A PRECODING AND EQUALISATION DESIGN BASED ON OVERSAMPLED FILTER BANKS FOR DISPERSIVE CHANNELS WITH CORRELATED NOISE

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Abstract. Oversampled filter banks (OSFBs) have recently been considered for channel coding since their redundancy introduced into the transmitted signal permits more freedom in the design of joint transmitter and receiver. Further specifically, they can be exploited to transmit over low noise subspaces or even mitigate dispersiveness of the channel. In this paper we propose a joint precoding and equalisation design using OSFBs, which find a compromise between transmitting over the low-noise subspace of channel noise's polyphase components, and the high-gain subspace of the channel's polyphase components. Polynomial building blocks are permitted and the minimisation of the mean square error (MSE) at the receiver output is achieved. We describe the design, and highlight the communalities and differences of this approach to existing methods. Simulation results show the benefit of the proposed system design compared to existing design approaches.

1. INTRODUCTION

In recent years, oversampled filter banks (OSFBs) have been considered for channel coding applications, which aim to harness the degrees of freedom (DOFs) due to the redundancy released by oversampling to minimise the influence of channel noise on the received signal. Labeau has shown that the redundancy due to oversampling injected into the transmission can be exploited to find that solution amongst a manifold of reconstructing filter banks that projects onto the low-noise subspace [4]. Further in [5] the receive filter bank has been designed to minimise channel noise under the only constraint of paraunitarity, which provides more DOFs than [4]. The transmit filter bank is easily derived due to the paraunitarity of the design. In [6], we have shown that this can yield good results in a PLC scenario. However, these methods are only suitable for non-dispersive channels or after equalisation.

For the mitigation of both channel dispersion and channel noise, filterbank based transmitter and receiver designs that exploit redundancy have been investigated by Scaglione et al. in a number of publications, see e.g. [1, 2], and Mertins [3]. While these methods are more general than [4, 6], they are block-based and significant design effort in terms of DOFs is invested into inter-block interference (IBI) cancellation, particularly for low oversampling ratios. Further, the design requires the number of polyphase components, or block size, to be larger than the channel length in order for the design to be viable in the case of [2] or converge in the case of [3].

Therefore in this paper, we propose a novel OSFB design for precoding and equalisation which addresses both channel noise and dispersion but is not restricted to block transmission. Similar to [3], the precoder is set to minimise the mean square error (MSE) made in estimating the transmitted signal, while the extended Wiener solution provides the design for the equaliser. However, a novel design based on a recently proposed broadband eigenvalue decomposition for polynomial matrices which enables us to admit polynomial precoder and equalisers. The designed filter banks find a compromise between transmitting over the low-noise subspace of channel noise's polyphase components, and the high-gain subspace of the channel's polyphase components. Furthermore, we consider a waterfilling algorithm [7] to maximise the capability of the precoder and equaliser design for data transmission. Simulations are based on a PLC noise model as reported in [11], and demonstrate that greater design freedom and enhanced performance, particularly for low oversampling ratios, can be achieved.

The paper is organised as follows. The channel model and general system setup is outlined in Sec. 2, while the design of the proposed precoder and equalisation blocks is briefly introduced in Sec. 3. Simulation results and a comparison to [3] is presented in Sec. 4.

2. CHANNEL MODEL AND SYSTEM SETUP

Fig. 1 shows a general set up of the proposed channel coding system comprising of a precoder and an equaliser stage($\mathbf{Q}$ is the waterfilling component which will be introduced later). In order to describe the overall system, it is advantageous to represent the signals and systems in the form of decomposed polyphase components [8]. Thus, in Fig. 1, the SISO channel transfer function $C(z)$ is replaced by a pseudo-circulant matrix $\mathbf{C}(z)$ comprising of K polyphase components of $C(z)$ as

$$\mathbf{C}(z) = \begin{bmatrix} C_0(z) & z^{-1}C_{K-1}(z) & \cdots & z^{-1}C_1(z) \\ C_1(z) & C_0(z) & \cdots & z^{-1}C_2(z) \\ \vdots & \vdots & \ddots & \vdots \\ C_{K-1}(z) & z^{-1}C_{K-2}(z) & \cdots & z^{-1}C_0(z) \end{bmatrix}, \quad (1)$$

where $C_k(z) = \sum_n c(nK+k)z^{-n}$. Alternatively, $\mathbf{C}(z)$ can be written as a polynomial of matrices $\mathbf{C}(z) = \sum_n z^{-n}\mathbf{C}_n$. This description is reached by demultiplexing the channel input and output into K polyphase components [3, 8]. Similarly, the channel noise, $\underline{W}(z)$ is decomposed into K polyphase components. In the transmitter, N demultiplexed polyphase input signal vector $\underline{X}(z)$ is passed through the precoder $\mathbf{H}(z)$, while in the receiver the reconstructed signal vector is obtained from the equaliser $\mathbf{G}(z)$ output. Obviously, oversampling implies $N < K$, i.e. the whole system only rely on N subchannels for transmission. Also, the desired property $\hat{x}(n) = x(n-n_0)$ can be obtained in the noise-free case if $\mathbf{H}(z)$ and $\mathbf{G}(z)$ are chosen such that the perfect reconstruction (PR) condition

$$\mathbf{G}(z)\mathbf{C}(z)\mathbf{H}(z) = z^{-n_0+1}\mathbf{I}_{N\times N} \quad (2)$$

holds.

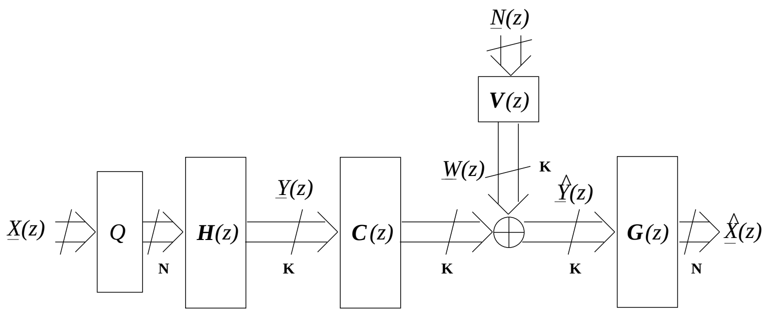

Figure 1: General model of the proposed OSFBs channel coding system.

3. PRECODING AND EQUALISATION DESIGN

The work in [2] is based on oversampled filter banks as precoding and equalisation blocks, whereby inter-block interference (IBI) is eliminated by dedicating to this task the first L DOFs for a channel of order L. As a result, the channel matrix $\mathbf{C}(z)$ as well as the noise covariance matrix are no longer dependent on z. Therefore, non-polynomial matrices $\mathbf{H}$ and $\mathbf{G}$ can be found for the precoder and equaliser. Mertins [3] suggested a Wiener solution, but then restricted the precoder to a non-polynomial type in order to facilitate the design. Although the design is expected to work for arbitrary channel orders, the number of polyphase components K has to be larger than L in order to obtain a system with satisfactory performance. Other filter bank based coders produce precoders and equalisers of polynomial type to minimize the impact of channel noise [4, 5, 6], but are not suitable for dispersive channels. Therefore, we want to focus on a design method for noisy and dispersive channels that admits polynomial precoders and equalisers.

In this section, we first describe the assumption of the data and the applied noise type, and then we explain the precoder and equaliser desgin step by step.

3.1 *Data Assumption and Noise Description*

The data process $\underline{x}[n] \circ\!\!-\!\!\bullet \underline{X}(z)$ is assumed to be white, zero-mean, wide sense stationary with variance σ_x^2, such that its power spectral matrix, $R_{xx}(z) \bullet\!\!-\!\!\circ R_{xx}[\tau] = \mathscr{E}\left\{\underline{x}[n]\underline{x}^{\mathrm{H}}[n-\tau]\right\}$, is given by $R_{xx}(z) = \sigma_x^2\mathbf{I}$. In contrast, we assume that the noise may be colored and spectrally, spatially correlated, which can be described via its power spectral matrix $R_{ww}(z)$ as:

$$R_{ww}(z) = \sum_m R_{ww}[m] z^{-m} \tag{3}$$

with

$$R_{ww}[m] = \mathscr{E}\left\{\underline{w}[n]\underline{w}^{\mathrm{H}}[n+m]\right\}. \tag{4}$$

3.2 *MSE Under Optimal Receive Filters*

For any selected precoder $\mathbf{H}(z)$, and a given channel impulse response $c(n)$, the optimal MMSE receive filters can be found by the Wiener solution [3] as:

$$\begin{aligned}\mathbf{G}(z) &= z^{-n_0+1}\sigma_x^2 \\ &\times \left[\mathbf{I} + \sigma_x^2 \tilde{\mathbf{H}}(z)\tilde{\mathbf{C}}(z)\mathbf{R}_{ww}^{-1}(z)\mathbf{C}(z)\mathbf{H}(z)\right]^{-1} \\ &\times \tilde{\mathbf{H}}(z)\tilde{\mathbf{C}}(z)\mathbf{R}_{ww}^{-1}(z) \quad ,\end{aligned} \tag{5}$$

where $\tilde{\mathbf{H}}(z) = \mathbf{H}^H(z^{-1})$ denotes the parahermitian operation. If $\mathbf{G}(z)$ is selected as in (5), the MSE is then given by the trace of the power spectral matrix $\mathbf{R}_{ee}(m) = E\{\mathbf{e}[n]\mathbf{e}^H[n+m]\}$ of the error $\underline{e}(n) \circ\!\!-\!\!\bullet \underline{E}(z) = \underline{X}(z) - \hat{\underline{X}}(z)$,

$$\begin{aligned}&\mathbf{R}_{ee}[m] \circ\!\!-\!\!\bullet \mathbf{R}_{ee}(z) = \\ &\quad \sigma_x^2\left[\mathbf{I} + \sigma_x^2\tilde{\mathbf{H}}(z)\tilde{\mathbf{C}}(z)\mathbf{R}_{ww}^{-1}(z)\mathbf{C}(z)\mathbf{H}(z)\right]^{-1}\end{aligned} \tag{6}$$

The challenge is now to design the precoder $\mathbf{H}(z)$ such that $\mathrm{tr}\{\mathbf{R}_{ee}[m]\}|_{m=0}$ is minimised.

3.3 *Precoder and Equaliser Design*

Obviously, minimising $\mathrm{tr}\{\mathbf{R}_{ee}[m]\}|_{m=0}$ can be accomplished by maximising the term

$$\tilde{\mathbf{H}}(z)\tilde{\mathbf{C}}(z)\mathbf{R}_{ww}^{-1}(z)\mathbf{C}(z)\mathbf{H}(z) \tag{7}$$

in (6). Our proposed design in this paper is based on a broadband eigenvalue decomposition (BEVD) algorithm [9, 10], which can factorise the middle term $\tilde{\mathbf{C}}(z)\mathbf{R}_{ww}^{-1}(z)\mathbf{C}(z)$, such that

$$\tilde{\mathbf{C}}(z)\mathbf{R}_{ww}^{-1}(z)\mathbf{C}(z) = \mathbf{U}(z)\Gamma(z)\tilde{\mathbf{U}}(z) \tag{8}$$

where $\mathbf{U}(z)$ is paraunitary, i.e. lossless with $\mathbf{U}(z)\tilde{\mathbf{U}}(z) = \mathbf{I}$. The matrix $\Gamma(z) \in \mathbb{C}^{K\times K}$ should be diagonal,

$$\Gamma(z) = \mathrm{diag}\{\Gamma_0(z), \Gamma_1(z), \cdots \Gamma_{K-1}(z)\} \tag{9}$$

and spectrally majorised with $\Gamma_0(e^{j\Omega}) \geq \Gamma_1(e^{j\Omega}) \geq \cdots \geq \Gamma_{K-1}(e^{j\Omega}) \quad \forall\Omega$. This decomposition is also called strong decorrelation. Now, the term we want to maximise becomes

$$\tilde{\mathbf{H}}(z)\mathbf{U}(z)\Gamma(z)\tilde{\mathbf{U}}(z)\mathbf{H}(z) \quad . \tag{10}$$

By further denoting the columns of the paraunitary matrix $\mathbf{U}(z)$ as

$$\mathbf{U}(z) = [\underline{U}_0(z) \;\; \underline{U}_1(z) \;\; \cdots \;\; \underline{U}_{K-1}(z)] \quad \in \mathbb{C}^{K\times K}(z) \quad , \tag{11}$$

we can obviously extract the precoder $\mathbf{H}(z)$ from $\mathbf{U}(z)$ to select the higher N elements on the main diagonal of $\Gamma(z)$,

$$\mathbf{H}(z) = \left[\underline{U}_0(z) \;\; \underline{U}_1(z) \;\; \cdots \;\; \underline{U}_{N-1}(z)\right] \quad \in \mathbb{C}^{K\times N}(z) \quad , \tag{12}$$

whereby the polyphase matrix $\mathbf{H}(z)$ defines an oversampled filter bank with K channels decimated by $N < K$. Note that due to $\tilde{\mathbf{H}}(z)\mathbf{U}(z) = [\mathbf{I}_N \;\; \mathbf{0}_{N\times K-N}]$ the presence of the only N

strongest eigenmodes ensure that the MSE $\mathrm{tr}\{\mathbf{R}_{ee}[m]\}|_{m=0}$ is minimised.

Further to the upper design, it needs to be pointed out that the inversion of the power spectral matrix $\mathbf{R}_{ww}(z)$ is also achieved from BEVD decomposition. Since the paraunitary property of $\mathbf{U}(z)$, it is straightforward that $\mathbf{R}_{ww}(z) = \mathbf{U}_{ww}(z)\Lambda_{ww}(z)\tilde{\mathbf{U}}_{ww}(z)$ leads to

$$\mathbf{R}^{-1}_{ww}(z) = \mathbf{U}_{ww}(z)\Lambda^{-1}_{ww}(z)\tilde{\mathbf{U}}_{ww}(z) \quad , \qquad (13)$$

which is considerably easier than the inversion afforded via the Smith-MacMillan decomposition [8].

3.4 *Power Allocation Design*

According to the above description, it is easy to understand the performance of each selected subchannel that system relies on is different, i.e. sequenced by the SNR value. Therefore, it is unfair to allocate the same power for all the subchannels from the capacity maximising point of view. To solve this problem, we simply introduce a waterfilling component according to [7] into our design.

Assuming that $\{\lambda_1, \dots, \lambda_N > 0\}$ corresponds to the first N elements of the zero-lag slice of $\Lambda(z)$, a waterfilling algorithm can be applied to pour the total power into each subchannel with the amount of $\{q_1, \dots, q_N\}$. Then, the power allocation matrix before precoder can be formed as

$$\mathbf{Q} = \mathrm{diag}\ [q_1,\ q_2,\ \dots\ , q_N] \quad , \qquad (14)$$

which can be seen from Fig. 1.

As far as the equaliser part is concerned, $\mathbf{G}(z)$ can be constructed from equation (5) as long as $\mathbf{H}(z)$ in (5) equals to $\mathbf{H}(z)$ in (12) times $\mathbf{Q}$ in (14).

4. SIMULATIONS AND RESULTS

4.1 *Power Line Noise Model*

Power line communication (PLC) environment is rather unfavorable, and subject to highly correlated additive Gaussian noise, which can be modeled as [11] $S_n(f) = a + b|f|^c$ dBm/Hz, where $a = 0, b = 38.75, c = -0.72$ are adapted corresponding to a worst case scenario described in [11].

4.2 *BER Performance Comparison and System Estimation Error Discussion*

By introducing the power line noise, in the following we try to compare the performance between our proposed design and Mertins' method. Fig. 2 shows the BER comparison between our OSFB_MMSE design and Mertins' method under a rather severely dispersive channel with its impulse response as: $C(z) = 1 + (0.8 + 0.9j)z^{-1} + 0.3z^{-2}$, whereby $N = 6$ and $K = 10$ are selected. In both method, total transmit power is set to be identical for fair. From the figure, we can view that although K is much larger, even $K - N$ is larger than the channel order, Mertins' method still can not perform well due to the highly correlated noise, while our method can achieve better BER performance and prove working well. In the later simulation described in 4.3, we will further explore the merits of our proposed design.

Fig. 3 characterises the polynomial mean square error matrix $\mathbf{R}_{ee}(z)$ in our method for $SNR = 13dB$. It clearly shows us the error caused in each subchannel increases along with the subchannel number. This could also motivate us to use different modulation techniques in each subchannel so that the total throughput is increased while the affordable error rate remains.

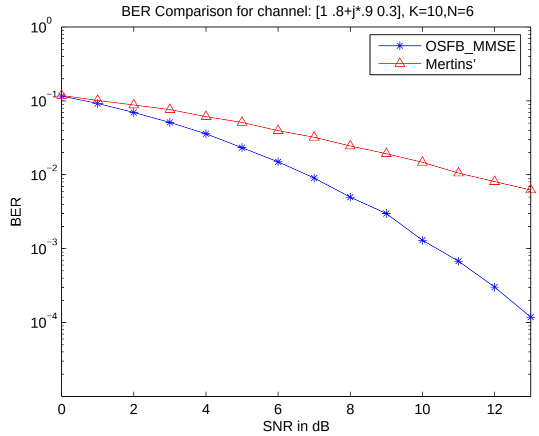

Figure 2: SNR_BER comparison between our proposed OSFB_MMSE design and Mertins' for channel=[1 .8+j*.9 .3] with PLC noise, K=10,N=6.

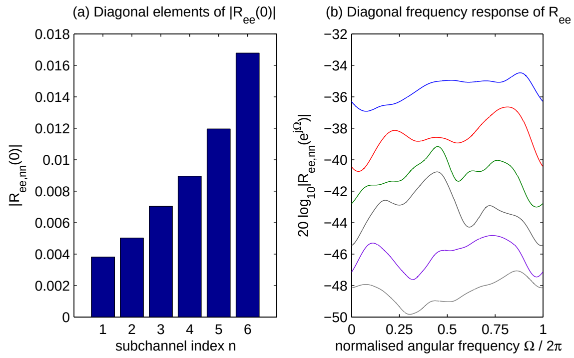

Figure 3: (a) Main diagonal elements of the system estimation error $\mathbf{R}_{ee}(z)$. (b) Frequency response of each subchannel estimation error $\mathbf{R}_{ee}(z)$.

4.3 *FB Magnitude Responses*

In order to provide a clear demonstration of the selection of each subband filters and their relations to the noise and channel, the following simulations try to show the magnitude responses of the selected subchannel filters according to the given channel and noise. For simplicity, both the channel and noise are assumed to be lowpass which occupies 50% and 25% of the total bandwidth, respectively and the order are set to be both 31.

Since both the channel and noise filter order are 31, we have to set $K = 32$ for Mertins' method to converge and $N = 8$ for a ratio of $N/K = 1/4$ to gain the selected subchannel. Meanwhile, for our design, $K = 8, N = 2$ are used for the same ratio. In Fig. 4 and Fig. 5, (a) and (b) plot the magnitude response of the noise and channel in the system respectively, while (c) in both figure shows the magnitude responses of N selected subchannel filters in the precoder.

As we can see, due to the permission of utilising poly-

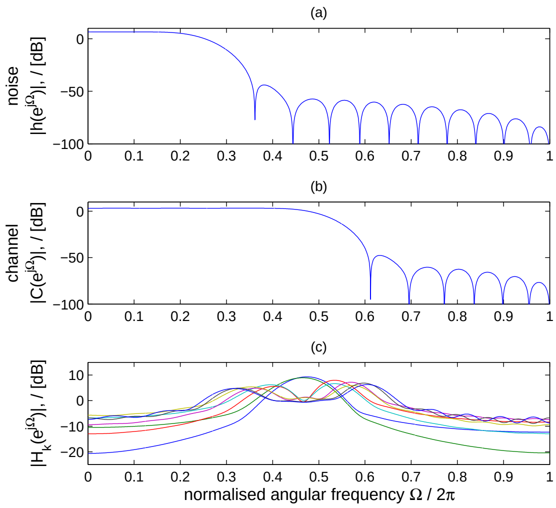

Figure 4: Magnitude responses of (a) lowpass noise (b) lowpass channel (c) N selected subband filters of the precoder for Mertins' design. $K = 32, N = 8$ filters.

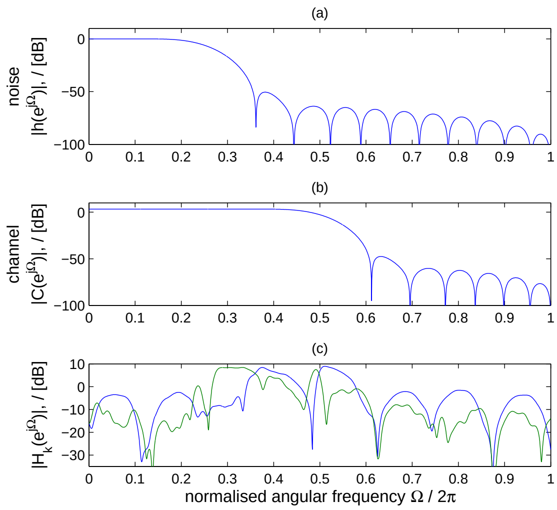

Figure 5: Magnitude responses of (a) lowpass noise (b) lowpass channel (c) N selected subband filters of the precoder for our OSFB disign. $K = 8, N = 2$ filters.

nomial matrix in the precoder, although smaller polyphase component are selected (even smaller than the channel order), our method still present a better selectivity compared to Mertins'. Also, accoding to further simulations, Mertins' disign gives exact solution only when $K - N$ is larger than the channel order and preformance decreases dramatically when $K - N$ becomes smaller while our method remains similar.

5. CONCLUSIONS

We have proposed an oversampled filter bank design for a polynomial precoding and equalisation system, which is based on an extended Wiener filter solution presented by [mertins03a]. Different from previous realisations, the proposed design admits true polynomial matrices for precoder and equaliser. The precoder is designed to minimise the mean square error made in estimating the transmitted signal, while the extended Wiener solution provides the design for the equaliser. Capacity maximising problem has been considered as well by the waterfilling algorithm. The proposed system minimises the influence from both channel dispersion and channel noise, and compares very favourably to work reported in the literature based on a non-polynomial precoding mechanism.

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