NEW SPREADING CODES AND DETECTION ALGORITHMS FOR LOW COMPLEXITY POST-CODED OFDM SYSTEMS

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ABSTRACT

Coded OFDM systems are generally employed to overcome the symbol recovery problem in uncoded OFDM systems that is caused due to loss of diversity in uncoded OFDM systems. In this paper, we extend our recent research work on low complexity post-coded OFDM (PC-OFDM) systems. We consider new spreading codes and detection algorithms for PC-OFDM in this paper. PC-OFDM systems introduce frequency diversity in an efficient manner so that the overall computation cost of the system can be significantly reduced. We discuss the design principles of spreading codes for PC-OFDM systems and obtain code construction criterion for minimum error performance. We propose two spreading codes based on these principles. We also investigate different choices for receiver structures suitable for PC-OFDM and compare their performance through simulations on IEEE UWB channels.

1. INTRODUCTION

While OFDM systems convert a multipath fading channel into a series of equivalent flat fading channels, they lack the inherent diversity available in multipath channels. Theoretically, an uncoded OFDM system needs a simple receiver due to ISI free channel but their performance deteriorates severely in the presence of channel frequency nulls [1]. This deterioration can be avoided by employing explicit diversity or redundancy (coding) in the OFDM symbols. Different coded OFDM systems have been reported that employ some form of channel coding [2] or precoding [1, 3]. In [1], it is shown that complex field coding is better than Galois field coding as it produces the codes that are better suited for fading channels.

We noted in our recent work [4, 5] that complex field precoded OFDM systems [1] provide superior performance but their implementation complexity is quite high. We, therefore, proposed low-complexity coded OFDM systems in [4, 5]; we used PC-OFDM as the acronym for these systems that will be explained shortly. Different from our earlier work, we introduce some new spreading codes and detection algorithms for PC-OFDM systems in this paper. In short, PC-OFDM systems introduce frequency diversity in an efficient manner so that the overall computation cost of the system can be significantly reduced. The computation savings in PC-OFDM come from two fronts: 1) small size FFT/IFFT as compared to frequency domain precoding, and 2) special structures of encoding matrices that requires $\mathcal{O}(N)$ operations instead of $\mathcal{O}(N^2)$ operations.

The paper is organized as follows. Section 2 presents the system details and formulates the problem mathematically. In Section 3, we discuss the design of PC-OFDM encoder and outline the design principles of spreading codes and obtain conditions of minimum error rate codes. We list specific examples. Section 4 discusses the simplified receiver architecture for PC-OFDM and different choices of detectors. We present simulation results in Section 5 and conclude the paper in Section 6.

2. SYSTEM DETAILS AND PROBLEM FORMULATION

Consider an uncoded OFDM system that is implemented using an N-point IFFT/FFT. Let $\mathbf{F}_N$ be the $N \times N$ FFT (fast Fourier transform) matrix with (n,k)th entry as $[\mathbf{F}_N]_{n,k} = (1/\sqrt{N}) \times \exp\{-j2\pi(n-1)(k-1)/N\}$. It is well known that the use of cyclic prefix (CP) in OFDM systems convert a multipath fading channel into a set of parallel flat-frequency channels such that the $N \times 1$ vector of received OFDM symbol $\mathbf{u}$ can be expressed as:

$$\mathbf{u} = \mathbf{H}_D \mathbf{b} + \eta. \tag{1}$$

where $\mathbf{H}_D := \mathrm{diag}[\mathbf{F}_N \tilde{\mathbf{h}}]$ with $\tilde{\mathbf{h}}$ is obtained from the concatenation of L_h channel taps, $\{h_l\}_{l=1}^{L_h}$, and $N - L_h$ zeros. $\mathbf{b}$ is the $N \times 1$ vector of modulated information symbols and η represents additive white Gaussian noise.

The existing techniques encode the data before IFFT operation and can be termed as frequency domain precoded OFDM or FP-OFDM in short. In contrast, we claim that the system complexity can be significantly reduced if precoding is applied on OFDM symbols after performing the IFFT operation. Since we are precoding the time domain OFDM symbols, we will refer to this scheme as Time Domain Postcoded OFDM (PC-OFDM). The term 'postcoded' emphasizes the fact that we encode the symbols after performing IFFT operation. For FP-OFDM, the transmitted symbol is given by $\mathbf{y} := \frac{1}{\sqrt{K/N}} \mathbf{F}_K^{\mathcal{H}} \mathbf{A}_f \mathbf{b}$, where $\mathbf{A}_f$ is the frequency domain precoding matrix and $1/\sqrt{K/N}$ is used for normalization. In contrast, the transmitted symbol for PC-OFDM is given by $\mathbf{y} := \mathbf{A}_t \mathbf{F}_N^{\mathcal{H}} \mathbf{b}$. In both cases, we consider complex field coding *i.e.*, $\mathbf{A}_f$ (or $\mathbf{A}_t$) $\in \mathbb{C}^{K \times N}$ with $K \geq N$, instead of Galois field as it provides more degrees of freedom [1]. In its simplest form, the design of PC-OFDM requires K to be an integer multiple of N. In the remainder of this paper, we assume that $K = NL$ where L is an integer. This should not be considered as a limitation of PC-OFDM systems because this requirement can be waived with additional complexity. It is important to note that any postcoding scheme can be made equivalent to a precoding scheme by selecting

$$\mathbf{A}_t = \frac{1}{\sqrt{L}} \mathbf{F}_{NL}^{\mathcal{H}} \mathbf{A}_f \mathbf{F}_N, \tag{2}$$

however, the converse is not true since the precoding matrix corresponding to a post-coded scheme is necessarily circulant as explained in the section.

3. PC-OFDM ENCODER DESIGN

To overcome the symbol recovery problem in OFDM systems at frequency nulls in the channel, we propose PC-OFDM systems with frequency diversity in the following manner:

1. *Explicit Frequency Diversity:* This can be achieved by simple repetitive coding that corresponds to a low cost upsampling operation in the time domain.
2. *Implicit Frequency Diversity:* In general, repetitive coding alone does not enhance the system performance significantly and we need to spread data symbols across different subcarriers that results in implicit diversity.

The spreading operation seems similar to multi-carrier code division multiple access (MC-CDMA) except that instead of multiuser we have multiple streams of data from single user. We achieve implicit diversity through the use of *spreading codes* in the complex field. Mathematically, the two forms of diversity can be embedded in the frequency domain precoding matrix $\mathbf{A}_f$ such that

$$\mathbf{A}_f = \underbrace{\begin{bmatrix}\mathbf{I}_N \\ \vdots \\ \mathbf{I}_N\end{bmatrix}}_{NL\times N} [\mathbf{B}_f] \tag{3}$$

where the concatenated identity matrices $\mathbf{I}_N$ account for repetitive coding and $\mathbf{B}_f$ represents the spreading matrix both in frequency domain. As PC-OFDM performs postcoding in time domain, we substitute $\mathbf{A}_f$ from (3) into (2) to get

$$\mathbf{A}_t = \frac{1}{\sqrt{L}}\mathbf{F}_{NL}^{\mathcal{H}} \underbrace{\begin{bmatrix}\mathbf{I}_N \\ \vdots \\ \mathbf{I}_N\end{bmatrix}}_{NL\times N} \mathbf{B}_f\mathbf{F}_N \tag{4}$$

Defining a time domain spreading matrix $N\times N$ as:

$$\mathbf{B}_t := \mathbf{F}_N^{\mathcal{H}}\mathbf{B}_f\mathbf{F}_N, \tag{5}$$

we can rewrite (4) as:

$$\mathbf{A}_t = \frac{1}{\sqrt{L}}\mathbf{F}_{NL}^{\mathcal{H}} \underbrace{\begin{bmatrix}\mathbf{F}_N \\ \vdots \\ \mathbf{F}_N\end{bmatrix}}_{NL\times N} \mathbf{B}_t \tag{6}$$

Since the IFFT of an $N\times N$ matrix that is repeated L times is simply the N-point IFFT of the matrix followed by upsampling by L. Thus, manipulating the FFT matrices on the right side of (6) results in a $NL\times N$ degenerated identity matrix of the form:

$$\tilde{\mathbf{I}}_{NL} := \frac{1}{\sqrt{L}}\mathbf{F}_{NL}^{\mathcal{H}} \underbrace{\begin{bmatrix}\mathbf{F}_N \\ \vdots \\ \mathbf{F}_N\end{bmatrix}}_{NL\times N} = \begin{bmatrix}\mathbf{e}_1 & \mathbf{e}_{1+L} & \cdots & \mathbf{e}_{1+(N-1)L}\end{bmatrix} \tag{7}$$

where $\mathbf{e}_i$ is the standard $NL\times 1$ column vector with '1' at ith row and '0' otherwise. For instance, with $N=2$ and $L=2$ the degenerated identity matrix is $\tilde{\mathbf{I}}_4 = \begin{bmatrix}1&0\\0&0\\0&1\\0&0\end{bmatrix}$. It is obvious that $\tilde{\mathbf{I}}_{NL}$ can be obtained by upsampling the identity matrix $\mathbf{I}_N$ by L, i.e.,

$$\tilde{\mathbf{I}}_{NL} = (\uparrow L)\,\mathbf{I}_N, \tag{8}$$

and we can write (6) in the form

$$\mathbf{A}_t = (\uparrow L)\,\mathbf{B}_t \tag{9}$$

where $(\uparrow L)$ represents upsampling by L. This shows that PC-OFDM provides explicit frequency diversity in a very low-complexity manner by simply upsampling the time domain OFDM symbols. In the following subsection, we outline the guidelines for the design of the spreading matrix $\mathbf{B}_t$.

3.1 Spreading Codes for PC-OFDM

While designing spreading codes, we limit ourselves to the case where the spreading matrix $\mathbf{B}_f$ satisfies the following conditions [6]:

C1. *Square shape*: To assure bandwidth efficiency

C2. *Orthogonality*: To keep the Eucledian distance unchanged among symbols after spreading.

C3. *Computationally efficient*: In general, the complexity of spreading operation is $\mathcal{O}(N^2)$ but it can be reduced if efficient structures are chosen for the spreading matrix.

Consider a PC-OFDM system that employs time domain postcoding with $\mathbf{B}_t$ as time domain spreading matrix. From (2), the equivalent spreading matrix in frequency domain will be

$$\mathbf{B}_f = \mathbf{F}_N\mathbf{B}_t\mathbf{F}_N^{\mathcal{H}}. \tag{10}$$

Then to meet C1-C3, we propose our design of spreading matrix for PC-OFDM in the following proposition:

Proposition 1. *(a) For low complexity, we choose $\mathbf{B}_f$ to be circulant of the form:*

$$\mathbf{B}_f = \mathrm{circ}\left[\{c(k)\}_{k=1}^N\right] \tag{11}$$

where we refer to the sequence $\mathbf{c} := \{c(k)\}_{k=1}^N$ as spreading codes. With this choice of $\mathbf{B}_f$, the time domain postcoding matrix $\mathbf{B}_t$ will be a diagonal matrix of the form:

$$\mathbf{B}_t = \mathrm{diag}\left[\mathbf{F}_N^{\mathcal{H}}\mathbf{c}\right] \tag{12}$$

The diagonal structure of $\mathbf{B}_t$ reduces the complexity of spreading operation to $\mathcal{O}(N)$.

(b) Define the diagonal elements of $\mathbf{B}_t$ as $\mathbf{d} := \mathbf{F}_N^{\mathcal{H}}\mathbf{c}$. Then for unitary spreading transform, a possible choice is to select $d(n) = e^{j\phi(n)}$.

Proof: To prove 1(a), note that (12) follows from the diagonalization property of the Fourier matrix along with (10) and (11). To prove 1(b), observe the fact that $\mathbf{B}_f^{\mathcal{H}}\mathbf{B}_f = \mathbf{I}_N$ to fulfill the orthogonality condition (*C2*). This results in $\mathbf{B}_t^{\mathcal{H}}\mathbf{B}_t = \mathbf{I}_N$ according to (10). Since $\mathbf{B}_t$ is diagonal, the magnitude of the diagonal elements of $\mathbf{B}_t$ must be unity or, in general, $d(n) = e^{j\phi(n)}$ for $n = 1,\cdots,N$.

Remark 1. It seems that the circulant structure of $\mathbf{B}_f$ restricts the degrees of freedom in the selection of spreading matrix but as we will discuss later that careful selection of $\mathbf{c}$ can attain the same performance as the one without circulant restriction.

Remark 2. It is important to note that Proposition 1-[b] only provides a starting point in the search of spreading codes that results in extremely low implementation cost.

Figure 1 shows a block diagram of PC-OFDM transmitter incorporating the explicit diversity in the form of upsampling by a factor of L and implicit diversity according to the spreading codes $d(n)$ specified by Proposition 1-[b]. It is obvious that a particular choice of the phase pattern $\phi(n)$ of the spreading codes $d(n) = e^{j\phi(n)}$ will affect the spectrum of $\mathbf{d}$ or simply the frequency domain spreading.

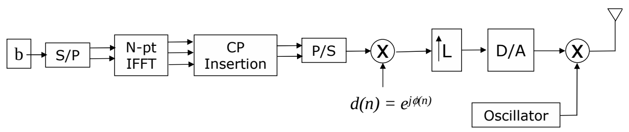

Figure 1: PC-OFDM transmitter block diagram

3.2 Minimum Bit Error Rate (BER) Spreading Codes

The bit error rate (BER) analysis of PC-OFDM systems is identical to that of space-time coded systems that has been studied extensively. We use the results of pairwise error probability (PEP) analysis from [7, 1] to design the spreading codes with minimum BER. For maximum likelihood detector (will be discussed in Sec 4.2), the PEP of PC-OFDM systems depend on [1, 5]:

- *Diversity gain (G_d)*, that is related to the rank of $\mathbf{B}_f$.
- *Coding gain (G_c)*, that is related to the determinant of a matrix defined as $(\mathbf{D}_e\mathbf{V})^{\mathcal{H}}\mathbf{D}_e\mathbf{V}$ in [1] where $\mathbf{V}$ is truncated FFT matrix and $\mathbf{D}_e := \mathrm{diag}[\mathbf{B}_f(\mathbf{b}-\mathbf{b}')]$.

To design spreading codes with minimum BER, we seek to maximize G_d and G_c. To achieve maximum G_d, we need to have $\mathbf{B}_f$ full rank that can be fairly easily achieved since $\mathbf{B}_f$ is circulant. For coding gain, we depart from the approach of [1] and relate G_c to the determinant of $(\mathbf{D}_e\mathbf{F}_N)^{\mathcal{H}}\mathbf{D}_e\mathbf{F}_N$ assuming $\mathbf{B}_f$ as full rank. This particular approach helps us factor the determinants because of square matrices, and G_c can be shown to be:

$$G_c \propto \det(\mathbf{D}_e). \tag{13}$$

Since we are only interested in comparing the relative performance of different codes, it suffices to use (13) as metric. To simplify (13) further, let us consider binary phase shift keying (BPSK) modulation where G_c can be expressed as:

$$G_c \propto \prod_{k=1}^{L_h} c(k) \tag{14}$$

We limit the product to L_h because diversity gain cannot be more than L_h and assume that $|c(k+1)| \geq |c(k)|\ \forall\, k$.

Using the above approach, we numerically evaluated the coding gains for different sequences and picked the two sequences (spreading codes) for this paper that gives the best coding gain. These sequences are:

1. *Maximally flat spreading codes (or Chu's Code)*: The first sequence we tried to maximize the coding gain is the one that has flat spectrum. To design codes with flat spectrum, we make use of the stationary-phase concept (a popular concept in the field of non-linear frequency modulation [8]) that states that the magnitude spectrum of the signals of the form $d(n) = e^{j\phi(n)}$ is proportional to second derivative of $\phi(n)$ with respect to n. Thus, the phase pattern $\phi(n)$ proportional to n^2 will result in flat magnitude spectrum. Later we found that these codes are similar to Chu's code [8] that also contains an n^2 term. Thus, we use $d(n) = e^{j\pi n^2/N}$ for $n = 1, \cdots, N$

2. *Costas Sequence:* Costas sequence [8] that refers to a particular permutation of numbers from $1, \ldots, N$ is another candidate for spreading codes that possesses good autocorrelation properties. An example of Costas sequence for $N = 8$ is [2 6 3 8 7 5 1 4]. The Costas sequence we used in this paper is of the form $d(n) = e^{jn}$ where n refers to the Costas permutation pattern. One limitation of Costas sequences is that they do not exist for every N [8].

4. PC-OFDM RECEIVER DESIGN

4.1 Simplified Receiver Architecture

In [5], we applied the multirate signal processing concepts to obtain a simplified architecture for PC-OFDM receiver. The simplification is achieved due to the cascade of upsampling and filtering (transmission through the channel) operation that can be equivalently expressed as a polyphase decomposition of the channel. The polyphase decomposition of channel leads us to design a dual system with downsampling and delay operations at the receiver as shown in Fig. 2 where each branch contains a polyphase decomposition $H_p(z) = \sum_{l=0}^{L_h-1} h(lL+p)z^{-l}$ of the channel. After simple low cost operations discussed in [5], the receiver is capable of separating the L diversity branches. After performing N-point FFT operation on each of the branches, the received symbols at the pth branch are given by:

$$\mathbf{u}_p = \mathbf{H}_{pD}\mathbf{B}_f\mathbf{b} + \eta_p \quad \text{for} \quad p = 1, \cdots, L \tag{15}$$

where $\mathbf{u}_p$ is the $N \times 1$ vector of received symbols and $\mathbf{H}_{pD} := \mathrm{diag}\,[\mathbf{F}_N\mathbf{h}_p]$ with $\mathbf{h}_p$ representing the pth phase of the channel $\{h_l\}_{l=0}^{L_h-1}$ that is zero-padded to make it $N \times 1$. The symbols received at different diversity branches at the receiver can be combined using any of the conventional diversity combining methods like maximum ratio combining (MRC) or equal gain combining. We discuss diversity combining for PC-OFDM in the following subsection.

4.2 Detection Algorithms

In PC-OFDM system, the task of the detection algorithm is two-fold: 1) to combine different diversity branches (*diversity combining*) at the receiver, and 2) to unfold the spreading operation (*equalization*). In this paper, we consider the algorithms that perform these operations jointly. The optimal detector minimizes the average probability of error. This is achieved by maximum likelihood (ML) that detects the transmitted symbols based on the following minimization:

$$\hat{\mathbf{b}} = \arg\min_{\mathbf{b}\in\mathcal{B}} ||\mathbf{u} - \mathbf{H}\mathbf{B}_f\mathbf{b}||^2 \tag{16}$$

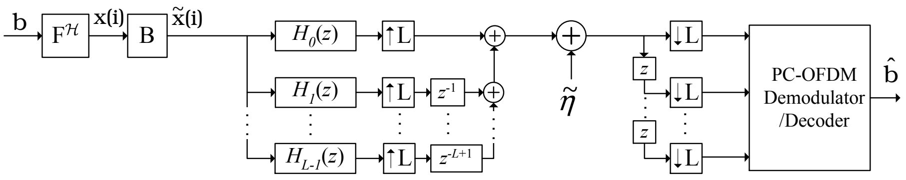

Figure 2: Equivalent model of PC-OFDM system with polyphase decomposition of channel

where $||.||$ represents l_2 norm and $\mathcal{B}$ is the finite set of signal constellation. ML detection though optimum is a costly operation and is practically not feasible for large N. Here we explore the use of three suboptimal detectors that can be implemented with reduced complexity.

4.2.1 *Zero Forcing (ZF) Detector*

A simple suboptimal detector is the zero forcing (ZF) detector. For PC-OFDM receiver, it performs ZF diversity combining and ZF equalization in a joint manner. Mathematically, ZF detector solves the unconstrained least-squares problem and obtains an estimate of the data in the form:

$$\hat{\mathbf{b}}_{ZF} = \mathbf{B}_f^{\mathcal{H}} \left[\left(\sum_{p=1}^{L} \mathbf{H}_{pD}^{\mathcal{H}} \mathbf{H}_{pD} \right)^{-1} \sum_{p=1}^{L} \mathbf{H}_{pD}^{\mathcal{H}} \mathbf{u}_p \right]. \quad (17)$$

The terms inside the square brackets represent the ZF diversity combining of L diversity branches. The data symbols are subsequently detected from the estimate $\hat{\mathbf{b}}_{ZF}$ using hard decision according to the modulation scheme used.

4.2.2 *Successive Interference Cancellation (SIC)*

We found through simulations that the performance of ZF is quite poor. A possible low complexity solution is to apply the idea of successive interference cancellation (SIC) that was first proposed for space-time codes [9]. In successive interference cancellation, we detect a symbol that corresponds to the maximum channel gain using ZF detector of (17). Assuming we made correct decision, the effect of the detected symbol is subtracted from the vector of received symbols and the process is iterated such that we form a better estimate of each of the symbols at the end of the iteration. We refer to this detector as ZF-SIC. If we construct an $NL \times 1$ vector $\mathbf{r}$ by concatenating the received vectors from each branch then from (15)

$$\mathbf{r} = \mathbf{G}\mathbf{b} + \eta \quad (18)$$

where

$$\mathbf{G} := \begin{bmatrix} \mathbf{H}_{1D} \\ \vdots \\ \mathbf{H}_{LD} \end{bmatrix} \quad \text{and} \quad \mathbf{B}_f := [\mathbf{g}_1 \cdots \mathbf{g}_N]$$

with $\mathbf{g}_i$ as ith column. Assuming that $\mathbf{G}$ is ordered according to channel gain, we can summarize ZF-SIC algorithm as shown in *Algorithm* 1.

4.2.3 *Quasi Maximum Likelihood (Q-ML)*

The non-linear optimization in (16) is commonly referred to as an *integer least-squares* problem that is known to be unsolvable in polynomial time. An approximate solution to the optimization in (16) can be found by transforming the problem to convex optimization. In [10], semi-definite programming is used to obtain the quasi maximum likelihood (Q-ML) solution of (16); the complexity of Q-ML detector is $\mathcal{O}(N^{3.5})$. In our simulations, we used the MATLAB scripts[1] for Q-ML provided by the authors of [10].

Algorithm 1 ZF-SIC Detector

initialization; $\mathbf{G}^{(0)} = \mathbf{G}$, $\mathbf{r}^{(0)} = \mathbf{r}$.
for $i = 1$ to N **do**
  Using $\mathbf{G}^{(i-1)}$, obtain ZF estimate $\hat{\mathbf{b}}_{ZF}$.
  Use hard decision detector to obtain $\hat{b}_i$
  Compute $\mathbf{r}^{(i)} = \mathbf{r}^{(i-1)} - \mathbf{g}_i \hat{b}_i$.
  Update: $\mathbf{G}^i = [\mathbf{g}_{i+1} \cdots \mathbf{g}_N]$
end for

5. SIMULATION RESULTS

We perform simulations to compare the bit error rate (BER) of different spreading codes and detection algorithms discussed in the paper. We use BPSK modulated symbols and transformed them to OFDM symbols with $N = 16$ and $L = 2$ that results in code rate of $1/2$. For Figs. 3 and 4, we use Rayleigh fading channel with five taps that are generated according to the Jakes model. We compare the effect of different spreading codes on system performance and the results are shown in Fig. 3. It is found that the performance of Costas sequence and maximally flat codes is similar except at high signal to noise ratio (SNR) when Costas gives better performance. The performance of different receiver structures for PC-OFDM systems is shown in Fig. 4. It is clear from the figure that ZF-SIC is a low complexity alternative to Q-ML at a slightly higher error rate. In Fig. 5, we compare the BER performance of different coded OFDM systems over UWB Channels for $N = 128$ and $L = 2$ using Q-ML. To compare with precoded OFDM systems, we employ the complex precoders proposed in [1]. We also obtain the BER performance of pulsed-OFDM [11] and the results are shown in Fig. 5. The slope of the curve shows that pulsed-OFDM could not achieve the full diversity order available in the system. The comparison between precoded and PC-OFDM systems shows that the low complexity design of PC-OFDM systems does not result in any performance loss.

6. CONCLUSIONS

We discussed some new designs of spreading codes and detection algorithms for low complexity PC-OFDM systems we proposed earlier. The analysis of PC-OFDM transmitter

[1] available at http://www.ece.umn.edu/users/luozq/software/sw_about.html

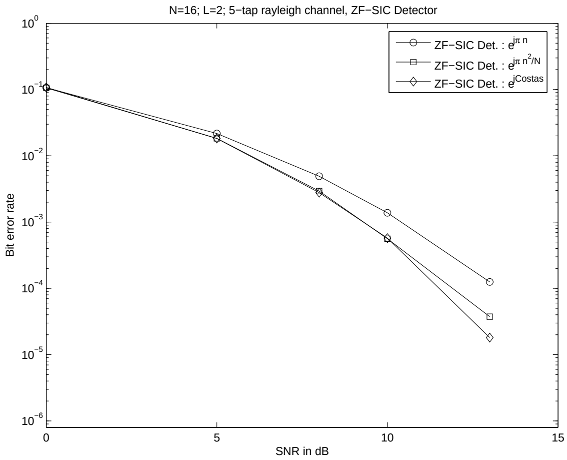

Figure 3: BER of PC-OFDM systems with different spreading codes

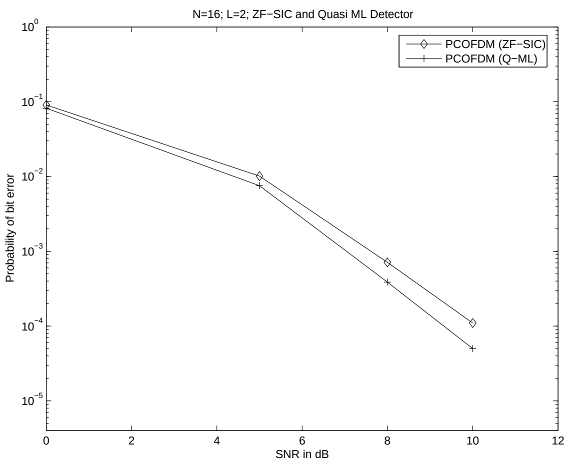

Figure 4: BER performance comparison of different receivers

provided us with guidelines to design the spreading codes. We explored new detection algorithms and compare their perforamce through simulations. It is obvious from the discussion, that the PC-OFDM implements coding in OFDM systems at significantly low computation cost. We omit the computational complexity comparison in this paper due to space limitations.

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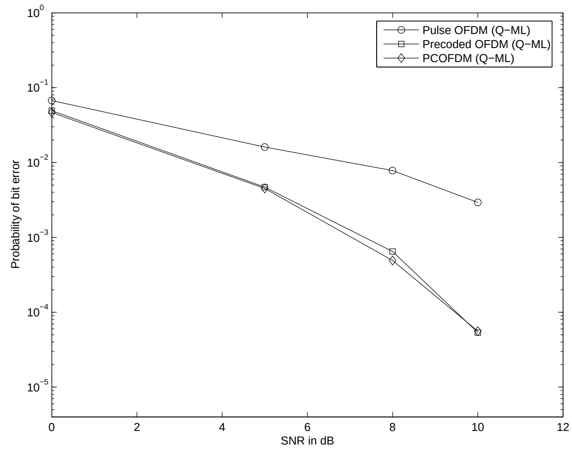

Figure 5: BER of different coded OFDM systems over UWB channel

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