SUBSPACE TRACKING BASED ON THE CONSTRAINED PROJECTION APPROXIMATION APPROACH

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ABSTRACT

In this paper, we present an algorithm for tracking the signal subspace recursively. It is based on an interpretation of the signal subspace as the solution of a minimization of a constrained projection approximation task. We show that we can apply the matrix inversion lemma to solve this problem recursively. Proposed algorithm avoids orthonormalization process after each update for post-processing algorithms which need orthonormal basis of the signal subspace. Simulation results in the direction of arrival (DOA) tracking context depict high performance of this algorithm in comparison with other algorithms.

1. INTRODUCTION

The interest in subspace-based methods stems from the fact that they consist of splitting the observations into a set of desired and a set of disturbing components, which can be viewed in terms of signal and noise subspaces respectively. Subspace-based high-resolution methods have been applied in numerous analyses such as the MUSIC, the minimum-norm, the ESPRIT, and the weighted subspace fitting (WSF) methods for estimating frequencies of sinusoids or directions of arrival (DOA) of plane waves impinging on an antenna array. The estimation of the signal subspace is commonly based on the traditional eigenvalue decomposition (EVD) or singular value decomposition (SVD). However, the main drawback of these decompositions is their inherent complexity.

In order to overcome this difficulty, a large number of approaches have been introduced for fast subspace tracking in the context of adaptive signal processing. Most of these techniques can be grouped into three families. In the first one, classical batch methods for EVD/SVD like QR algorithm, Jacobi rotation, power iteration, and Lancoz method have been modified to fit adaptive processing [1]-[3]. In the second family, variations and extensions of Bunch's rank-one updating algorithm [4] such as subspace averaging [5] have been proposed. The third class of algorithms considers the EVD/SVD as a constrained or unconstrained optimization problem, for which the introduction of a projection approximation hypothesis leads to fast subspace tracking methods (see, e.g., the PAST [6] and NIC [7] algorithms).

Some of these approaches add orthonormalization step to achieve orthonormal eigenvectors [8], which increases the computational load. The necessity of orthonormalization depends on the post-processing method which uses the signal subspace estimate to extract the desired signal information. If we are using MUSIC or minimum-norm method for calculating DOA's or frequencies from the signal subspace, for which orthonormal basis of the signal subspace is required, orthonormalization step is crucial.

In this paper we present a recursive algorithm for tracking the signal subspace spanned by the eigenvectors corresponding to the r largest eigenvalues, where r is the dimension of signal subspace. This algorithm relies on an interpretation of the signal subspace as the solution of an approximated projection based on a constrained optimization problem whose solution gives the orthonormal basis. We will derive both exact and recursive solutions for this problem. We call our approach as constrained projection approximation subspace tracking (CPAST). This algorithm avoids the orthonormalization step. Simulation results are given to compare performance of the CPAST algorithm with PAST and approximated power iteration (API) [9] algorithms in the context of adaptive DOA estimation.

2. SIGNAL MATHEMATICAL MODEL

Consider the samples $\mathbf{x}(t)$, recorded during the observation time on the n sensor outputs of an array, satisfying the following model:

$$\mathbf{x}(t) = \mathbf{A}(\theta)\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{x} \in C^n$ is the vector of sensor outputs, $\mathbf{s} \in C^r$ is the vector of complex signal amplitudes, $\mathbf{n} \in C^n$ is an additive noise vector, $\mathbf{A}(\theta)=[\mathbf{a}(\theta_1),\mathbf{a}(\theta_2),\ldots,\mathbf{a}(\theta_r)] \in \mathrm{C}^{n\times r}$ is the matrix of the steering vectors $\mathbf{a}(\theta_j)$, and θ_j, j=1,2,…,r is the parameter of the jth source, for example its DOA. It is assumed that $\mathbf{a}(\theta_j)$ is a smooth function of θ_j and that its form is known (i.e. the array is calibrated). We assume that the elements of $\mathbf{s}(t)$ are stationary random processes, and the elements of $\mathbf{n}(t)$ are zero-mean stationary random processes which are uncorrelated with the elements of $\mathbf{s}(t)$. The covariance matrix of the sensors' outputs can be written in the following form:

$$\mathbf{R} = E\{\mathbf{x}(t)\mathbf{x}^H(t)\} = \mathbf{ASA}^H + \mathbf{R}_n$$

where $\mathbf{S} = E\{\mathbf{s}(t)\mathbf{s}^H(t)\}$ is the signal covariance matrix assumed to be nonsingular ("H" denotes Hermitian transposition), and $\mathbf{R}_n$ is the noise covariance matrix. A large number of methods such as SVD or EVD use covariance matrix of data to estimate the signal subspace.

3. SIGNAL SUBSPACE INTERPRETATION

Let $\mathbf{x} \in C^n$ be a complex valued random vector process with the autocorrelation matrix $\mathbf{C} = E\{\mathbf{xx}^H\}$ which is assumed to be positive definite. The orthonormal eigenvectors and the positive eigenvalues of $\mathbf{C}$ are denoted by $\mathbf{u}_i$ and λ_i (i=1,2,…,n) respectively. Equivalently, we have $\mathbf{C}=\mathbf{U}\sum\mathbf{U}^H$ with $\mathbf{U}=[\mathbf{u}_1,\mathbf{u}_2,\ldots,\mathbf{u}_n]$ and $\sum=\text{diag}(\lambda_1, \lambda_2,\ldots, \lambda_n)$.

We consider the following minimization problem:

$$\underset{\mathbf{W}}{\text{minimize}} \quad J(\mathbf{W}) = E\left\|\mathbf{x} - \mathbf{WW}^H\mathbf{x}\right\|^2 \tag{2}$$

where $\mathbf{W}$ is a $n\times r$ ($r\leq n$) full rank matrix. It can be shown that $J(\mathbf{W})$ has a global minimum and the columns of the solution of the above problem are orthonormal and span the signal subspace (see [6]). Thus, the use of an iterative algorithm to minimize $J(\mathbf{W})$ will always converge to an orthonormal basis of the signal subspace without any orthonormalization operations during the iteration. Although the capability of gradient based subspace update approaches is clear to us, it is not the aim of this paper to use these approaches. Instead, we replace the expectation in (2) with an exponentially weighted sum as follows:

$$\underset{\mathbf{W}}{\text{minimize}} \quad J(\mathbf{W}(t)) = \sum_{i=1}^{t}\beta^{t-i}\left\|\mathbf{x}(i) - \mathbf{W}(t)\mathbf{W}^H(t)\mathbf{x}(i)\right\|^2 \tag{3}$$

and we will try to solve this problem recursively. All sample vectors available in the time interval $1\leq i\leq t$ are involved in estimating the signal at the time instant t. The use of the forgetting factor $0<\beta\leq 1$ is intended to ensure that data in the distant time are downweighted in order to afford the tracking capability when the system operates in a nonstationary environment. $J(\mathbf{W}(t))$ is a fourth-order function of elements of $\mathbf{W}(t)$. The key issue of the projection approximation subspace tracking (PAST) approach is to approximate $\mathbf{W}^H(t)\mathbf{x}(i)$ in (3), the unknown projection of $x(i)$ onto the columns of $\mathbf{W}(t)$, by the expression $y(i)=\mathbf{W}^H(i\text{-}1)\mathbf{x}(i)$, which can be calculated for $1\leq i\leq t$ at the time instant t. This results in a modified cost function:

$$J'(\mathbf{W}(t)) = \sum_{i=1}^{t}\beta^{t-i}\left\|\mathbf{x}(i) - \mathbf{W}(t)\mathbf{y}(i)\right\|^2 \tag{4}$$

which is quadratic in the elements of $\mathbf{W}(t)$. This results in the following minimization problem:

$$\underset{\mathbf{W}}{\text{minimize}} \quad J'(\mathbf{W}(t)) = \sum_{i=1}^{t}\beta^{t-i}\left\|x(i) - \mathbf{W}(t)\mathbf{y}(i)\right\|^2$$

The solution to this problem (the PAST solution) is as follows [6]:

$$\mathbf{W}(t) = \mathbf{C}_{\mathbf{xy}}(t)(\mathbf{C}_{\mathbf{yy}}(t))^{-1}$$

where

$$\mathbf{C}_{\mathbf{xy}}(t) = \sum_{i=1}^{t}\beta^{t-i}\mathbf{x}(i)\mathbf{y}^{\mathbf{H}}(i) = \beta\mathbf{C}_{\mathbf{xy}}(t-1) + \mathbf{x}(t)\mathbf{y}^{\mathbf{H}}(t) \tag{5}$$

$$\mathbf{C}_{\mathbf{yy}}(t) = \sum_{i=1}^{t}\beta^{t-i}\mathbf{y}(i)\mathbf{y}^{\mathbf{H}}(i) = \beta\mathbf{C}_{\mathbf{yy}}(t-1) + \mathbf{y}(t)\mathbf{y}^{\mathbf{H}}(t)$$

We note that the PAST algorithm is derived by minimizing the modified cost function in (4) instead of the original one in (2). Hence, the columns of $\mathbf{W}(t)$ are not exactly orthonormal. The deviation from the orthonormality depends on the signal to noise ratio (SNR) and the forgetting factor β. This lack of orthonormality affects seriously the performance of post-processing algorithms which are dependant on orthonormality of the basis. To overcome this problem, we define the following constrained optimization problem:

$$\underset{\mathbf{W}}{\text{minimize}} \quad J'(\mathbf{W}(t)) = \sum_{i=1}^{t}\beta^{t-i}\left\|\mathbf{x}(i) - \mathbf{W}(t)\mathbf{y}(i)\right\|^2 \quad \textit{subject to} \quad \mathbf{W}^H(t)\mathbf{W}(t) = \mathbf{I}_r \tag{6}$$

where $\mathbf{I}_r$ is the $r\times r$ identity matrix and it is clear that the constraint in (6) guarantees the orthonormality. To solve this constrained problem we use Lagrange multipliers method. So, after expanding the expression for $J'(\mathbf{W}(t))$, we can replace (6) with the following problem:

$$\underset{\mathbf{W}}{\text{minimize}} \quad h(\mathbf{W}) = tr(\mathbf{C}) - 2tr(\sum_{i=1}^{t}\beta^{t-i}\mathbf{x}(i)\mathbf{y}^H(i)\mathbf{W}^H(t)) + tr(\sum_{i=1}^{t}\beta^{t-i}\mathbf{y}(i)\mathbf{y}^H(i)\mathbf{W}^H(t)\mathbf{W}(t)) + \lambda\left\|\mathbf{W}^H\mathbf{W} - \mathbf{I}_r\right\|_F^2$$

where $tr(\boldsymbol{C})$ is the trace of the matrix $\boldsymbol{C}$, $\|.\|_F$ denotes the Frobenius norm, and λ is the Lagrange multiplier. Let $\nabla h = 0$, where ∇ is the gradient operator with respect to $\mathbf{W}$, then we have:

$$-\sum_{i=1}^{t}\beta^{t-i}\mathbf{x}(i)\mathbf{y}^H(t) + \sum_{i=1}^{t}\beta^{t-i}\mathbf{W}(\mathbf{t})\mathbf{y}(i)\mathbf{y}^H(t) + \lambda[-2\mathbf{W}(\mathbf{t}) + 2\mathbf{W}(\mathbf{t})\mathbf{W}^H(\mathbf{t})\mathbf{W}(\mathbf{t})] = 0 \tag{7}$$

If we obtain $\mathbf{W}$ from aforementioned equation and use it in $\mathbf{W}^H\mathbf{W} = \mathbf{I}_r$, after some manipulations we obtain:

$$\sum_{i=1}^{t}\beta^{t-i}\mathbf{y}(i)\mathbf{y}^H(i)-2\lambda\mathbf{I}_r+2\lambda\mathbf{W}^H(t)\mathbf{W}(t)=$$
$$[\sum_{i=1}^{t}\beta^{t-i}\mathbf{y}(i)\mathbf{x}^H(i)\sum_{i=1}^{t}\beta^{t-i}\mathbf{x}(i)\mathbf{y}^H(i)]^{\frac{1}{2}} \tag{8}$$

where $(.)^{1/2}$ denotes the square root of a matrix.
Now, using (8), we can remove λ from equation (7) and attain the following solution:

$$\mathbf{W}(t)=\mathbf{C_{xy}}(t)(\mathbf{C}^H_{\mathbf{xy}}(t)\mathbf{C_{xy}}(t))^{\frac{-1}{2}} \tag{9}$$

This constrained projection approximation subspace tracking (CPAST) algorithm guarantees the orthonormality of the columns of **W**(t). The general form of solution of CPAST algorithm is similar to PAST except its square root.

4. ADAPTIVE SUBSPACE TRACKING

Subspace tracking methods have applications in numerous domains, including the fields of adaptive filtering, source localization, and parameter estimation. In many of these applications we have a continuous stream of data. Thus, developing adaptive algorithms is very useful for these applications. An efficient and numerically robust recursive solution for (6) can be obtained by using the matrix inversion lemma to compute the inverse of $\mathbf{C}^H_{\mathbf{xy}}(t)\mathbf{C_{xy}}(t)$ in (9).
The matrix inversion lemma can be written as follows:

$$(\mathbf{A}+\mathbf{BCD})^{-1}=\mathbf{A}^{-1}-\mathbf{A}^{-1}\mathbf{B}(\mathbf{DA}^{-1}\mathbf{B}+\mathbf{C}^{-1})^{-1}\mathbf{DA}^{-1} \tag{10}$$

We define matrix $\mathbf{\Phi}(t)$ as below:

$$\mathbf{\Phi}(t)=\mathbf{C}^H_{\mathbf{xy}}(t)\mathbf{C_{xy}}(t) \tag{11}$$

We replace the first $\mathbf{C_{xy}}(t)$ term in (11) with its recursive formula in (5), to obtain:

$$\mathbf{\Phi}(t)=\beta\mathbf{C}^H_{\mathbf{xy}}(t-1)\mathbf{C_{xy}}(t)+\mathbf{y}(t)\mathbf{x}^H(t)\mathbf{C_{xy}}(t)$$

Now, we define matrix **A**, vectors **B** and **D**, and scalar C in the following form:

$$\mathbf{A}=\beta\mathbf{C}^H_{\mathbf{xy}}(t-1)\mathbf{C_{xy}}(t) \tag{12}$$
$$\mathbf{B}=\mathbf{y}(t) \tag{13}$$
$$C=1 \tag{14}$$
$$\mathbf{D}=\mathbf{x}^H(t)\mathbf{C_{xy}}(t) \tag{15}$$

Then, using (10) and (12-15), we have:

$$\mathbf{P}(t)=\mathbf{\Phi}^{-1}(t)=\mathbf{A}^{-1}-\frac{\mathbf{A}^{-1}\mathbf{y}(t)\mathbf{x}^H(t)\mathbf{C_{xy}}(t)\mathbf{A}^{-1}}{\mathbf{x}^H(t)\mathbf{C_{xy}}(t)\mathbf{A}^{-1}\mathbf{y}(t)+1} \tag{16}$$

Now, we define matrix $\mathbf{A}'$, vectors $\mathbf{B}'$ and $\mathbf{D}'$, and scalar $\mathbf{C}'$, in the following form:

$$\mathbf{A}'=\beta^2\mathbf{\Phi}(t-1) \tag{17}$$
$$\mathbf{B}'=\beta\mathbf{C}^H_{\mathbf{xy}}(t-1)\mathbf{x}(t) \tag{18}$$
$$\mathbf{C}'=1 \tag{19}$$
$$\mathbf{D}'=\mathbf{y}^H(t) \tag{20}$$

Substituting $\mathbf{C_{xy}}(t)$ from (5) into (12), we have:

$$\mathbf{A}=\beta^2\mathbf{\Phi}(t-1)+\beta\mathbf{C}^H_{\mathbf{xy}}(t-1)\mathbf{x}(t)\mathbf{y}^H(t) \tag{21}$$

Then, using (17-20) and applying MIL to (21), we obtain the inverse of A as follows:

$$\mathbf{A}^{-1}=\mathbf{P}(t-1)\mathbf{E}(t) \tag{22}$$

where

$$\mathbf{E}(t)=\beta^{-2}\mathbf{I}_r-\frac{\beta^{-2}\mathbf{C}^H_{\mathbf{xy}}(t-1)\mathbf{x}(t)\mathbf{y}^H(t)\mathbf{P}(t-1)}{\mathbf{y}^H(t)\mathbf{P}(t-1)\mathbf{C}^H_{\mathbf{xy}}(t-1)\mathbf{x}(t)+\beta}$$

and $\mathbf{I}_r$ is the r by r identity matrix. Substituting (22) into (16), we have:

$$\mathbf{P}(t)=\mathbf{P}(t-1)\mathbf{E}(t)-\mathbf{P}(t-1)\mathbf{E}(t)\mathbf{y}(t)\mathbf{K}(t) \tag{23}$$

where

$$\mathbf{K}(t)=\frac{\mathbf{x}^H(t)\mathbf{C_{xy}}(t)\mathbf{P}(t-1)\mathbf{E}(t)}{\mathbf{x}^H(t)\mathbf{C_{xy}}(t)\mathbf{P}(t-1)\mathbf{E}(t)\mathbf{y}(t)+1} \tag{24}$$

Using (24) and (23), it can be shown that:

$$\mathbf{K}(t)=\mathbf{x}^H(t)\mathbf{C_{xy}}(t)\mathbf{P}(t) \tag{25}$$

If we substitute $\mathbf{C_{xy}}(t)$ from (5) into (25), we have:

$$\mathbf{K}(t)=\beta\mathbf{x}^H(t)\mathbf{C_{XY}}(t-1)+\mathbf{x}^H(t)\mathbf{x}(t)\mathbf{y}^H(t) \tag{26}$$

It follows from (26) that:

$$\mathbf{Q}(t)=\mathbf{x}(t)\mathbf{y}^H(t)\mathbf{P}(t)=\mathbf{x}(t)\frac{\mathbf{K}(t)-\beta\mathbf{x}^H(t)\mathbf{C_{xy}}(t-1)\mathbf{P}(t)}{\mathbf{x}^H(t)\mathbf{x}(t)} \tag{27}$$

Now, using (5), (9) and (16), we can write:

$$\mathbf{W}(t)=\mathbf{C_{xy}}(t)\mathbf{P}^{\frac{1}{2}}(t)=(\beta\mathbf{C_{xy}}(t-1)+\mathbf{x}(t)\mathbf{y}^{\mathbf{H}}(t))\mathbf{P}^{\frac{1}{2}}(t) \tag{28}$$

To achieve a recursive form for **W**(t), we multiply both sides of (28) by $\mathbf{P}^{\frac{H}{2}}(t)$ and use (27) to obtain:

$$\mathbf{W}(t)\mathbf{P}^{\frac{H}{2}}(t)=\beta\mathbf{C_{xy}}(t-1)\mathbf{P}(t)+\mathbf{Q}(t)$$

Now we replace **P**(t) with the right hand side of (23) to obtain:

$$\mathbf{W}(t)\mathbf{P}^{\frac{H}{2}}(t)=\beta\mathbf{C_{xy}}(t-1)\mathbf{P}(t-1)\mathbf{E}(t)-\beta\mathbf{C_{xy}}(t-1)\mathbf{P}(t-1)\mathbf{E}(t)\mathbf{y}(t)\mathbf{K}(t)+\mathbf{Q}(t) \tag{29}$$

Combining (28) and (29), we can write the following recursive equation:

Table 1. The CPAST algorithm for tracking the signal subspace

Computation Process
Choose appropriate values for $\mathbf{P}(0)$, $\mathbf{W}(0)$, and $\mathbf{C_{xy}}(0)$
FOR $t = 1,2,...$ DO
$\mathbf{y}(t) = \mathbf{W}^H(t-1)\mathbf{x}(t)$
$\mathbf{C_{xy}}(t) = \beta\mathbf{C_{xy}}(t-1) + \mathbf{x}(t)\mathbf{y}^H(t)$
$\mathbf{\Phi}(t) = \mathbf{C}^H_{\mathbf{xy}}(t)\mathbf{C_{xy}}(t)$
$\mathbf{a}(t) = \mathbf{y}^H(t)\mathbf{P}(t-1)$
$\mathbf{b}(t) = \mathbf{C}^H_{\mathbf{xy}}(t-1)\mathbf{x}(t)$
$\mathbf{E}(t) = \beta^{-2}\left[\mathbf{I}_r - \mathbf{b}(t)\mathbf{a}(t)/(\mathbf{a}(t)\mathbf{b}(t)+\beta)\right]$
$\mathbf{g}(t) = \mathbf{x}^H(t)\mathbf{C_{xy}}(t)\mathbf{P}(t-1)\mathbf{E}(t)$
$\mathbf{K}(t) = \mathbf{g}(t)/(\mathbf{g}(t)\mathbf{y}(t)+1)$
$\mathbf{F}(t) = \mathbf{E}(t)(\mathbf{I}_r - \mathbf{y}(t)\mathbf{K}(t))$
$\mathbf{P}(t) = \mathbf{P}(t-1)\mathbf{F}(t)$
$\mathbf{Q}(t) = \mathbf{x}(t)\left(\mathbf{k}(t) - \beta\,\mathbf{b}^H(t)\mathbf{P}(t)\right)/(\mathbf{x}^H(t)\mathbf{x}(t))$
$\mathbf{W}(t)\mathbf{P}^{\frac{H}{2}}(t) = \beta\mathbf{W}(t-1)\mathbf{P}^{\frac{H}{2}}(t-1)\mathbf{F}(t) + \mathbf{Q}(t)$
$\mathbf{W}(t) = [\mathbf{W}(t)\mathbf{P}^{\frac{H}{2}}(t)]\mathbf{\Phi}^{\frac{H}{2}}(t)$

$$\mathbf{W}(t)\mathbf{P}^{\frac{H}{2}}(t) = \beta\mathbf{W}(t-1)\mathbf{P}^{\frac{H}{2}}(t-1)\mathbf{E}(t) - \beta\mathbf{W}(t-1)\mathbf{P}^{\frac{H}{2}}(t-1)\mathbf{E}(t)\mathbf{y}(t)\mathbf{K}(t) + \mathbf{Q}(t)$$

The above equation is used for updating $\mathbf{W}(t)\mathbf{P}^{\frac{H}{2}}(t)$ in each iteration. Finally $\mathbf{W}(t)$ is obtained as follows:

$$\mathbf{W}(t) = [\mathbf{W}(t)\mathbf{P}^{\frac{H}{2}}(t)]\mathbf{\Phi}^{\frac{H}{2}}(t) \qquad (30)$$

Table 1 summarizes this recursive CPAST algorithm for tracking the signal subspace. It can be shown that computational complexity of this algorithm is $O(nr^2)$ which is much less than the direct computation of SVD or EVD.

Appropriate initial values should be chosen for $\mathbf{P}(0)$ and $\mathbf{W}(0)$. $\mathbf{P}(0)$ must be a Hermitian positive definite matrix and $\mathbf{W}(0)$ should contain r orthonormal vectors. The choice of these initial values affects the transient behavior but not the steady state behavior of the algorithm. The simplest way is to set $\mathbf{P}(0)$ to the $r\times r$ identity matrix and the columns of $\mathbf{W}(0)$ to the first r columns of the $n\times n$ identity matrix.

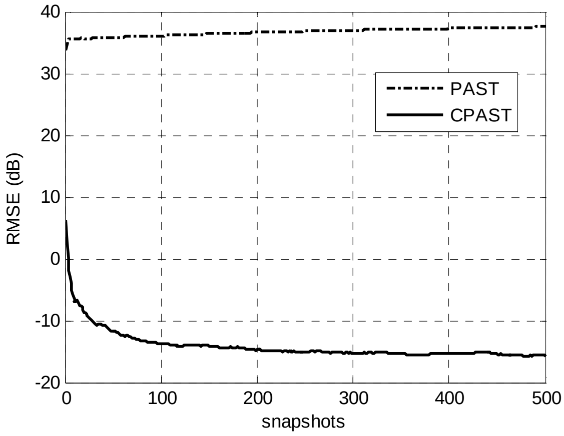

Figure 1. Root mean square error in DOA estimation vs. snapshots

5. SIMULATION RESULTS

In this section, we use simulations to demonstrate the applicability and performance of the CPAST algorithm. To do so, we consider the proposed algorithm in DOA estimation context. We use MUSIC algorithm for finding the DOA's of signal sources impinging on an array of sensors. Let $\{\mathbf{s}_i\}_{i=1}^n$ denote the orthonormal eigenvectors of covariance matrix $\mathbf{R}$ and let $\mathbf{S} = (\mathbf{s}_1,...,\mathbf{s}_r)$. We assume that the corresponding eigenvalues of $\mathbf{R}$ are sorted in descending order. We know that consistent estimates of the DOA's can be determined as the minimizing arguments of the following cost function:

$$f_{MUSIC}(\theta) = \mathbf{a}^H(\theta)\left(\mathbf{I}_n - \mathbf{SS}^H\right)\mathbf{a}(\theta)$$

where $\mathbf{S}$ is the orthonormal basis of the signal subspace and $\mathbf{I}_n$ is the identity matrix of dimension n. The definition of $\mathbf{S}$ shows that the CPAST algorithm can be used for estimating it. Once $\mathbf{S}$ was estimated, it can be used in the MUSIC algorithm for finding the desired DOA's.

We consider a uniform linear array where the number of sensors is m=17 and the distance between adjacent sensors is equal to half wavelength. We use the forgetting factor β=0.97. To illustrate the effect of orthonormal basis on the MUSIC algorithm, we consider two signal sources which are in directions (-5^o, 5^o) and their SNR's are 0 dB. Figure 1 shows the root mean square error of DOA estimates obtained by using PAST and CPAST algorithms. This figure shows the sensitivity of MUSIC to orthonormality of basis. In all figures in this section, except figure 2, the number of simulation runs used for obtaining each point is equal to 100.

Principal angle is a measure of the difference between the subspaces spanned by the columns of $\mathbf{S}(t)$ and of the matrix $\mathbf{A}$ in the signal model (1) [10]. The principal angles are zero if the compared subspaces are identical. In figure 2, we have depicted all (r=2) principal angles for the previously defined DOA estimation problem. This figure is depicted for one simulation run. Figure 2 shows the low

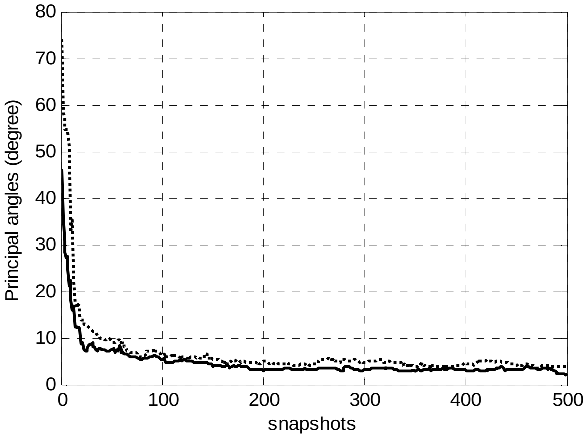

Figure 2. Principal angles vs. snapshots for CPAST estimates

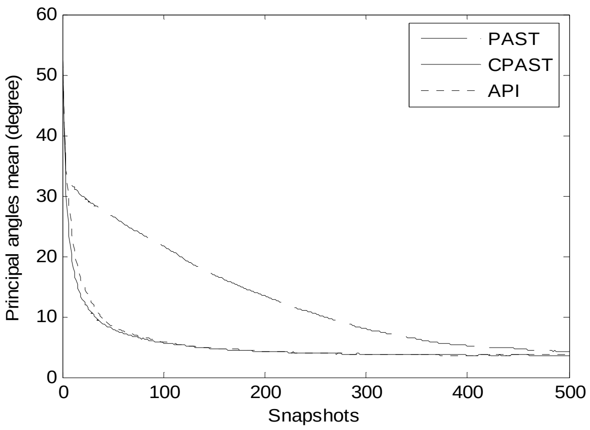

Figure 3. Principal angles mean vs. snapshots

variability of CPAST estimates with time.

Figure 3 compares mean of principal angles for three subspace tracking methods PAST, CPAST and API.

It turns out from this figure that CPAST gives much better performance than PAST.

The deviation of the subspace weighting matrix $\mathbf{W}(t)$ from orthonormality can be measured by means of the following error criterion [9]:

$$20\log\left(\left\|\mathbf{W}^{H}(t)\mathbf{W}(t)-\mathbf{I}_r\right\|_F\right)$$

Figure 4 shows the mean of the above error criterion for the three algorithms PAST, CPAST and API when applied to the previously defined DOA estimation problem. It can be seen that CPAST provides full or nearly full orthonormality of the basis. In spite of PAST, CPAST shows very good performance even in the first snapshots.

6. CONCLUSION

In this paper, we introduced an interpretation of the signal subspace as the solution of a constrained optimization problem. Then, we derived the solution of this problem and discussed the applicability of the so-called CPAST algorithm for tracking the subspace. We derived a recursive formulation of this solution for adaptive implementation. Our algorithm avoids the orthonormalization of basis in each update. Simulation results in DOA tracking context showed the good performance of the propose algorithm.

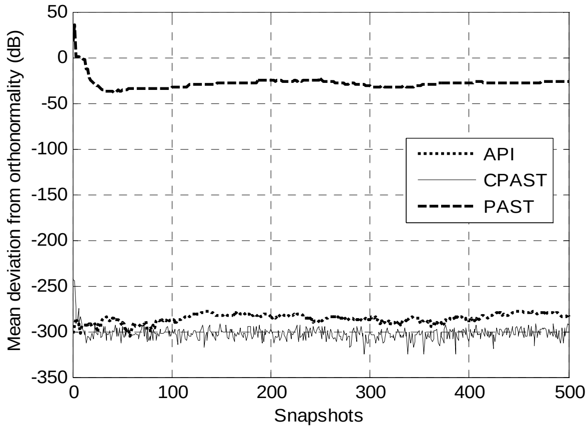

Figure 4. Deviation from orthonormality vs. snapshots

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