OPTIMUM MULTILEVEL CHAOTIC SEQUENCES FOR ASYNCHRONOUS DS-CDMA SYSTEMS OVER RICIAN SELECTIVE FADING CHANNEL

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ABSTRACT

The BER performances of asynchronous DS-CDMA systems using conventional matched-filter receiver and multilevel chaotic spreading are estimated over a Rician fading selective channel. The paper completes the results of previous works dedicated to the optimization of system's BER performances by controlling the chaotic sequence generator. It is showed that a family of PWAM (Piece-Wise Affine Markov) maps named (n, t) – tailed shifts can generate sequences that optimize the performances of the DS-CDMA system under different load and channel fading conditions. Even though these works considered the multilevel quantization for the system performance analysis, the optimum case was identified only for the binary sequences, stating that the multilevel sequences offer no improvement over the classical sequences. The present paper shows that in the case of (n, t) – tailed shifts sequences the optimum performances are reached not only for a binary quantization, but for any number of quantizing levels.

1. INTRODUCTION

In DS-CDMA (Direct Sequence – Code Division Multiple Access) system the user generated data signal is multiplied by a code sequence uniquely associated to that user. For each user a receiver that is matched to the transmitted signal is extracting each data symbol from the received signal by correlating it to the same synchronized spreading sequence as in the transmitter. Assuming an ideal channel with no fading, and no MAI (Multiple Access Interference) from the other users, then the single user matched-filter receiver is optimum. However, the real channels present these distortions and other solutions have to be found to optimize the system performances. A solution for reducing these distortions effects is considering only the spreading sequences set design that optimizes the system performances. This is possible due to the fact that in DS-CDMA systems the disturbances introduced by fading and MAI are depending on the correlation properties of the chosen spreading sequences. Assuming the SGA (Standard Gaussian Approximation) these disturbances powers determine the system's BER performances.

Chaotic sequences generated by non-linear dynamical systems proved to be an efficient spreading solution for DS-CDMA systems. Due to the increased dynamics and their sensitive dependence on the initial state these sequences present pseudorandom properties, enhanced security, and in some cases offer large sets with specified correlation properties. Several papers are proving that these chaotic sequences also offer better system performances than the classical sequences. Among these works, Mazzini et al. presented in [1], and [2] thorough system performance analysis and optimization methods for a particular family of chaotic sequences generated by PWAM maps. In [1] some statistical tools are introduced for correlation properties analysis of quantized PWAM sequences. According to this tensorial algebra based theory, the higher order moments of the PWAM sequences can be expressed in a convenient exponential form.

When a Rician selective fading channel is considered for the DS-CDMA communication under the SGA assumption the BER performances are depending on the MAI variance and the self interference variance determined by the faded delayed versions of the user signal. The same authors showed in [2] that when chaotic PWAM maps are employed over an exponential Rician channel model, than the MAI and self interference variances are depending on the second-, third-, and fourth-order moments of the multilevel quantized sequences. A special map named (n,t) – tailed shifts was found to allow BER performance optimization under different system load and channel fading conditions. In [2] it was stated that only the binary quantized (n,t) – tailed shifts sequences offer a performance improvement over the classical pseudo-random sequences, like Gold and Kasami. In our opinion, the authors missed the fact that the multilevel sequences have a maximum auto-correlation value that is different from the unity, as the binary sequences have, and in order to make its comparable with the binary sequences, a simple normalization of the sequence samples with this maximum auto-correlation term is needed. In this paper we reconsidered the same analysis as in [2] using the normalization above to analyze the DS-CDMA system performances by using multilevel sequences that were first presented in [3] and [4]. The theoretical and simulation results of this analysis led us to the conclusion that the properly normalized multilevel (n, t) – tailed shifts sequences can reach for any number of levels the same optima performances as were reported in [2] only for the binary quantized sequences. The facts that motivated our search for optimum multilevel sequences are that multilevel sequences offer also enhanced security, and faster sequence acquisition over the binary ones. Nevertheless, the generation process of multilevel sequences is more complex.

The paper is organized as follows. The second section is presenting the DS-CDMA asynchronous system and exponential fading channel models used for the analysis. In section three the BER performances are estimated for the system introduced in section two. The fourth section is dedicated to the second-, third, and fourth-order moments estimation for optimum multilevel quantized (n, t) – tailed shifts sequences, and the resulted system BER performances. The fifth part presents some simulation results compared to the theoretical performances under different system load and channel fading conditions. Finally, some conclusions are drawn.

2. DS-CDMA SYSTEM AND CHANNEL MODEL

The asynchronous DS-CDMA system over a Rician selective fading channel is the same as in [2]. The linear time-varying channel impulse response for any user k is given by:

$$h_k(t) = \beta_0 \delta(t) + \sum_{u=1}^{\infty} \beta_u A_u^{(k)} e^{-j\varphi_u^{(k)}} \delta(t - uT_c) \quad (1)$$

where $\delta(t)$ is the Dirac pulse, the spreading sequence chip interval $T_c=T/N$, T is the data symbol interval, N is the sequence period, $\beta_u^2\left[A_u^{(k)}\right]^2$ is the instantaneous random power attenuation of the uth secondary ray in the multipath, while $\varphi_u^{(k)}$ and uT_c are its phase and delay. The secondary rays will be considered to fade independently and are modelled by Rayleigh distributed random variables $A_u^{(k)}$, normalized to have a unity mean power $E\{[A_u^{(k)}]^2\} = 1$. The fading phases $\varphi_u^{(k)}$ are considered as uniformly distributed in the interval $[0, 2\pi]$. The β_u and β_0 represent the uth secondary ray, and the first ray constant fading factors.

Denoting the Rice factor by R, i.e. the ratio between the power of the first ray and the remaining rays, we have

$$\beta_0 = \sqrt{R \sum_{u=1}^{\infty} \beta_u^2} \quad (2)$$

The power attenuation will be considered exponential as in [2], so that two constants A and B exist such that

$$\beta_u = Ae^{-Bu}, u \ge 1 \quad (3)$$

The same condition as in [2] is ensuring that the power of all the rays in the channel is constant, so that:

$$(1+R) \sum_{u=1}^{\infty} \beta_u^2 = 1 \quad (4)$$

and then we have the following linking formulas between the constants A, B, β_0, and the Rice factor:

$$A^2 = \frac{e^{2B} - 1}{1+R}; \quad \beta_0 = \frac{R}{1+R} \quad (5)$$

The signal transmitted by the kth user is given by

$$s_k(t) = \sqrt{2P} a_k(t) b_k(t) e^{j(\omega_c t + \theta_k)} \quad (6)$$

where P represents the common signal power, $a_k(t)$ and $b_k(t)$ are the spreading waveform and the data waveform, respectively corresponding to the discrete-time samples sequences $\{a_{k,j}\}, \{b_{k,j}\}$. The carrier angular frequency is ω_c, and the initial phase in the kth user carrier is θ_k. Due to the asynchronous transmission each user k signal is experiencing a random propagation delay $0 \le \tau_k < T$.

The received signal for the kth user is given by

$$y_k(t) = \mathrm{Re}\{s_k(t) \otimes h_k(t)\} \quad (7)$$

where $\otimes$ denotes the convolution product.

The sum of all these received signals is entering the matched-filter receiver that computes the correlation with a synchronized replica of the spreading sequence for each user. Assuming the ith user among all K users in the system the correlator's output estimated over a data symbol interval T is composed by three parts [2]:

$$Z_i = \Omega_i + \Xi_i + \Psi_i \quad (8)$$

where each of the three terms in (8) has the following significance: Ω_i – the useful component, corresponding to the ith user current data symbol value, Ξ_i – distortion component caused by all the secondary rays carrying the previous data symbols of the ith user fading channel. This term corresponds to the self-interference, and Ψ_i – distortion component caused by all the primary and the secondary rays carrying data symbols transmitted by the other users in the system. This term corresponds to the MAI and fading.

It is demonstrated [2-4] that the first term in (8) is given by

$$E[\Omega_i] = \sqrt{\frac{P}{2}} \beta_0 b_{i,0} T \quad (9)$$

where $b_{i,0}$ the ith user current data symbol value.

Following the procedure in [2] the second term in (8) is:

$$\Xi_i = \sqrt{\frac{P}{2}} \sum_{u=1}^{\infty} \beta_u A_u^{(i)} \cos[\omega_c \tau_i + \omega_c u T_c + \varphi_u^{(i)} - \theta_i] \cdot \left[b_{i,-\lfloor uT_c/T \rfloor - 1} R_{i,i}(uT_c) + b_{i,-\lfloor uT_c/T \rfloor} \hat{R}_{i,i}(uT_c)\right] \quad (10)$$

where $b_{i,-\lfloor uT_c/T \rfloor -1}$ and $b_{i,-\lfloor uT_c/T \rfloor}$ represent two previous symbols transmitted by the ith user that propagated over the uth path and are distorting the current data symbol. The terms denoted by $R_{i,i}(t)$ and $\hat{R}_{i,i}(t)$ denote the continuous time partial correlation functions given by

$$\begin{aligned} R_{k,i}(t) &= C_{k,i}(l-N)T_c + [C_{k,i}(l+1-N) - C_{k,i}(l-N)](t - lT_c) \\ \hat{R}_{k,i}(t) &= C_{k,i}(l)T_c + [C_{k,i}(l+1) - C_{k,i}(l)](t - lT_c) \end{aligned} \quad (11)$$

and estimated as autocorrelation functions for $k=i$. The notations introduced in (11) are $l = \left\lfloor \frac{Nt}{T} \right\rfloor$ and $C_{k,i}(l)$ that is the discrete aperiodic cross-correlation function for the sequences $(a_{k,j})$ and $(a_{i,j})$, defined as:

$$C_{k,i}(l) = \begin{cases} \sum_{j=0}^{N-1-l} a_{k,j} a_{i,j+l}, \text{for } 0 \le l \le N-1 \\ \sum_{j=0}^{N-1+l} a_{k,j-l} a_{i,j}, \text{for } 1-N \le l < 0 \end{cases} \quad (12)$$

and $C_{k,i}(l) = 0, \text{for } |l| \ge N$.

The third term in (8) is composed by two different parts:

$$\Psi_i = \Psi_i^P + \Psi_i^S \quad (13)$$

where Ψ_i^P is determined by the primary rays of the other users $k \ne i$, and Ψ_i^S defines the secondary rays contributions of the other users $k \ne i$. The first term in (13) defines the MAI contribution for the system without fading [2-4]:

$$\Psi_i^P = \sqrt{\frac{P}{2}} \beta_0 \sum_{\substack{k=1 \\ k \ne i}}^{K} [b_{k,-1} R_{k,i}(\tau_k) + b_{k,0} \hat{R}_{k,i}(\tau_k)] \cos(\omega_c \tau_k - \theta_k) \quad (14)$$

where $R_{i,i}(t)$ and $\hat{R}_{i,i}(t)$ were defined in (11) and $b_{k,-1}$ and $b_{k,0}$ represent the kth user current and previous symbol interfering with the useful symbol. Following the same procedure results that the second term in (13) is given by

$$\begin{aligned} \Psi_i^S = \sqrt{\frac{P}{2}} \sum_{\substack{k=1 \\ k \ne i}}^{K} \sum_{u=1}^{\infty} \beta_u A_u^{(k)} \cos[\omega_c \tau_k + \omega_c u T_c + \varphi_u^{(k)} - \theta_k] \cdot \\ \cdot \left[b_{k,-\lfloor (uT_c+\tau_k)/T \rfloor -1} R_{k,i}(uT_c + \tau_k) + \right. \\ \left. + b_{i,-\lfloor (uT_c+\tau_k)/T \rfloor} \hat{R}_{k,i}(uT_c + \tau_k)\right] \end{aligned} \quad (15)$$

where all the terms have the same significance as in (10).

3. BER PERFORMANCE ESTIMATION

Under the SGA assumption the mean and the variance of the correlator output Z_i expressed by (8) is given by [1-4]:

$$E\{Z_i\} = E[\Omega_i] = \sqrt{\frac{P}{2}}\beta_0 b_{i,0}T;\ \mathrm{var}\{Z_i\} = \sigma^2_{\Xi_i} + \sigma^2_{\Psi_i^P} + \sigma^2_{\Psi_i^S} \quad (16)$$

Following the same method as in [2] we can average the self-interference variance over the number of users in the system:

$$\sigma^2_{\Xi} = E\left[\sigma^2_{\Xi_i}\right] = \frac{PT^2}{4N^2}\cdot \frac{A^2}{1-e^{-2BN}}\sum_{u=0}^{N-1}\left\{\underset{i}{E}\left[C^2_{i,i}(N-u)\right] + \underset{i}{E}\left[C^2_{i,i}(u)\right]\right\}e^{-2Bu} \quad (17)$$

The mean MAI variance can be computed as [1-4]:

$$\sigma^2_{\Psi^P} = \underset{i}{E}\left[\sigma^2_{\Psi_i^P}\right] = \frac{PT^2}{12N^3}\beta_0^2\left(\sum_{\substack{k=1\\k\neq i}}^{K}\underset{i}{E}\left[r_{k,i}\right]\right) \quad (18)$$

where $r_{k,i}$ represents the interference term corresponding to the interfering user k. The interference term $r_{k,i}$ from (18) can be written in terms of the cross-correlation as [2], [3]:

$$r_{k,i} = 2\sum_{l=1-N}^{N-1} C^2_{k,i}(l) + \sum_{l=1-N}^{N-1} C_{k,i}(l)C_{k,i}(l+1) \quad (19)$$

The average variance of the second term in (13) that denotes the secondary rays MAI is given by [2]:

$$\sigma^2_{\Psi^S} = \underset{i}{E}\left[\sigma^2_{\Psi_i^S}\right] = \frac{PT^2}{12N^3}\sum_{u=1}^{\infty}\beta_u^2\left(\sum_{\substack{k=1\\k\neq i}}^{K}\underset{i}{E}\left[r_{k,i}\right]\right) \quad (20)$$

Using the power condition in equation (4) it can be seen that the average variance of Ψ_i is having the expression:

$$\sigma^2_{\Psi} = \sigma^2_{\Psi^P} + \sigma^2_{\Psi^S} = \frac{PT^2}{12N^3}\sum_{\substack{k=1\\k\neq i}}^{K}\underset{i}{E}\left[r_{k,i}\right] \quad (21)$$

which is not depending on the fading channel parameters. In fact, due to the power condition in (4) the variance expression in (21) is identical to that of the MAI variance in DS-CDMA system over non-faded channel [2-4].

The mean BER performance averaged over the whole set of spreading sequences is given by [2]:

$$BER \approx Q\left(\frac{\sqrt{\frac{P}{2}}\beta_0 b_{i,0}T}{\sqrt{\sigma^2_{\Xi} + \sigma^2_{\Psi}}}\right) \quad (22)$$

4. TAILED SHIFTS OPTIMUM SEQUENCES

One of the best known family of PWAM maps M: $X \to X$, X = [0, 1] that generates chaotic sequences is the (n, t) – *tailed shifts* map, defined in [1], for $t < n/2$:

$$M(x) = \begin{cases} \left((n-t)x\right)_{(\mathrm{mod}\,(n-t)/n)} + \dfrac{t}{n}, & \text{for } 0 \le x < \dfrac{n-t}{n} \\ \left(t\left(x - \dfrac{n-t}{n}\right)\right)_{(\mathrm{mod}\,t/n)}, & \text{otherwise} \end{cases} \quad (23)$$

4.1 Second, third and fourth moments estimations

Using the tensorial algebra as in [1] it can be demonstrated that for the (n, t) – *tailed shifts* map M-levels uniformly quantized sequences the second order moment has the following approximate expression, for any user k [4]:

$$A_{k,2}(l) = \underset{\{a_k\}}{\mathbf{E}}[f(a_{k,0})f(a_{k,l})] \cong \begin{cases} \dfrac{M+1}{3(M-1)}, & \text{for}\, l=0; \\ \dfrac{\left[\dfrac{1+g}{M}\cdot G\cdot(G-1) + M + 1 - 2\cdot G\right]^2}{g^2(M-1)^2}(-g)^l, & \text{for}\, l \neq 0 \end{cases} \quad (24)$$

where $g = t/(n-t)$ is a correlation parameter, $G = \left\lceil \frac{M}{1+g} \right\rceil$, and the function $f(x)$ is the M levels uniform quantization function [4].

For a constant power level at the output of the matched-filter receiver in the DS-CDMA system it is necessary to normalize all the shifted values of the second order moment by its non-shifted value. In fact, all the sequences will be normalized by this maximum autocorrelation value. Hence, the equation (24) is rewritten in the normalized form as:

$$\tilde{A}_{k,2}(l) = \frac{A_{k,2}(l)}{A_{k,2}(0)} = \frac{\underset{\{a_k\}}{\mathbf{E}}[f(a_{k,0})f(a_{k,l})]}{\underset{\{a_k\}}{\mathbf{E}}\left\{[f(a_{k,0})]^2\right\}} \cong \begin{cases} 1, & \text{for}\, l=0; \\ G_2(-g)^l, & \text{for}\, l \neq 0, \forall k, \end{cases} \quad (25)$$

where G_2 is a constant value with time resulting from (24) and (25) depending on the number of levels M, and on the sequence generator parameter g.

Following the same procedure as in [1] the third order moment of the sequences normalized to the maximum autocorrelation value has the following expression, for any user k:

$$A_{k,3}(l) = \frac{\underset{\{a_k\}}{\mathbf{E}}[f(a_{k,0})f^2(a_{k,m})f(a_{k,l})]}{[A_{k,2}(0)]^2} \cong (-g)^l G_3, \text{for}\,\forall k,l,m \quad (26)$$

where G_3 is a constant value with time given by:

$$\begin{aligned} G_3 = -\frac{G_2}{g}A_{k,2}(0)\Bigg\{&\left[1 + 4\frac{(G-1)^2}{(M-1)^2} - 4\frac{G-1}{M-1}\right]\cdot \\ &\cdot\left[\frac{2}{1+g} - \frac{2}{M}G + \frac{1}{M}\right] + \frac{1}{M}\Bigg[2G - M - 1 + \\ &+ \frac{2}{3(M-1)^2}\left[G(G-1)(2G-1) - \right. \\ &\left. - (M-G)(M-G+1)(2M-2G+1)\right] + \\ &+ \frac{2}{M-1}\left[(M-G)(M-G+1) - (G-1)G\right]\Bigg]\Bigg\} \end{aligned} \quad (27)$$

Also, the fourth order moment of the sequences normalized to the maximum autocorrelation value has the following expression, for any user k:

$$A_{k,4}(l,m,n) = \frac{\underset{\{a_k\}}{\mathbf{E}}[f(a_{k,0})f(a_{k,l})f(a_{k,m})f(a_{k,n})]}{[A_{k,2}(0)]^2} \quad (28)$$

$$\cong (-g)^{n-m+l}G_4, \qquad \text{for}\,\forall k,l,m,n, n > m > l > 0$$

where G_4 is a constant value with time given by:

$$G_4 = G_2^2 \quad (29)$$

4.2 DS-CDMA system performance estimation

Considering the normalized second order moment in (24) into the mean MAI variance expression in (21), the latter can be demonstrated to have the following expression [4]:

$$\sigma_{\Psi}^{2}(g,M) \cong \frac{PT^{2}(K-1)}{6N}\left[1-G_{2}g+2G_{2}^{2}g^{2}+G_{2}^{2}\frac{g^{3}(2g-1)}{1-g^{2}}\right] \quad (30)$$

Following again the complex computational procedure as in [2], which starts from equation (17) and uses second-, third- and fourth-order moments' expressions from (24)-(29), we derived the expression of mean self-interference variance as

$$\begin{aligned}
\sigma_{\Xi}^{2}(g,M) = {} & \frac{PT^{2}A^{2}}{4N^{2}\left(1-e^{-2BN}\right)}\Big\{NF_{0}\left(e^{-2B},N\right)+ \\
& + 2G_{4}\left[\frac{Ng^{2}}{1-g^{2}}-\frac{2g^{2}}{\left(1-g^{2}\right)^{2}}\right]\left[F_{0}\left(e^{-2B},N-1\right)-e^{-2B}\right]+ \\
& + NC_{1}e^{-2BN}+\left(N^{2}-N\right)e^{-2BN}+ \\
& + 2C_{2}Ne^{-2BN}F_{0}(-g,N)-2C_{2}e^{-2BN}F_{1}(-g,N)+ \\
& + g^{2N}(4G_{3}-4NG_{4}-2G_{4})\cdot\left[F_{1}\left(g^{-2}e^{-2B},N\right)-\right. \\
& \left. - F_{1}\left(g^{-2}e^{-2B},N-N_{1}\right)\right]+4G_{4}g^{2N}\frac{g^{-2}}{1-g^{-2}}\cdot \\
& \cdot\left[F_{1}\left(g^{-2}e^{-2B},N-1\right)-F_{1}\left(g^{-2}e^{-2B},N-N_{1}\right)\right]- \\
& - 2g^{2N}\left[NG_{3}+NG_{4}\frac{g^{-2}}{1-g^{-2}}-G_{4}\frac{g^{-2}}{\left(1-g^{-2}\right)^{2}}\right]\cdot \\
& \cdot\left[F_{0}\left(g^{-2}e^{-2B},N-1\right)-F_{0}\left(g^{-2}e^{-2B},N-N_{1}\right)\right]+ \\
& + 2G_{4}\frac{g^{2N+2}}{\left(1-g^{2}\right)^{2}}\left[F_{0}\left(g^{-2}e^{-2B},N-1\right)-F_{0}\left(g^{-2}e^{-2B},N_{1}+1\right)\right]+ \\
& + 4G_{4}g^{2N}\left[F_{2}\left(g^{-2}e^{-2B},N-1\right)-F_{2}\left(g^{-2}e^{-2B},N-N_{1}\right)\right]+ \\
& + C_{2}(-g)^{N}F_{1}\left(-g^{-1}e^{-2B},N\right)+C_{2}NF_{0}\left(-ge^{-2B},N\right)- \\
& - C_{2}F_{1}\left(-ge^{-2B},N\right)+N(NG_{4}-G_{4}+2G_{3})\cdot \\
& \cdot F_{0}\left(g^{2}e^{-2B},N_{1}+1\right)+2G_{4}\frac{g^{-2}}{1-g^{-2}}\left(N+\frac{1}{1-g^{-2}}\right)\cdot \\
& \cdot\left[F_{0}\left(g^{2}e^{-2B},N_{1}+1\right)-g^{2}e^{-2B}\right]+2G_{4}\frac{g^{2}}{\left(1-g^{2}\right)^{2}}\cdot \\
& \cdot\left[F_{0}\left(g^{2}e^{-2B},N-N_{1}\right)-g^{2}e^{-2B}\right]-2(2NG_{4}-G_{4}+2G_{3})\cdot \\
& \cdot F_{1}\left(g^{2}e^{-2B},N_{1}+1\right)-4G_{4}\frac{g^{-2}}{1-g^{-2}}\left[F_{1}\left(g^{2}e^{-2B},N_{1}+1\right)-\right. \\
& \left. - g^{2}e^{-2B}\right]+4G_{4}F_{2}\left(g^{2}e^{-2B},N_{1}+1\right)\Big\}
\end{aligned} \quad (31)$$

where $N_{1}=\left\lfloor\frac{N-1}{2}\right\rfloor$, F_0, F_1, and F_2 are the same auxiliary functions that were used in [2], while the constants C_1, and C_2 have the following expressions:

$$\begin{aligned}
C_{1} &= \frac{18\,[1+3^{4}+5^{4}+...+(M-1)^{4}]}{M\left(M^{2}-1\right)^{2}} \\
C_{2} &= \frac{1}{g^{2}M^{2}\left(M^{2}-1\right)^{2}}\left[-(g+1)^{2}D^{2}+4M(g+1)D\cdot\right. \\
&\cdot\left(M^{2}+3M+2-6MG+6G^{2}-6G\right)+48M^{2}G(M-G+1)\cdot \\
&\left.\cdot\left(M^{2}+3M+2-3MG+3G^{2}-3G\right)-4M^{2}(M+1)^{2}(M+2)^{2}\right]
\end{aligned} \quad (32)$$

where $D=2G(G-1)(4G-3M-2)$

5. NUMERICAL RESULTS

The asynchronous DS-CDMA system performances presented above for (n, t) – tailed shifts sequences are investigated. The analysis is performed for a spreading factor N=63 and simulations are run using a Monte-Carlo method.

The mean MAI variance per user from (30) is depicted in figure 1 as a function of the parameter g, considering three values for the number of quantizing levels M. It is obvious from figure 1 that for any value of M, the minimum mean MAI variance value is reached, but for different g values.

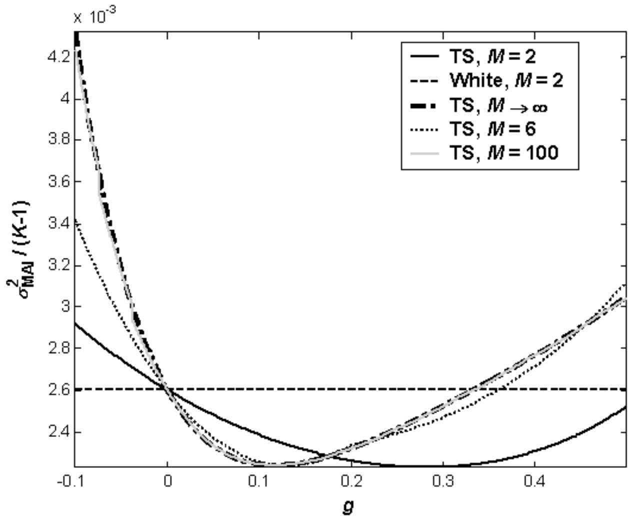

Fig.1. MAI variance per user as a function of g.

In figure 1 there are also presented for comparison the MAI variance for the systems using classical white sequences and chaotic optimum non-quantized sequences.

In figure 2 the self-interference variance from equation (31) is represented as a function of the channel dispersion parameter B, with R=1, for three distinct values of M. It is obvious from figure 2 that for any value of M the self-interference takes the same values with B. The simulations under the same scenarios provide good approximations for the theoretical results. The self-interference variance is represented also in figure 2 for the classical Gold and Kasami sequences, together with the non-correlated (n, t) – tailed shifts sequences (g=0). The comparison between these cases reveals that the correlated sequences that minimize the MAI variance, offer also increased values for the self-interference variance, while non-correlated sequences minimize the self-interference variance, determining MAI variance increase.

Having the analytical exact expressions of the MAI and self-interference variance expressions then an optimization of the system's BER performance can be done as in [2]. As it was shown in section 4.2 the key performance parameters σ_{Ψ}^{2} (from equation (30)) and σ_{Ξ}^{2} (from equation (31)) depend on the sequence generator parameter g and on the number of levels M. This allows the BER minimization under the SGA assumption, which is equivalent to maximizing the signal-to-interference ratio from equation (22), by controlling g and M. For example, in figure 3 the optimum (minimum) BER is represented on the right side of the graph as a function of the number of users K for two different fading scenarios, B=1 and B=0.1. This BER minimization is performed for any K value with respect to the parameter g. The resulted optimizing values, g_{opt} are represented on the left side in figure 3 as a function of the corresponding number of users, K.

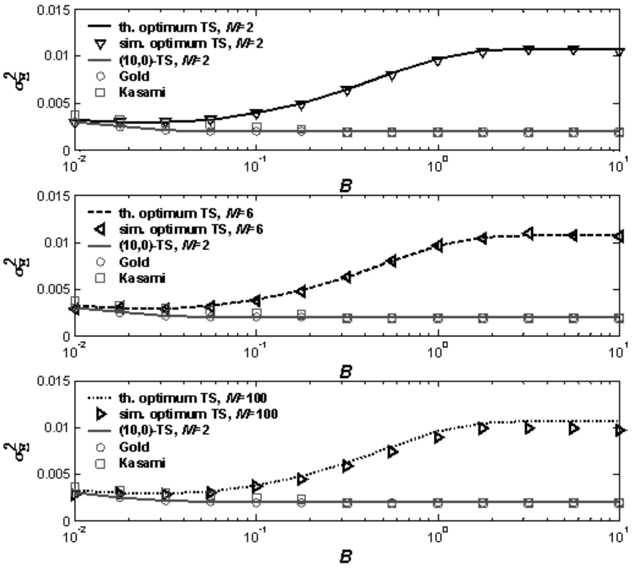

Fig.2. Self-interference variance as a function of B, R=1, and three values for M.

Similar results are depicted in figure 4 where the optimum BER is represented together with its corresponding g_{opt} values, as functions of channel dispersion B, for two different system loadings, K=8 and K=10.

The results in figures 3 and 4 are identical to those presented in [2] only that the same BER values are obtained for any value of M, as plotted for M=2, M=4, and M=100. In fact, the BER curves are perfectly overlapping, for any M value. However, these identical BER values are obtained for different optimum g_{opt} values. In order to keep the same optimum BER performances when increasing the number of levels M, one has to decrease the correlation parameter g_{opt} values. The BER is represented also in figure 3 and 4 for the non-correlated (n, t) – tailed shifts sequences (g=0). From these figures results that chaotic optimum sequences are always offering better BER performances than classical sequences, for any number of levels M.

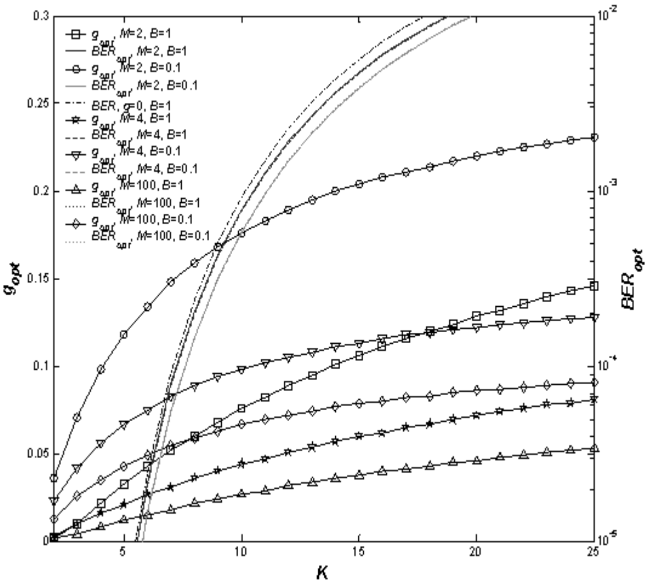

Fig.3. Optima BER_{opt} and g_{opt} as function of K and M, for R=1 and B=1 or B=0.1.

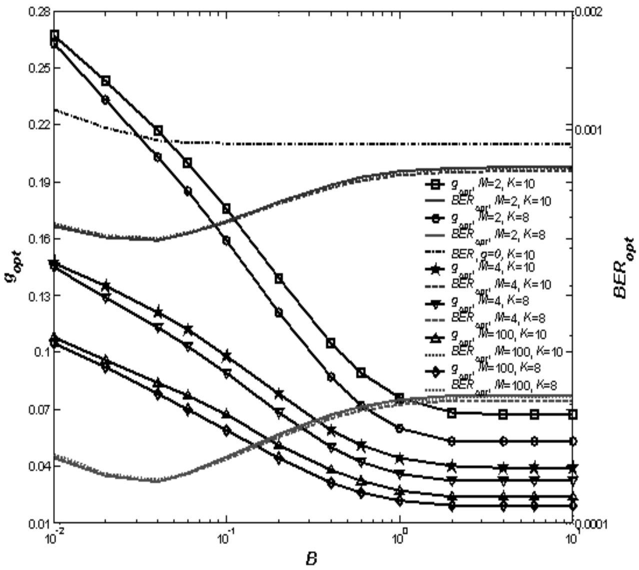

Fig.4. Optima BER_{opt} and g_{opt} as function of B and M, for R=1 and K=10 or K=8.

6. CONCLUSIONS

The paper adds a new result to the chaotic spreading DS-CDMA system performance analysis and optimization procedures presented by Mazzini et al. The fact that the optimum (n, t) – tailed shifts spreading sequences that minimize the BER under the SGA assumption can be designed for any number of quantizing levels, extends the applicability of these chaotic sequences to DS-CDMA systems. It was showed that under the same channel fading and system loading scenarios, the multilevel sequences can offer the same BER performances than the binary sequences, previously known as unique optimum sequences.

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