REGISTRATION OF MULTIPLE REGIONS DERIVED FROM THE OPTICAL FLOW MODEL AND THE ACTIVE CONTOUR FRAMEWORK

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ABSTRACT

In this paper, we present a new paradigm to carry out the non-rigid registration of multiple regions with a dense deformation field derived from the optical flow model and the active contour framework. The method can merge different tasks such as registration, segmentation and incorporation of prior knowledge into a single framework. The technique is based on finding the deformation field which minimizes an energy functional designed to be minimal when the curves selected in the moving image have reached the target objects. This way, we obtain registration results that are accurate on the considered contours with a deformation field smooth over the whole image. Finally, the registration of multiple regions is handled with a label function. A well-suited application of our registration model is the atlas-based segmentation in medical imaging. Results on 2D synthetic and real data show the performance of our registration algorithm.

1. INTRODUCTION

Atlas-based segmentation of medical images has become a standard paradigm for exploiting prior anatomical knowledge in image segmentation. In the majority of approaches proposed so far to register an atlas to a patient image, the objective of the transformation is to optimize some global intensity-based correspondence measure like gray-level differences or mutual information. The main limitation of these methods is that they often lead to a compromise between the accuracy of the registration and the smoothness of the deformation. When at some places the registration is not accurate enough, a widely-used solution is to globally or locally allow more variability in the registration model in order to obtain more local deformation but with the risk of creating irregularities in the deformation field. Moreover, this does not assure that the desired level of precision will be obtained. To cope with this problem, more local constraints have to be included in the registration process. These constraints should allow for more importance to be given to the registration of relevant structures (to be robust regarding possible inconsistencies between both images) and to introduce more prior knowledge, such as the intensity distribution or the admissible shapes of the objects selected to drive the registration. Thus, we introduce in this paper a registration algorithm able to make an accurate registration, driven by the objects present in the image while ensuring a regular deformation field all over the image. This algorithm has been developed in the active contour framework [2] because it is particularly well suited to define and implement local constraints, as we will see later in this paper.

The active contour technique was initially designed for image segmentation. It looks for the optimal boundary of an object by minimizing an energy functional. That way, one gets accurate segmentation results. So far, the active contour model has been used to segment objects in an image using boundary-based, region-based or shape-based information. The registration technique we present here will benefit from all of these families of active contours in determining a dense deformation field defined on the whole image. We propose a formulation adapting the active contour-based segmentation framework to registration. This formalism is derived from the combination of the general level set approach [3] with the optical flow model [4]. Thus, we will see that different tasks such as segmentation, incorporation of prior knowledge and registration can be merged into a single framework. In this new framework, the deformation field becomes the important variable and no longer the contour as in the standard active contour approach. However, our registration method keeps the same advantages of level set-based segmentation in terms of accuracy and prior knowledge. In fact we will show that the evolution terms designed for the image segmentation process can be re-used for the image registration task.

This work is not the first attempt at integrating the active contours framework in a non-rigid registration context. The main source of inspiration for this work is the non-parametric approach proposed by Vemuri et al. in [11]. They propose a registration algorithm based on the level sets of the moving image. In this paper, the level sets correspond to the contours naturally present in images. In [14] we propose to take the idea of level sets registration from [11] by recovering the dense deformation field from the level set function of Osher and Sethian [3], instead of the gray level intensities of the image [11]. This way, we were able to derive benefit from the numerous segmentation models developed in the level set framework. However one level set function can represent several objects in an image but the contours of these objects have to be non-connected. Beside, in the Vemuri's model, connected and non connected can be modeled with only one function. In this paper, we propose to combine the advantages of [11] with our previous model [14] by modeling the active contour by a label function. We note that in the active contours framework, the idea of using labels to perform a multiphase segmentation has recently been proposed by Cremers et al. in [13].

The paper is organized as follows. First, the general evolution equation for level sets-based registration based on the optical flow model demonstrated in [14] is presented. After, we describe the method we propose to perform the registration of multiple regions based on a label function. Then, we present the use of a region-based term arising directly from the standard active contour framework in the proposed registration approach. Finally we illustrate the performance of our algorithm on synthetic and real 2D data.

2. REGISTRATION METHOD

2.1 Active Contours in the Level Set Framework

In the active contour framework, two types of representation were defined to model the contour.

In the first type of representation, "the snake method" [2] proposes a parametric representation of the contour by a linear combination of basis functions (e. g. splines, wavelets, ...) as follows:

$$C(q) = \sum_{i=0}^{K} p_i b_i(q), \tag{1}$$

where $C(q)$ is the curve parameterized by q, p_i are the control points, b_i the basis functions and K the number of control points/basis functions. The evolution of the snake is given by the displacement vectors of its control points such as:

$$\frac{\partial C(q,t)}{\partial t} = -FN, \tag{2}$$

where F is the velocity of the flow (or speed function) and N is the unit normal to the curve C.

In the second type of representation, "the level set method" ([3], [7], [10]) proposes an implicit representation of the contour with a level set function which is usually given by a signed distance map as follows:

$$\phi(x) = \begin{cases} 0 & x \in C \\ -d(x) & x \in \Omega_{in} \\ +d(x) & x \in \Omega \setminus \Omega_{in}, \end{cases} \tag{3}$$

where $\phi(x)$ is the level set function at the image point x, d is the Euclidean distance to the closest contour point on C, Ω is the image domain, Ω_{in} is the image area inside the contour and $\Omega \setminus \Omega_{int}$ is the image area outside the contour.

In order to derive the motion equation, the level set values of the evolving contour $C(t)$ are constrained to always remain zero, which means:

$$\phi(C(t),t) = 0. \tag{4}$$

By deriving (4) with the chain rule and combining the result with (2), we obtain the standard active contour evolution equation in the level set representation:

$$\underbrace{\frac{\delta\phi}{\delta C}}_{\nabla\phi}\underbrace{\frac{\partial C}{\partial t}}_{-FN} + \frac{\partial \phi}{\partial t} = 0 \Rightarrow \phi_t = F|\nabla\phi|, \tag{5}$$

where $\phi_t = \frac{\partial\phi}{\partial t}$. Note that the unit normal vector N is here defined for each isophote, i.e. curve with the same level, as follows:

$$N = \frac{\nabla\phi}{|\nabla\phi|}. \tag{6}$$

Due to its implicit representation and global contour description, we have postulated in [14] that the level set representation is a good support for estimating a deformation field defined on the whole image domain.

2.2 Deformation Field Derived from the Optical Flow and the Level Set Framework

In [14], the dense deformation field needed in registration is computed by tracking the motion of the level set function, Equation (5), with the optical flow method. Optical flow is a well-known pixel-based image registration technique (for a detailed survey, see Barron et al. [4]).

The optical flow method is based on the assumption that the brightness of the moving image, here the isophotes of the level set function $\phi(x,t)$, stays constant for small displacements, and for a short period of time:

$$\phi(x,t) = \phi(x+u, t+dt) \Rightarrow d\phi(x,t) = 0, \tag{7}$$

where u is the deformation vector field and $d\phi$ is the total derivative of ϕ.

The optical flow constraint (7) can thus be rewritten as:

$$\bigtriangledown\phi \frac{\partial u(x,t)}{\partial t} + \frac{\partial\phi}{\partial t} = 0. \tag{8}$$

As a result, we get the evolution equation of the vector flow from (8):

$$\frac{\partial u(x,t)}{\partial t} = -\frac{\phi_t}{|\bigtriangledown\phi|}\frac{\bigtriangledown\phi}{|\bigtriangledown\phi|}, \tag{9}$$

where ϕ_t, given by Equation (5), represents the variation of the level set function according to the desired application such as segmentation, registration, regularization, etc. Thus, by introducing (5) in (9), we obtain the following formula *merging the active contour segmentation framework with the image registration task*:

$$\frac{\partial u(x,t)}{\partial t} = -FN, \tag{10}$$

where N represents the unit normal vector of level sets. Thus Equation (10) generates deformations radial to the active contours.

The speed function F and the normal vector field N in (10) are determined from the level set function $\phi(x,t)$ and the deformation field $u(x,t)$. The level set function ϕ at time t is given by the deformation field $u(x,t)$ and the initial level set function $\phi(x,0)$ such that:

$$\phi(x,t) = \phi(x+u(x,t),0), \tag{11}$$

which ensures that the evolution of the level set function exactly corresponds to the current deformation.

Then, introducing Equation (11) in (10) yields:

$$\frac{\partial u(x,t)}{\partial t} = -F(\phi(x+u(x,t),0))\frac{\bigtriangledown\phi(x+u(x,t),0)}{|\bigtriangledown\phi(x+u(x,t),0)|}, \tag{12}$$

Equation (11) is modified to stabilize the numerical computation when $\bigtriangledown\phi$ is close to zero as follows:

$$\frac{\partial u(x,t)}{\partial t} = -F(\phi(x+u(x,t),0))\frac{\bigtriangledown\phi(x+u(x,t),0)}{\sqrt{\bigtriangledown\phi(x+u(x,t),0)^2+\varepsilon^2}}, \tag{13}$$

ε is a small positive constant. This equation makes the level sets move along their respective normal with a speed $F(x)$. The direction of the gradient is depending of the signed convention chosen to distinguish the inside and outside of the object. With the level set function described in (3), the gradient always goes trough the inside of the contour. Thus a contour point move out if $F(x) > 0$ and in if $F(x) < 0$. We want to underline here that the active contour framework rely their forces on a polarity (+/-).

Note that, as mentioned in [11], the existence and the uniqueness of the result for PDEs of the same type of (13) is difficult to prove and has to be further investigated.

2.3 Constraints on the computation of the forces

In a registration framework, the computation of the attraction speed term [2] F has to respect the two following points:

- Keeping the signed distance function:
  In [6], Gomez et al. have demonstrated that when F is computed at each point of a common level set function $\phi(x,t)$, all isphotes evolve independently toward the target contour to minimize their energy. This implies the well-known effect that the level set function lost progressively its properties of signed distance map during its evolution. The distance between two consecutive isophotes decreases when they reach the target contour. Thus, in registration, F has to be computed in a narrowband of one pixel-width around the active contours. The displacement of other points in the image will be found by interpolation.
- Canceling the segmentation force at convergence:
  When an active contour reaches its target, the forces computed around the contour are not zero but reversed. Thus we need a condition to cancel the segmentation force at convergence.

In the two next sections, we will show how to introduce these two constraints in the evolution equation (13).

[2] As attraction term, we consider a term that attracts the active contour to its corresponding contour in the target image.

2.4 Registration of Multiple Regions

In [11], Vemuri et al. present a non rigid registration algorithm based on the level sets of the moving image. They work with the intensity function I of the moving image. Hence, the level sets of interest in (13) correspond to the intensity contours naturally present in the given image. They use the following evolution equation:

$$\frac{\partial u(x,t)}{\partial t} = (I_T(x) - I(x+u(x,t))) \cdot \frac{\nabla G_\sigma * I(x+u(x,t),0)}{\sqrt{\nabla G_\sigma * I(x+u(x,t),0)^2 + \varepsilon^2}}, \quad (14)$$

where I_T is the target image, G_σ is a Gaussian kernel with standard deviation of σ and $*$ is the convolution operator. We note that Equation (14) is very similar to (13) when $F = (I(x+u,0) - I_T)$ and $\phi = I$. But in (14) I needs to be convolved with a Gaussian kernel G prior because the gradient computation is very sensitive to noise. As the level sets of ϕ in (13), the level sets of I in (14) move along their respective normal with a speed F. Besides, in (14) the direction of the gradient depends on the local intensity levels of the image. Thus, the gradient goes from the highest intensity to the lowest intensity. In (13), due to the level set representation (Equation (3)), the gradient is always oriented through the inside of the active contour. The advantage of the Vemuri's model is that *connected, non connected, close and open contours* can be modeled with only one function. The limitation is that all contours of the moving image are considered for the registration. This can create mismatching in presence of inconsistencies between the moving and the target image as this will be illustrated in the results section 3. Also we will show in the next section that the speed terms designed for the active contour segmentation models cannot be used in the Vemuri's model.

One level set function ϕ can represent several objects in an image but these objects can not be connected. To represent connected contours, Vese and Chan propose in [8] to use a second level set function.

In this paper, we drop the level set function ϕ that we use to represent objects in [14]. We propose to use a label function L to define an arbitrary number of connected or non connected closed objects. This idea combine the advantages of our model (13) with the Vemuri's method [11]. Function L represents a labelled closed region selected in the moving image I as follows:

$$L : x \in \Omega \rightarrow L(x) = k, k \in [1,..,N] \text{ if } x \in \Omega_k, \quad (15)$$

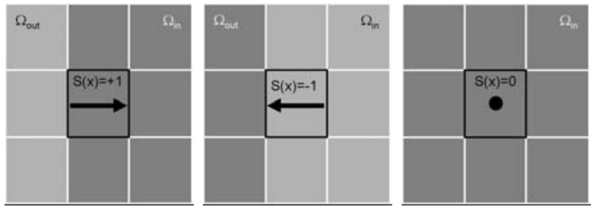
where Ω_k is the k^{th} labelled region and N is the arbitrary number of regions.

In the L representation, the level sets of interest correspond to the borders between two regions. As L is not a continuous function across the borders, L is convolved with a Gaussian kernel G_σ prior to the gradient computation.

Speed terms coming from the active contour framework can be used again in the registration process if the gradient of L is oriented inside the region. We define the following function S to determine the desired gradient direction based on the label values:

$$S(x) = \begin{cases} +1 & \text{if } \max_i L(x+x_i) > L(x) \\ -1 & \text{if } \min_i L(x+x_i) < L(x) \\ 0 & otherwise \end{cases}, \quad (16)$$

where $x + x_i$, $i \in [1,8]$ correspond to the 8-connected neighbors of pixel x. $\max_i L(x+x_i)$ and $\min_i L(x+x_i)$ are respectively the maximum and minimum values of function L among the 8 neighbors of x. Figure 1 illustrates the function S. Each panel shows the current pixel (enhanced in bold) surrounding by its 8 neighbors. The black arrow shows the direction of the gradient. If neighbors have values larger or equal to $L(x)$, the gradient is already in the good direction so $S(x) = 1$ (Figure 1(a)). If one neighbor has a value inferior to $L(x)$, the gradient direction will be change with $S(x) = -1$ (Figure 1(b)). Finally if the neighborhood has the same value of $L(x)$, the gradient has to be null which means $S(x) = 0$ (Figure 1(c)). The latter condition allows to compute the forces only on a narrowband of one pixel-width around the contours selected to drive the registration and independently of the gaussian convolution. This satisfies the first constraint described in section 2.3.



(a) (b) (c)

Figure 1: Illustration of Function S. a) $S(x) = 1$: Gradient is in the right direction. b) $S(x) = -1$: Gradient direction has to be changed. c) $S(x) = 0$: Gradient is null.

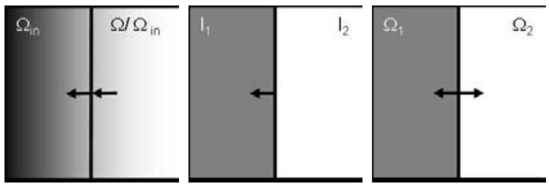
Thus, $\phi(x)$ is replaced by $L(x)$ in Equation (13) and the constraints on the gradient through function S is added such that we obtain the following evolution equation:

$$\frac{\partial u(x,t)}{\partial t} = -S(x) F(L(x+u(x,t),0)) \cdot \frac{\nabla G_\sigma * L(x+u(x,t),0)}{\sqrt{\nabla G_\sigma * L(x+u(x,t),0)^2 + \varepsilon^2}}, \quad (17)$$

where F is the speed function given by the active contour segmentation model.

The representation by a label function allows first to select the contours in the moving image that have to drive the registration, then to model closed and connected objects by only one function.

Figure 2, 3 and 4 compare the gradient direction, the attraction force to a target contour (represented by a dotted line) and the force at convergence (when the active contour reaches its target contour) on the three function ϕ (Panel 1), I (Panel 2) and L (Panel 3). The active contour is represented by the black line. For the Figures 3 and 4, the target contour corresponds to the dotted line.



(a) (b) (c)

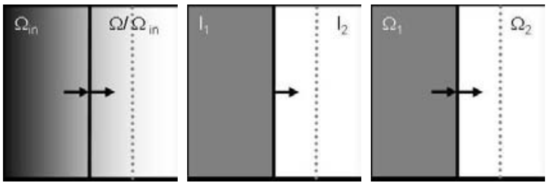
Figure 2: Illustration of the gradient direction on the functions ϕ, I and L: a) $\nabla\phi$ on a narrowband of 1 pixel-width around the zero level set. b) ∇I on a contour. c) ∇L on a narrowband of 1 pixel-width around the border between two regions.

2.5 Model of Simultaneous Registration and Segmentation

In this section, we show how evolution terms originally designed for the active contour-based segmentation can be used in our registration model. We illustrate our model with a region-based term based on the probability density function (pdf) of intensities and on the entropy measure. Note that other region-based segmentation models like [9, 15] can be introduced in our registration model in the same way.

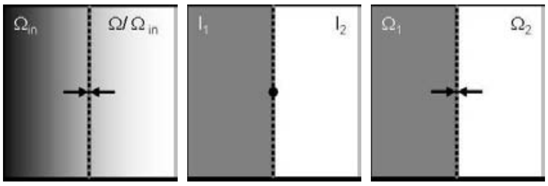
2.5.1 Region-based speed function

In our approach, the moving image is considered as a reference image or atlas. It contains spatial prior knowledge about the target



(a) (b) (c)

Figure 3: Illustration of an attraction force trough a contour (dotted line) on the functions ϕ, I and L: a) $F\nabla\phi$ on a narrowband of 1 pixel-width around the zero level. b) $F\nabla I$ on a contour. c) $F\nabla L$ on a narrowband of 1 pixel-width around the border between two regions.



(a) (b) (c)

Figure 4: Forces at convergence on the functions ϕ, I and L: a) $F\nabla\phi$ on a narrowband of 1 pixel-width around the zero level set. b) $F\nabla I$ on a contour. c) $F\nabla L$ on a narrowband of 1 pixel-width around the border between two regions.

image such as a good initial position for the active contours driving the registration process. Other kinds of prior knowledge can be extracted from the reference image like prior intensity distributions concerning the structures to be segmented in the target image. We choose to illustrate our registration method with an approach inspired by Herbulot et al. in [12]. This approach is presented in detail in a joint paper [15]. It is based on the following energy designed to be minimal when the entropy of a region Ω defined in the target image by the evolving label function is close to the entropy of the corresponding region in the reference image:

$$E(\Omega) = \frac{1}{|\Omega|}\int_{\Omega} -ln(q_{prior}(I(x),\Omega))dx, \qquad (18)$$

where q_{prior} is the prior density distribution of a given region extracted from the reference image and I is the intensity function of the target image.

We assume that corresponding regions between the reference and target images have similar intensity distributions. Note that q_{prior} does not evolve during the registration process. Hence it is computed once on the reference image in a pre-process step. This substantially reduces the computation complexity making this model fast.

The pdf of a given region in the reference image is estimated using the Parzen windows method [1]:

$$p(\alpha,\Omega) = \frac{1}{|\Omega|}\int_{\Omega} G_{\sigma}(\alpha - I(x))dx, \qquad (19)$$

where G is a Gaussian kernel with 0-mean and σ^2 variance.

The derivation of functional (18) leads to the following speed function:

$$F(q_{prior},x) \qquad = \frac{1}{|\Omega|}(ln(q_{prior}(I(x),\Omega))). \qquad (20)$$

This term is used in a region competition approach between the current region and the neighboring region as:

$$F(x) = F(q_{prior,in},x) + F(q_{prior,out},x)), \qquad (21)$$

where F is the speed term in (20), $q_{prior,in}$ is the reference pdf inside the current region and $q_{prior,out}$ is the reference pdf outside the current region. Competition regions can be found in the labelled image with the 8 neighbors of the current pixel x. When there are several possible regions for $q_{prior,out}$, we choose to consider the one that gives the highest probability for the pixel x, i.e. the one with the maximal value for $q_{prior,out}(x)$.

In the implementation, we speed up the algorithm using the following function K, which gives to $F(x)$ a constant magnitude speed while conserving the direction of propagation:

$$K(x) = k\text{Sign}(F(x)), \qquad (22)$$

where k is a positive constant and Sign() is the sign function. Finally, numerical stabilities are obtained when $k\Delta t$ is smaller or equal to the length of one pixel. The deformation field u is updated as follows:

$$u(x,t+\Delta t) = u(x,t) + \Delta t\frac{\partial u(x,t)}{\partial t} = u(x,t) - \Delta t F N, \qquad (23)$$

where Δt is the time step.

2.6 Interpolation and Regularization

Deformations computed on the active contour are then extended to the whole image by diffusion. The PDE corresponding to the linear diffusion is the well-known heat equation:

$$\begin{aligned} \frac{\partial v(x,t)}{\partial t} &= \Delta v(x,t), \\ v(x,t=0) &= u_*(x), \end{aligned} \qquad (24)$$

where u_* is the solution of Equation (23) at the point x and Δ is the Laplacian operator. This equation corresponds to the Gaussian regularization.

This diffusion has also a regularization effect. Therefore we do not use a regularization term in the F computation in equation (18). The Gaussian interpolation will also attenuate the contribution of the two lateral opposite forces around the active contour when it reaches its target.

A regular displacement field is obtained with a Gaussian filter applied to $u(x,t+\Delta t)$ at each iteration.

Finally the registration is speed up with a multi-resolution approach.

3. RESULTS

Figure 5 shows the data set of 2D images used to illustrate the registration method. It contains synthetic images (Row 1) and neck 2D CT images (Row 2). The moving and target images are respectively shown in the first and second column. The synthetic images contain four connected regions. Three have a uniform intensity that stays constant between the moving and the target image. The last region has a similar distribution in both images but the orientation of the texture pattern is different. The neck images come from different subjects. They contain common structures as the trachea, the vertebra, the jaw, the external contour of the neck and structures that do not correspond and can thus perturb the registration as the arteria, muscles and fat (Figure 5(f)). The target contours are copied onto all of these images to visualize the initial differences. Column 3 shows the label function defining the regions that our algorithm has to consider. The label function of the CT image was obtained by thresholding the moving image 5(d). Figure 6 shows the registration results obtained on these data with the Vemuri's model (Column 1) and our registration algorithm (Column 2). The target contours are copied onto all of these images to visualize the quality of the registration. The deformed grid of Column 3 helps to visualize the deformation defined on the whole image domain by our algorithm. The results obtained on synthetic images (Row 1) show that both algorithms give the same quality of registration on uniform regions.

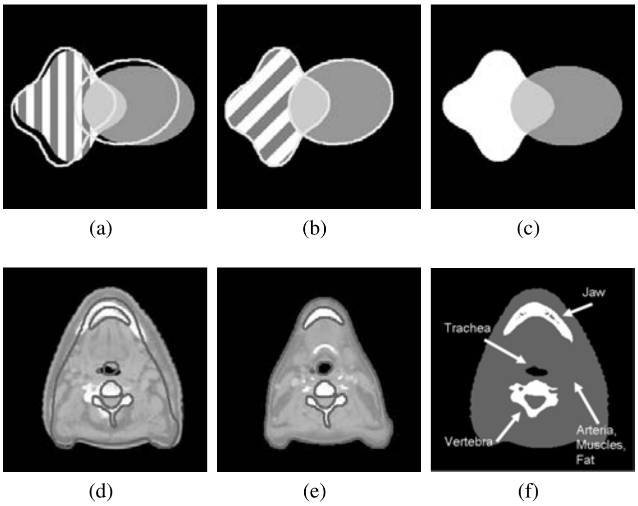


Figure 5: Data set used to illustrate the registration method. Rows: 1) Synthetic 2D images. 2) Neck 2D CT. Columns: a) Moving Image. 2) Fixed image. 3) Labelled function.

However, concerning the texture area, due to its very local speed term (difference of intensity), the Vemuri's algorithm is trying to register the details of the texture pattern. Since our algorithm performs a global registration of this region, the texture pattern is thus following the deformation of the region contours. In the neck images, the Vemuri's algorithm is trying to register the structures that do not correspond. This has for consequences to disturb the registration of common structures as the trachea, the vertebra or the jaw. The proposed method registers the most common structures.

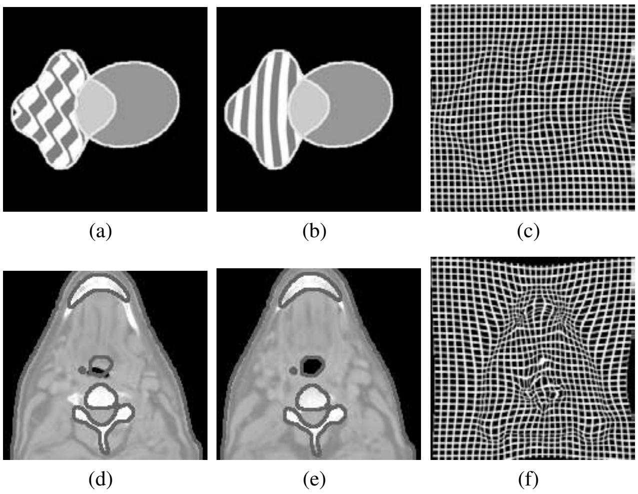


Figure 6: Comparison between the Vemuri's model [11] and our algorithm. Rows: 1) Results on synthetic image. 2) Results on CT neck image. Columns: 1) Zoom on Vemuri's algorithm result. 2) Zoom on our algorithm result 3) Deformed grid to visualize the deformations computed on the whole image domain by our algorithm.

4. DISCUSSION AND CONCLUSIONS

In this paper, we present a general formulation for adapting the active contour framework to image registration. It allows to perform segmentation and registration at the same time, which fits particularly well the atlas registration method. We propose a scheme for multiple regions registration based on the modeling of active contours by a label function. We illustrate the registration method with a region-based attraction term initially designed for active contours. The closest work to the non rigid registration algorithm we propose is probably the well-known Demons algorithm of Thirion [5]. The common points between both algorithms are first that they are designed to match contours. Contour points are generally more important than other points in the image. Furthermore, using only the contour points leads to a faster algorithm. Then both algorithms rely their forces on a polarity (+/-). However in the Thiron's algorithm this polarity depends on intensity differences and in our algorithm it depends on the objects to register. Future work is mainly focused on the study of the similitude between our algorithm and the Thirion's algorithm. Moreover the performance of our region-based registration algorithm will be analyzed on 3D images and compared to the non rigid registration algorithms currently used for atlas-registration.

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