LOW COMPLEXITY COVARIANCE-BASED DOA ESTIMATION ALGORITHM

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ABSTRACT

The aim of this work is to present an alternative method for estimating the direction-of-arrival (DoA), that is, the incoming angle, of a signal impinging on an antenna array. The proposed method is similar to ESPRIT (estimation of signal parameters via rotational invariance techniques) algorithm, which is the most widely used technique for this application. The new algorithm exploits the structural similarities between ESPRIT and the Tong-Xu-Kailath method for blind channel equalization. The result is an ESPRIT-like algorithm for DoA estimation with substantially reduced computational complexity. Simulation results are included to verify the properties and performance of the new covariance-based DoA algorithm, in comparison to ESPRIT and to the theoretical Cramer-Rao lower bound.

1. INTRODUCTION

The use of antenna arrays has emerged as a very powerful technique for improving the receiver performance in digital communications. Several signal processing applications are employed for estimating some parameters or the whole waveform of the received signals [1]. In some situations, the estimation of the incoming signals is equivalent to estimating the direction of the transmitting sources [2] [3]. Examples of applications of array techniques to mobile communications systems can be found in [2].

The earliest algorithms for estimating signals in a space-time framework were based on the maximum likelihood paradigm. However, such solutions are very computationally intense. Subsequently, lower complexity algorithms were developed, such as MUSIC (multiple signal classification) [4] for performing direction-of-arrival (DoA) estimation. Nevertheless, MUSIC does not take any advantage of the array geometrical configuration. Then, an algorithm for estimation of parameters via rotational invariance techniques (ESPRIT) [5] [6] was proposed, which uses an invariance property induced by a constant spacing between antennas in a doublet pair. ESPRIT presents substantially lower computational complexity than MUSIC for performing DoA estimation [3], which is one of the main reasons for its large popularity.

In this paper, we discuss the close relationship between ESPRIT and a blind-equalization technique, the so-called Tong-Xu-Kailath (TXK) [7] algorithm solely based on second-order statistics of the cyclostationary incoming signal. By exploiting the link between ESPRIT and the TXK algorithm, a new algorithm is proposed and referred to as the covariance-based DoA (CB-DoA). It is then verified that the CB-DoA algorithm presents performance comparing favorably to the standard ESPRIT method, with substantially lower computational complexity.

This work is organized as follows: Sections 2 and 3 describe the DoA framework and total least-squares (TLS) ESPRIT algorithm, respectively. Then, the CB-DoA algorithm is introduced in Section 4, by exploiting the underlying similarities between ESPRIT and the TXK algorithm, as discussed in Section 5. Section 6 compares the implementation aspects for both TLS-ESPRIT and CB-DoA. Experimental results presented in Section 7 address the CB-DoA performance in distinct setups, emphasizing its reduced computational load when compared to the TLS-ESPRIT algorithm. Besides that, mean-square error performance is compared to the theoretical Cramer-Rao lower bound. Finally, Section 8 draws some conclusions highlighting the main contributions of the paper.

2. DOA ESTIMATION

Consider a MIMO (multiple-input multiple-output) environment with M transmitting narrowband sources and $2N$ receiving antennas, with $N > M$, as represented in Fig. 1. It is assumed that each of the sub-channels has an additive white Gaussian noise (AWGN) as the only interference source. Also, the receiving antennas are grouped in pairs, as described in [5], with a constant displacement δ between the antennas in each pair.

At time t, let $s_m(t)$ represent the signal transmitted by the m^{th} antenna, with $0 \leq m < M$, and let $x_i(t)$ and $y_i(t)$ be the two signals in the i^{th} receiving-antenna doublet, with $0 \leq i < N$. Considering that the incoming signals reach the i^{th} antenna doublet with an angle denoted by θ_m, the gain provided by the antennas for such an angle is represented by $a_i(\theta_m)$. If $n_{x,i}(t)$ and $n_{y,i}(t)$ represent the noise components received by each antenna in the i^{th} doublet, the description of the received signals as functions of the transmitted signals is

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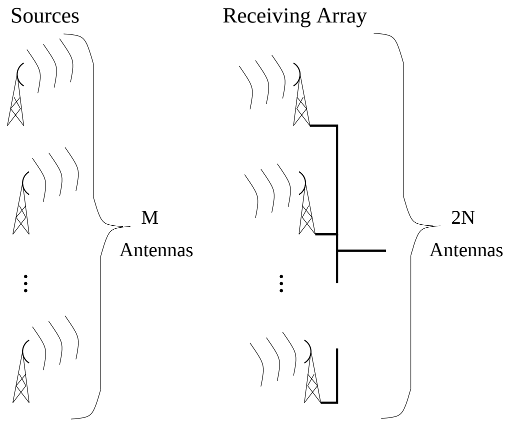

Figure 1: MIMO Environment.

given by [5]

$$x_i(t) = \sum_{m=0}^{M-1} s_m(t) a_i(\theta_m) + n_{x,i}(t), \qquad (1)$$

$$y_i(t) = \sum_{m=0}^{M-1} s_m(t) e^{\frac{j\omega\delta}{c}\sin(\theta_m)} a_i(\theta_m) + n_{y,i}(t), \qquad (2)$$

where $j = \sqrt{-1}$, ω is the frequency of the narrowband signal, and c is the speed of light.

By defining the auxiliary vectors and matrices in the discrete-time k domain as

$$\mathbf{x}(k) = \begin{bmatrix} x_0(k) & x_1(k) & \dots & x_{N-1}(k) \end{bmatrix}^{\mathrm{T}}, \qquad (3)$$

$$\mathbf{y}(k) = \begin{bmatrix} y_0(k) & y_1(k) & \dots & y_{N-1}(k) \end{bmatrix}^{\mathrm{T}}, \qquad (4)$$

$$\mathbf{n}_x(k) = \begin{bmatrix} n_{x,0}(k) & n_{x,1}(k) & \dots & n_{x,N-1}(k) \end{bmatrix}^{\mathrm{T}}, \qquad (5)$$

$$\mathbf{n}_y(k) = \begin{bmatrix} n_{y,0}(k) & n_{y,1}(k) & \dots & n_{y,N-1}(k) \end{bmatrix}^{\mathrm{T}}, \qquad (6)$$

$$\mathbf{s}(k) = \begin{bmatrix} s_0(k) & s_1(k) & \dots & s_{M-1}(k) \end{bmatrix}^{\mathrm{T}}, \qquad (7)$$

$$\mathbf{A} = \begin{bmatrix} a_0(\theta_0) & a_0(\theta_1) & \dots & a_0(\theta_{M-1}) \\ a_1(\theta_0) & a_1(\theta_1) & \dots & a_1(\theta_{M-1}) \\ \vdots & \vdots & \ddots & \vdots \\ a_{N-1}(\theta_0) & a_{N-1}(\theta_1) & \dots & a_{N-1}(\theta_{M-1}) \end{bmatrix}, \qquad (8)$$

$$\mathbf{\Phi} = \mathrm{diag}\left[e^{\frac{j\omega\delta}{c}\sin(\theta_0)}, e^{\frac{j\omega\delta}{c}\sin(\theta_1)}, \dots, e^{\frac{j\omega\delta}{c}\sin(\theta_{M-1})}\right] \qquad (9)$$

then, the input-to-output relationships given in Equations (1) and (2) can be rewritten as

$$\mathbf{x}(k) = \mathbf{A}\mathbf{s}(k) + \mathbf{n}_x(k), \qquad (10)$$

$$\mathbf{y}(k) = \mathbf{A}\mathbf{\Phi}\mathbf{s}(k) + \mathbf{n}_y(k). \qquad (11)$$

Matrix $\mathbf{A}$ is the so-called array manifold matrix [5], and its elements are the gains of the antenna array as a function of the incoming angle.

Considering that signal sources are uncorrelated to noise, then the covariance matrices of the received and transmitted signals are related by

$$E[\mathbf{x}(k)\mathbf{x}^H(k)] = \mathbf{R}_x(0) = \mathbf{A}\mathbf{R}_s(0)\mathbf{A}^H + \sigma^2\mathbf{R}_{n,x}(0), \qquad (12)$$

$$\begin{aligned} E[\mathbf{x}(k)\mathbf{y}^H(k)] &= \mathbf{R}_{xy}(0) = \\ &= \mathbf{A}\mathbf{R}_s(0)\mathbf{\Phi}^H\mathbf{A}^H + \sigma^2\mathbf{R}_{n,xy}(0), \end{aligned} \qquad (13)$$

where σ^2 is the noise variance. Besides that,

$$\mathbf{R}_s(0) = E[\mathbf{s}(k)\mathbf{s}^H(k)], \qquad (14)$$

$$\mathbf{R}_{n,x}(0) = E[\mathbf{n}_x(k)\mathbf{n}_x^H(k)], \qquad (15)$$

$$\mathbf{R}_{n,xy}(0) = E[\mathbf{n}_x(k)\mathbf{n}_y^H(k)]. \qquad (16)$$

3. ESPRIT ALGORITHM

By grouping together the doublet signals into a single vector $\mathbf{z}(k) = \begin{bmatrix} \mathbf{x}(k) \\ \mathbf{y}(k) \end{bmatrix}$, the transmission modeling becomes [5]

$$\mathbf{z}(k) = \bar{\mathbf{A}}\mathbf{s}(k) + \mathbf{n}_z(k), \qquad (17)$$

where $\bar{\mathbf{A}} = \begin{bmatrix} \mathbf{A} \\ \mathbf{A}\mathbf{\Phi} \end{bmatrix}$ and $\mathbf{n}_z(k) = \begin{bmatrix} \mathbf{n}_x(k) \\ \mathbf{n}_y(k) \end{bmatrix}$.

The ESPRIT algorithm performs a generalized eigendecomposition on the matrices

$$\begin{cases} \mathbf{R}_z(0) = E[\mathbf{z}(k)\mathbf{z}^H(k)] \\ \mathbf{\Sigma}_n(0) = E[\mathbf{n}_z(k)\mathbf{n}_z^H(k)] \end{cases}, \qquad (18)$$

such that

$$\mathbf{R}_z(0) - \sigma^2\mathbf{\Sigma}_n(0) = \bar{\mathbf{A}}\mathbf{R}_s(0)\bar{\mathbf{A}}^H. \qquad (19)$$

Hence, the generalized eigenvectors corresponding to the M largest generalized eigenvalues can be used as the columns of $\mathbf{U}_s$, determining

$$\mathbf{E}_s = \mathbf{\Sigma}_n(0)\mathbf{U}_s, \qquad (20)$$

where $\mathbf{E}_s$ and $\bar{\mathbf{A}}$ are related by a non-singular linear transformation $\mathbf{T}$ [5], such that

$$\mathbf{E}_s = \bar{\mathbf{A}}\mathbf{T} = \begin{bmatrix} \mathbf{A}\mathbf{T} \\ \mathbf{A}\mathbf{\Phi}\mathbf{T} \end{bmatrix} = \begin{bmatrix} \mathbf{E}_x \\ \mathbf{E}_y \end{bmatrix}. \qquad (21)$$

The ESPRIT algorithm then determines the following eigendecomposition,

$$\begin{bmatrix} \mathbf{E}_x^H \\ \mathbf{E}_y^H \end{bmatrix} \begin{bmatrix} \mathbf{E}_x & \mathbf{E}_y \end{bmatrix} = \mathbf{E}\mathbf{\Lambda}\mathbf{E}^H. \qquad (22)$$

Following, the resulting eigenvector matrix $\mathbf{E}$ is partitioned into sub-matrices, such that

$$\mathbf{E} = \begin{bmatrix} \mathbf{E}_{11} & \mathbf{E}_{12} \\ \mathbf{E}_{21} & \mathbf{E}_{22} \end{bmatrix}, \qquad (23)$$

allowing the definition of an auxiliary matrix

$$\mathbf{\Psi} = -\mathbf{E}_{12}\mathbf{E}_{22}^{-1}. \qquad (24)$$

At last, the ESPRIT algorithm performs an eigendecomposition on $\mathbf{\Psi}$ to estimate the desired DoA matrix $\mathbf{\Phi}$,

$$\mathbf{\Psi} = \mathbf{T}\mathbf{\Phi}\mathbf{T}^{-1}. \qquad (25)$$

The ESPRIT algorithm described in this section corresponds to the most popular implementation of ESPRIT, known as TLS-ESPRIT (total least-squares ESPRIT) [5].

4. COVARIANCE-BASED DOA ALGORITHM

Using a similar structure employed by the TXK algorithm [7], in the blind channel-equalization setup, one can perform an eigendecomposition directly on the matrix pencil $[\mathbf{R}_x(0)-\sigma^2\mathbf{R}_{n,x}(0)]$, yielding

$$\mathbf{R}_0 = \mathbf{R}_x(0)-\sigma^2\mathbf{R}_{n,x}(0) = \mathbf{U}\mathbf{\Sigma}^2\mathbf{U}^H. \quad (26)$$

As a result, one may form the matrices $\mathbf{\Sigma}_s^2$ and $\mathbf{U}_s$ with the M largest eigenvalues of $\mathbf{R}_0$ and their corresponding eigenvectors, respectively, such that matrix $\mathbf{A}$ satisfies

$$\mathbf{A} = \mathbf{U}_s\mathbf{\Sigma}_s\mathbf{V}. \quad (27)$$

Thus, one can define an auxiliary matrix $\mathbf{F}$ such that

$$\mathbf{F}\mathbf{A} = \mathbf{V}. \quad (28)$$

Therefore,

$$\mathbf{F} = \mathbf{\Sigma}_s^{-1}\mathbf{U}_s^H, \quad (29)$$

and, consequently, another auxiliary matrix $\mathbf{R}_1$ can be determined as

$$\mathbf{R}_1 = \mathbf{F}(\mathbf{R}_{xy}(0)-\sigma_{xy}^2\mathbf{R}_{n,xy})\mathbf{F}^H. \quad (30)$$

From Equations (27), (28), and (30), one has that the DoA matrix $\mathbf{\Phi}$ satisfies

$$\mathbf{R}_1 = \mathbf{V}\mathbf{\Phi}^H\mathbf{V}^H. \quad (31)$$

Since $\mathbf{\Phi}$ is a diagonal matrix, its conjugate transpose can be found by performing an eigendecomposition on $\mathbf{R}_1$. From the rotational invariance property, the elements of $\mathbf{\Phi}$ are known to have unit norm, then an improved estimate of $\mathbf{\Phi}$ is generated with normalized elements.

5. CB-DOA AND TXK

The new CB-DoA algorithm follows a similar structure as of the TXK algorithm for blind channel equalization. In fact, both algorithms use properties related to the received-signal second-order statistics, performing subspace decompositions on the associated covariance matrices.

Naturally, some differences between the TXK and the CB-DoA algorithms emerge due to the distinct setups associated to each application. While the TXK is based on a single receiver with uniform oversampling, the CB-DoA method uses multiple receivers, grouped in doublets, similarly to the ESPRIT algorithm. It can be shown that the uniform oversampling, in the channel equalization framework, yields the rank difference between the covariance matrices at lags zero and one [7]. Such rank reduction is exploited by the TXK technique for determining the multichannel matrix. Meanwhile, in the DoA estimation problem, the proposed algorithm takes advantage of the rotational invariance property, which guarantees the full-rank characteristic for both received-signal covariance matrices at lag zero [6].

The reciprocal relationship comprising ESPRIT and blind equalization is described in [8], where the rotational invariance of ESPRIT is exploited in a frequency-domain description of the blind equalization algorithm.

6. COMPUTATIONAL COMPLEXITY

In order to allow the comparison between the TLS-ESPRIT and the CB-DoA algorithms, the computational complexity of both methods is investigated in this section. For that purpose, Table 1 summarizes the basic operations for each algorithm. The acronyms ED and GE stand for eigendecomposition and generalized eigendecomposition, respectively. When referring to multiple lines or columns, Matlab notation was used. Recalling that M is the number of sources and $2N$ is the number of sensors, the number of operations for each algorithm can be determined.

Table 1: Short descriptions of TLS-ESPRIT and CB-DoA algorithms.

ESPRIT	**CB-DoA**
$[\mathbf{U}_s,\hat{\sigma}^2]=\mathrm{GE}(\hat{\mathbf{R}}_z(0),\hat{\Sigma}_n(0))$	$[\mathbf{U}_s,\hat{\sigma}^2]=\mathrm{GE}(\hat{\mathbf{R}}_x(0),\hat{\mathbf{R}}_{n,x}(0))$
$\mathbf{E}_s=\mathbf{\Sigma}_n(0)\mathbf{U}_s$	$\mathbf{F}=\mathbf{\Sigma}_s^{-1}\mathbf{U}_s^H$
$\mathbf{E}_x=\mathbf{E}_s(0:M-1,:)$	$\mathbf{R}_a=\hat{\mathbf{R}}_{xy}(0)-\hat{\sigma}_{xy}^2\hat{\mathbf{R}}_{n,xy}(0)$
$\mathbf{E}_y=\mathbf{E}_s(M:2M-1,:)$	
$\mathbf{E}_a=\begin{bmatrix}\mathbf{E}_x^H\\ \mathbf{E}_y^H\end{bmatrix}\begin{bmatrix}\mathbf{E}_x & \mathbf{E}_y\end{bmatrix}$	
$[\mathbf{E},\mathbf{\Lambda}]=\mathrm{ED}(\mathbf{E}_a)$	$\mathbf{R}_1=\mathbf{F}\mathbf{R}_a\mathbf{F}^H$
$\mathbf{E}_{12}=\mathbf{E}(0:M-1,M:\mathrm{end})$	$[\mathbf{V},\mathbf{\Phi}^H]=\mathrm{ED}(\mathbf{R}_1)$
$\mathbf{E}_{22}=\mathbf{E}(M:\mathrm{end},M:\mathrm{end})$	
$\mathbf{\Psi}=-\mathbf{E}_{12}\mathbf{E}_{22}^{-1}$	
$[\mathbf{T},\mathbf{\Phi}]=\mathrm{ED}(\mathbf{\Psi})$	

From Table 1, one verifies that the TLS-ESPRIT algorithm requires:

- 1 generalized eigendecomposition of a pair of $2N\times 2N$ matrices;
- 2 eigendecompositions (1 for a $2M\times 2M$ Hermitian matrix and 1 for an $M\times M$ non-Hermitian matrix);
- 1 full-matrix inversion of an $M\times M$ matrix;
- 6 matrix multiplications (5 for a pair of $M\times M$ matrices and 1 for the product of a $2N\times 2N$ and a $2N\times M$ matrices).

On the other hand, the new CB-DoA method requires:

- 1 generalized eigendecomposition of a pair of $N\times N$ matrices;
- 1 eigendecomposition of a $2N\times 2N$ Hermitian matrix;
- 1 diagonal-matrix inversion of an $M\times M$ matrix;
- 3 matrix multiplications of an $M\times M$ by an $M\times N$ matrices;
- 1 matrix subtraction of a pair of $N\times N$ matrices.

Although the computational cost of each method is highly implementation dependent, it is straightforward to verify that the proposed algorithm presents a smaller complex than ESPRIT algorithm. In fact, CB-DoA requires fewer matrix multiplications, includes a simpler (diagonal) matrix inversion, and requires a smaller generalized eigendecomposition, which is a very computationally intensive operation. In fact, according to [9], the generalized eigendecomposition of a pair of $2N \times 2N$ matrices is equivalent to one eigendecompostion of a $2N \times 2N$ matrix followed by an inversion of a matrix of equivalent dimensions. In Table 2, a detailed comparison is provided which shows the asymptotic complexity of the basic operations, according to [9]. In the next section, computer experiments are provided in order to quantify this computational complexity improvement.

Table 2: Comparison for the number of operations required by TLS-ESPRIT and CB-DoA.

Operation	Compl. [9]	ESPRIT	CB-DoA
Non-Herm. Eig.	$\mathcal{O}(25n^3)$	2	1
Herm. Eigendec.	$\mathcal{O}(n^2)$	1	1
Full Inversion	$\mathcal{O}(2n^3/3)$	2	1
Diag. Inversion	$\mathcal{O}(n)$	–	1
Multiplication	$\mathcal{O}(n^3)$	6	3
Subtraction	$\mathcal{O}(n^2)$	–	1

7. COMPUTER SIMULATIONS

7.1 Comparison between CB-DoA and ESPRIT

Some experiments were included to verify the performance of the CB-DoA algorithm. The symbols from each source were randomly generated from a Gaussian distribution with mean $\mu = 0.5$ and variance $\sigma^2 = 0.5$. Both the array manifold matrix and the DoA gain vector were randomly determined, following a similar Gaussian distribution as above. For estimating the covariance matrices $\mathbf{R}_x(0)$ and $\mathbf{R}_{xy}(0)$, 5,000 sample values were employed.

The metric used for performance assessment is the mean-square error (MSE), defined here as the arithmetical mean of the squared differences between the estimated and the actual arriving angles, $\hat{\theta}_i$ and θ_i, respectively, that is

$$\mathrm{MSE} = \frac{1}{M}\sum_{i=0}^{M-1} |\theta_i - \hat{\theta}_i|^2, \tag{32}$$

where the angles θ_i and its relation to the invariance matrix $\mathbf{\Phi}$ are defined in equation (9).

The MSE value, referring to an ensemble average over 300 runs, was determined for distinct values of the signal-to-noise ratio (SNR) measured at the receiver input.

Several distinct DoA setups were investigated in our simulations, including Setup 1 (with $M = 4$ signal sources and $N = 9$ receiving doublets) and Setup 2 (with $M = 7$ and $N = 12$). The MSE results for these two setups are depicted in Figs. 2 and 3, respectively, for the TLS-ESPRIT and CB-DoA algorithms. These figures indicate that both methods have similar MSE performances for a wide range of receiving SNR, and CB-DoA presents a slightly lower MSE. To assess

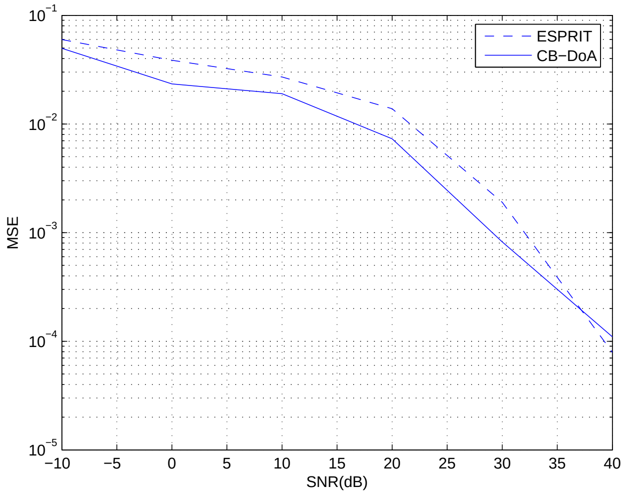

Figure 2: Estimate MSE for TLS-ESPRIT and CB-DoA algorithms as a function of the receiving SNR in Setup 1.

the computational complexity of each technique, the running time for several DoA setups was measured in a Pentium IV 3GHz PC, using a Matlab 7.0 platform on a Fedora Linux operating system. The results were averaged over 300 runs in the ensemble, for 5,000 samples. The running times were measured only for the algorithm themselves. Time spent on covariance estimations were not taken into account. The results are presented in Table 3. The column **Ratio** is defined as

$$\mathbf{Ratio} = \frac{\text{Time for CB-DoA}}{\text{Time for ESPRIT}}. \tag{33}$$

From Table 3, one can observe that in Setup 1 the CB-DoA running time was about 66% of the complexity associated to TLS-ESPRIT. This relationship improves even further favouring CB-DoA as the numbers of sources and sensors increase as also shown in Table 3.

Table 3: Average running time for TLS-ESPRIT and CB-DoA algorithms for distinct number of transmitting sources and receiving doublets.

Trans.	Doub.	TLS-ESPRIT	CB-DoA	Ratio
1	4	$1.17 \cdot 10^{-3}$s	$9.36 \cdot 10^{-4}$s	0.795
4	9	$1.92 \cdot 10^{-3}$s	$1.28 \cdot 10^{-3}$s	0.668
7	12	$3.64 \cdot 10^{-3}$s	$1.67 \cdot 10^{-3}$s	0.459
14	30	$1.34 \cdot 10^{-2}$s	$3.28 \cdot 10^{-3}$s	0.243
18	39	$2.91 \cdot 10^{-2}$s	$5.80 \cdot 10^{-3}$s	0.199

7.2 Comparison to the Cramer Rao Lower Bound

In order to assess the performance of CB-DoA algorithm in comparison to the theoretical limit represented by the Cramer-Rao Lower Bound (CRLB) [10], a new simulation environment was used, with $M = 1$ source and $N = 4$ uni-

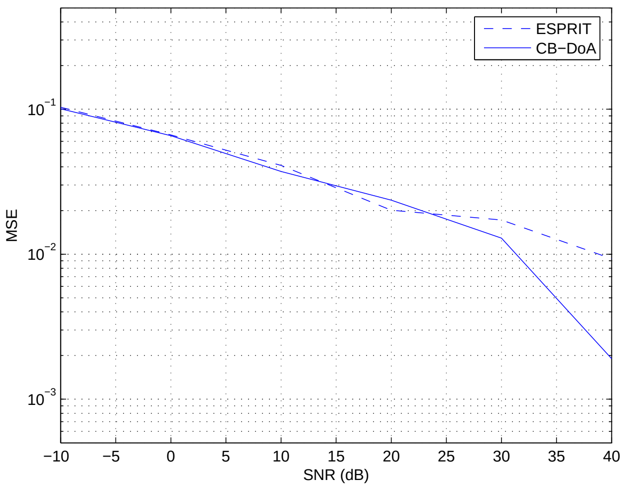

Figure 3: Estimate MSE for TLS-ESPRIT and CB-DoA algorithms as a function of the receiving SNR in Setup 2.

formly spaced sensors. The expression used for the CRLB is the approximation presented in [10] for $M = 1$ source and a large number of samples,

$$\mathrm{CRLB} = \frac{6}{\mathrm{SNR} \cdot N^3 K}, \quad (34)$$

where K denotes the number of samples and SNR is supposed to be equal in each sub-channel and represented in linear scale. MSE values were averaged over 300 runs in the ensemble. The simulation results are presented in Fig. 4.

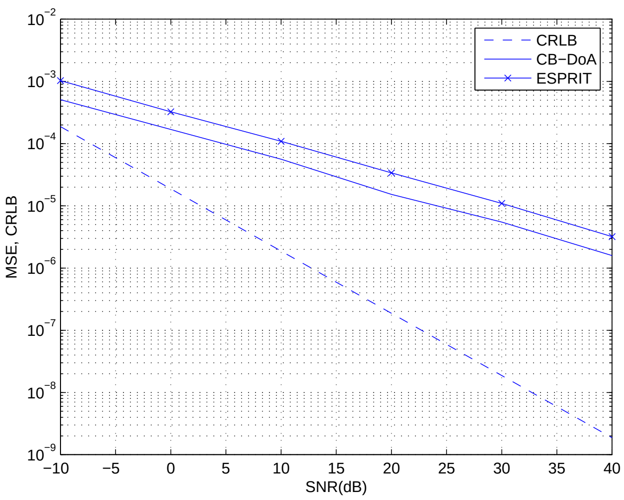

Figure 4: MSE Comparison for CB-DoA, ESPRIT, and approximate CRLB for $M = 1$ and $N = 4$.

8. CONCLUSIONS

A new method for estimating the direction-of-arrival (DoA) in an antenna array with the rotational invariance property between two subsets of antennas is described. The proposed covariance-based (CB) DoA algorithm originates from the TXK algorithm for the blind channel-equalization setup. The CB-DoA may be seen as an improved ESPRIT method, due to its lower computational complexity, while achieving equivalent performance for several receiving-SNR values. The improvement in computational complexity of ESPRIT is significant, since ESPRIT is known as a low-complexity algorithm for DoA estimation. Computer simulations confirm the CB-DoA reduced computational complexity and its robustness to the scalability of the DoA problem. Furthermore, the proposed CB-DoA presents an MSE performance closer to the approximate CRLB, given by Equation (34), than ESPRIT.

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