is operated in the presence of 3 signals with directions $(75°, 16.42°), (75°, 81.06°)$, and $(255°, 61.66°)$. From the MUSIC spectrum shown in Figure 1 it is clear that the number of estimated directions is 4 and not 3. Indeed using the approach presented in [12] this ambiguity is represented by the following ambiguous generator set of $(\underline{\theta}, \underline{\phi}) = \{(75°, 16.42°), (75°, 81.06°), (255°, 61.66°), (255°, 10.41°)\}$

To resolve this ambiguous situation, the proposed techniques are applied. First, four array patterns according to four different weight vectors are observed. Figure 2 shows the array pattern corresponding to a weight vector when the look–direction is at $(75°, 16.42°)$. The response gains from the directions $(75°, 81.06°)$, $(255°, 61.66°)$ and $(255°, 10.41°)$ are 3.526×10^{-4}, 7.18×10^{-5}, and 1.3032 respectively. Repeat the same procedure for other weight vectors. Table 1 summarizes array response gains for each direction. Based on a majority vote result, it indicates that the fourth signal from the direction $(255°, 10.41°)$ does not represent a real source and thus can be removed.

The result is confirmed in Figure 3 which shows signal powers at the output of the beamformer as a function of directions. The three highest peaks correspond to the true sources. Meanwhile the lowest estimated power at $(255°, 10.41°)$ is identified as a pseudo source. It is interesting to observe

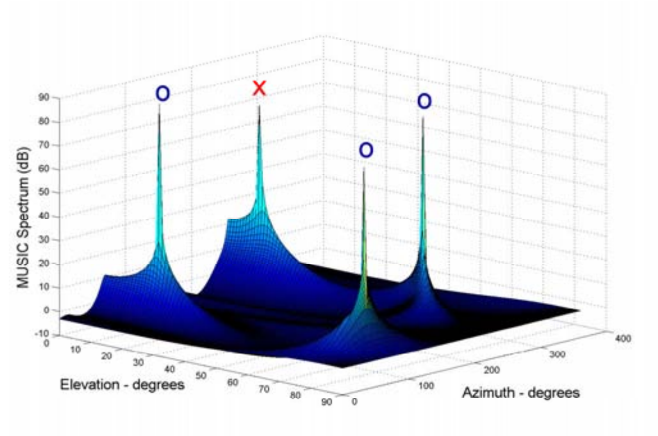

Figure 1: A manifoldly ambiguous situation for the planar array in (11). A pseudo peak at $(255°, 10.41°)$ (with a cross) does not correspond to a real source.

$\underline{w}$	$\lvert g(\theta_1, \phi_1)\rvert$	$\lvert g(\theta_2, \phi_2)\rvert$	$\lvert g(\theta_3, \phi_3)\rvert$	$\lvert g(\theta_4, \phi_4)\rvert$
$\underline{w}(\theta_1, \phi_1)$	1.0000	0.0004	0.0001	1.3032
$\underline{w}(\theta_2, \phi_2)$	0.0004	1.0000	0.0003	1.5387
$\underline{w}(\theta_3, \phi_3)$	0.0001	0.0003	1.0000	0.8984
$\underline{w}(\theta_4, \phi_4)$	0.2673	0.3156	0.1843	1.0000

Table 1: Array patterns according to 4 different weights vectors

that the estimated power at $(255°, 10.41°)$ is small, but non–zero. This is due to a linear dependence amongst a response from this direction and those from the other three directions. The values in the last row of Table 1 shows that the gain responses for the true directions cannot get completely suppressed when the look–direction is at $(255°, 10.41°)$. The presence of power at $(255°, 10.41°)$ is, therefore, a result of power leakage from other directions. Nonetheless, this power is very small and always less than the powers from the true directions, so it does not affect the identification.

It must be emphasized that plots of array patterns and signal powers shown in this paper are for illustrative purpose only. In practice, only samplings of the responses and powers according to the $(M + M_{amb})$ points of directions are to be evaluated.

The capability of the proposed techniques is now assessed in a situation where an ambiguous direction is significantly close to one of the true sources, under a finite number of snapshots (time samples of the received signal). The performance is to be compared with the model fitting method in [8]. In this simulation, sources are assumed to be uncorrelated, and the number of sources is determined prior to the estimation. Only a conventional scenario where a number of sources is less than the number of sensors is considered in this paper. Thus, consider again the planar array described by Equation (11), operating in the presence of four incoming signals with equal unit powers from $(\underline{\theta}, \underline{\phi}) = \{(75°, 16.42°), (75°, 81.06°), (255°, 61.66°), (258°, 7.00°)\}$. This generates an ambiguous peak at the direction $(255°, 10.41°)$, which is approximately 4.5° away from $(258°, 7.00°)$, shown in Figure 4. The noise power σ^2 is set to 30 dB below the signal power, and different sizes of observation intervals $L = \{30, 50, 100\}$ are examined. Using

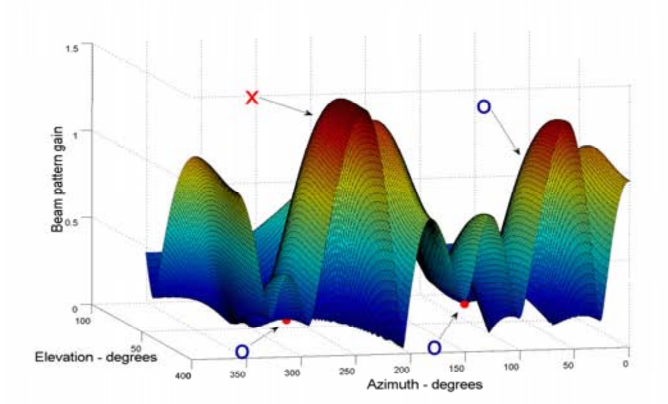

Figure 2: Illustration of the array patterns for a weight vector corresponding to the direction $(75°, 16.42°)$.

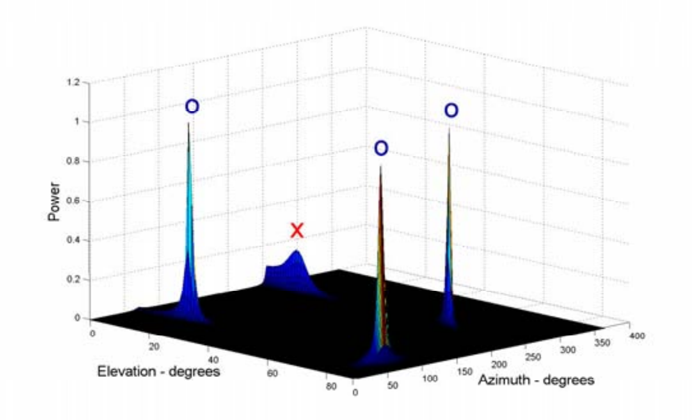

Figure 3: A plot of estimated signal powers from the output of a MVDR beamformer.

the practical covariance matrix

$$\hat{\mathbb{R}}_{xx} = \frac{1}{L}\sum_{l=1}^{L} \underline{x}(t_l)\underline{x}^H(t_l), \quad (12)$$

a set of data is collected over a number of trials, where $(M + M_{amb})$ highest peaks in MUSIC spectrum correspond to the specified directions, and no main peaks were merged. The number of regular trials was set to 5000. Notice a practical issue on how to determine M_{amb}. For example, there are several more peaks in Figure 4 that could be considered as candidate DOA's. In practice, M_{amb} is decided based on a magnitude threshold on the MUSIC spectrum. The threshold is set at 60 dB or above for the spectrum in Figure 4. It can be seen that the lower the threshold is, the larger the M_{amb} is expected.

The criterion examined here is the capability for the proposed techniques to correctly identify and remove the set of pseudo sources. The probability of correct identification is defined as the ratio of the number of trials that successfully resolve the ambiguities over a total number of trials. The probability of correct identification is shown in Table 2. As expected, the probability of correct identification is increased when snapshot size is larger. The proposed MVDR beamforming techniques show an excellent ambiguity resolving capability even when the snapshot size is very small. In fact,

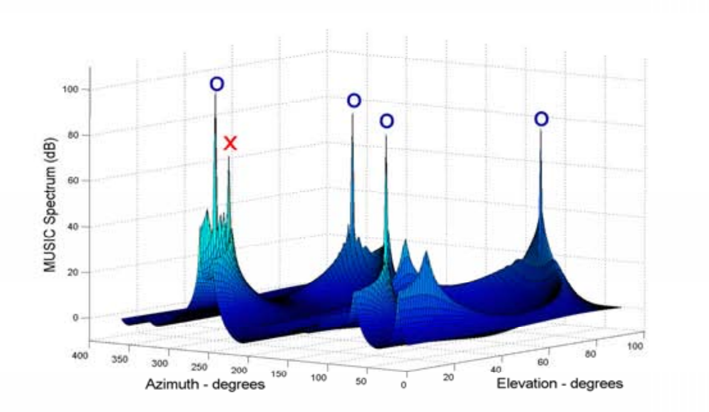

Figure 4: MUSIC spectrum for the array with 4 sources are present. The peak at $(255°, 10.41°)$ (with a cross) does not represent a real signal.

no case has been detected where the MVDR power estimation failed to resolve ambiguities.

Table 3 illustrates the average powers and standard deviations for each direction based on the model fitting and the MVDR beamforming techniques. Notice the output power from the pseudo source at $(255°, 10.41°)$ in the last column of the Table. The average power found by the beamformer is slightly larger than that by the model fitting technique, however it has a significantly small standard deviation. Figures 5 and 6 show the distributions of powers at the direction $(255°, 10.41°)$ when $L = 30$ for the model fitting and MVDR beamformer accordingly. Although, in most trials, the power estimated by model fitting technique converges to zero (3129 trials out of 5000), the distribution spreads in a wide range. This is because there were cases where the linear programming failed to converge, and resulted in a large value of powers. As a result, the estimated power of the pseudo source was higher than that from one of the true sources, so the technique fails to identify the correct pseudo source. On the other hand, the average power by MVDR beamformer is very concise around the mean. The maximum power is found to be less than 0.5.

Snapshot size (L)	30	50	100
Model Fitting	0.8380	0.9050	0.9742
MVDR– Pattern	0.9924	0.9996	1.0000
MVDR– Power	1.000	1.000	1.000

Table 2: Probability of correct identification

5. CONCLUSIONS

In this paper, two novel ambiguity resolving techniques based on the MVDR beamformer were proposed. The first approach is based on an observation of the array pattern where pseudo sources are intuitively identified by searching for directions where the gain responses do not get suppressed. The second approach is via an estimation of the signal power. Simulation results showed the improved performance in terms of identification capability over the previously proposed model fitting method [8]. The computational complexity is also kept at minimum as the methods involve a

	$(75°, 16.42°)$		$(75°, 81.06°)$		$(255°, 61.66°)$		$(258°, 7.00°)$		$(255°, 10.41°)$	
sample size (L)	avg	std	avg	std	avg	std	avg	std	avg	std
Model Fitting										
L=100	0.971	0.228	0.937	0.202	0.891	0.182	1.074	0.253	0.140	0.213
L=50	0.999	0.302	0.914	0.268	0.846	0.264	1.099	0.334	0.169	0.294
L=30	1.031	0.358	0.912	0.322	0.784	0.327	1.091	0.452	0.204	0.374
MVDR Power										
L=100	1.062	0.142	1.101	0.143	1.018	0.136	1.016	0.145	0.258	0.024
L=50	1.041	0.196	1.087	0.201	0.995	0.197	0.992	0.190	0.256	0.036
L=30	1.013	0.250	1.055	0.259	0.960	0.240	0.950	0.240	0.250	0.046

Table 3: Mean (avg.) and standard deviation (std.) of powers estimated by model fitting, and MVDR beam forming techniques.

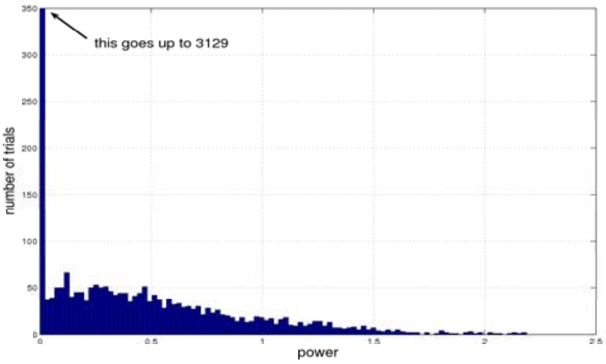

Figure 5: Histogram of the power distribution at $(255°, 10.41°)$ estimated by the model fitting method, $L = 30$

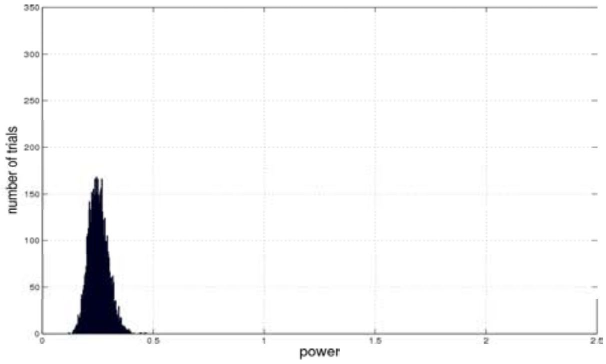

Figure 6: Histogram of the power distribution at $(255°, 10.41°)$ estimated by the MVDR beamforming technique, $L = 30$

one–step calculation, rather than a linear programming routine.

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