A NON-BLOCK BASED APPROACH TO THE BLIND EQUALIZATION OF SPACE-TIME BLOCK CODES OVER FREQUENCY SELECTIVE CHANNELS

Samir Bendoukha and Stephan Weiss

Institute of Communications and Signal Processing, University of Strathclyde
Glasgow, UK
phone: + (44) 141 548 2808, email: samir@eee.strath.ac.uk

ABSTRACT

Space-time block coding (STBC) and a number of derivative techniques have been developed to maximise the diversity gain of a multiple-input multiple-output (MIMO) channel. For frequency selective fading — i.e. dispersive — MIMO channels, solutions in the literature are fewer, and are in their majority block based, such as MIMO OFDM or time-reversal STBC. In order to ultimately achieve better tracking behaviour, in this paper we focus on the development of a non-block-based method. We utilise a constant modulus criterion on the detected signals together with the orthogonality contraint of the transmitted STBC signals, whereby for the later we derive novel criterion and an adaptive optimisation scheme. We demonstrate the proposed system in simulations and compare it to the performance of a TRSTBC system.

1. INTRODUCTION

Multi-input multi-output (MIMO) channels are known to increase the capacity of a transmission link. This can be exploited to increase either the multiplexing gain or the diversity gain, which leads to a higher data throughput or a better resilience of the link to fading, respectively.

In the pioneer work by Alamouti, [1], a transmitter diversity scheme, named Space-Time Block Coding (STBC), was derived which maximizes the level of diversity obtained in flat fading channels. This was later extended in [2, 3] for dispersive channels with inter-symbol interference (ISI) to a technique called time-reversal STBC (TRSTBC). In [2, 3], full channel state information (CSI) is assumed at the receiver, which usually implies the availability of a training sequence. This reduces the overall throughput of the system and may prove unrealistic in fast fading channels.

A blind receiver based on the Constant Modulus Algorithm (CMA) was derived in [4] for the blind equalization of TRSTBC systems. The block based receiver assumes stationarity of the channel over 2 consecutive blocks of data. Due to the slow convergence of the CM algorithm, this assumption will not hold when dealing with fast fading channels.

In this paper, a non-block based receiver, which is also based on the CM algorithm, is derived for use with Space-Time Block Coding. A new constraint is placed on the outputs of the equalizer to avoid identification of the same source at more than one output.

2. BLOCK BASED APPROACH

2.1 Time-Reversal STBC

The performance of STBC drops over frequency selective channels and the diversity level obtained is less than that of MRRC. A technique introduced in [2] and explained in more detail in [3], namely Time-Reversal STBC (TRSTBC), has been shown to achieve full diversity over multipath MIMO channels.

Figure 1 shows a TRSTBC system with 2 transmit and 2 receive antennas. The transmitted data is divided into two sets of symbols $s_1[n]$ and $s_2[n]$ and transmitted in sets of two bursts. During the first burst, $s_1[n]$ and $s_2[n]$ are transmitted from first and second antennas, respectively. During the second burst, the sequences are time reversed and conjugated and $-s_2^*[N-n]$ and $s_1^*[N-n]$ are transmitted from the first and second antennas, respectively, where N is the length of $s_1[n]$ and $s_2[n]$.

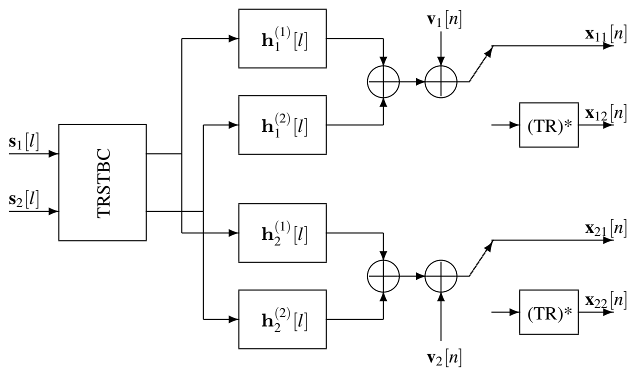

Figure 1: Data Model for a 2x2 TRSTBC system

Let $\mathbf{x}[n]$ be the received signal of dimension 4x1,

$$\mathbf{x}[n] = \left[x_{1,1}[n] \;\; x_{2,1}[n] \;\vdots\; x_{1,2}[n] \;\; x_{2,2}[n] \right]^T, \tag{1}$$

where $x_{j,1}[n]$ and $x_{j,2}[n]$ are the signals picked up by the j^{th} antenna during the regular and reverse modes of transmission, respectively. The $(\cdot)^T$ superscript denotes the matrix transposition operator. The vector $\mathbf{x}[n]$ can be written as

$$\mathbf{x}[n] = \sum_{l=0}^{L_i-1} \mathbf{H}[l]\mathbf{s}[n-l] + v[n], \tag{2}$$

where $v[n]$ is the Additive White Gaussian Noise (AWGN) vector,

$$\mathbf{s}[n] = \begin{bmatrix} s_1[n] \\ s_2[n] \end{bmatrix}, \tag{3}$$

and

$$\mathbf{H}[l] = \begin{bmatrix} \mathbf{h}_1[l] & \mathbf{h}_2[l] \\ \mathbf{h}_2^*[L_i{-}1{-}l] & -\mathbf{h}_1^*[L_i{-}1{-}l] \end{bmatrix} \tag{4}$$

$$\mathbf{h}_i[l] = [\mathbf{h}_{i,1}[l] \ \ \mathbf{h}_{i,2}[l]]^T, \tag{5}$$

where $\mathbf{h}_{i,j}[l]$ is the channel from the i^{th} transmit antenna to the j^{th} receive antenna. The length of the channels is assumed to be identical, denoted L_i.

Note that the signals received during the second phase of transmission, i.e. $x_{1,2}[n]$ and $x_{2,2}[n]$, are conjugated and time reversed, as shown in figure 1.

2.2 Tap-Constrained CMA

In [4], two separate equalizers are used to extract the two transmitted sequences $s_1[n]$ and $s_2[n]$. Each equalizer contains 4 sub-equalizers. At the n^{th} iteration, the weight vector of the i^{th} equalizer is given by

$$\mathbf{w}_i[n] = \left[w_{1,1}^{(i)}[n] \ \ w_{2,1}^{(i)}[n] \ \ w_{1,2}^{(i)}[n] \ \ w_{2,2}^{(i)}[n] \right], \tag{6}$$

and the corresponding output is

$$y_i[n] = \mathbf{w}_i^H[n]\tilde{\mathbf{x}}[n], \tag{7}$$

where

$$\tilde{\mathbf{x}}[n] = \left[\tilde{\mathbf{x}}_{1,1}^T[n] \ \ \tilde{\mathbf{x}}_{2,1}^T[n] \ \ \tilde{\mathbf{x}}_{1,2}^T[n] \ \ \tilde{\mathbf{x}}_{2,2}^T[n] \right]^T, \tag{8}$$

and $\tilde{\mathbf{x}}_{j,i}[n] = [x_{j,i}[n] \ \ x_{j,i}[n-1] \ \cdots \ x_{j,i}[n-L+1]]$.

The channel matrix for a 2x2 antenna TRSTBC system can be given by,

$$\mathbf{H}[l] = \begin{bmatrix} \mathbf{h}_{1,1}[l] & \mathbf{h}_{2,1}[l] \\ \mathbf{h}_{1,2}[l] & \mathbf{h}_{2,2}[l] \\ \mathbf{h}_{2,1}^*[L_i-1-l] & -\mathbf{h}_{1,1}^*[L_i-1-l] \\ \mathbf{h}_{2,2}^*[L_i-1-l] & -\mathbf{h}_{1,2}^*[L_i-1-l] \end{bmatrix}. \tag{9}$$

Filtering the received signals by the time reversed hermitian of $\mathbf{H}[l]$ gives

$$\begin{aligned} \mathbf{H}^H[L_i-1-l]x[n] &= \mathbf{H}^H[L_i-1-l]\mathbf{H}[l]\mathbf{s}[n] + \tilde{v}[n] \\ &= \mathbf{D}[l]\mathbf{s}[n] + \tilde{v}[n], \end{aligned} \tag{10}$$

where $\tilde{v}[n] = \mathbf{H}^H[M-1-l]v[n]$, and

$$\mathbf{D}[l] = \left(\sum_{j=1}^{2}\sum_{i=1}^{2} \mathbf{h}_{i,j}^*[L_i-1-l]\mathbf{h}_{i,j}[l] \right) I_2. \tag{11}$$

From (11), it can be observed that $\mathbf{D}[l]$ is diagonal, which means the signals radiated from the two transmit antennas, i.e. $s_1[n]$ and $s_2[n]$, can be ideally decoupled when full channel information is available at the receiver. Since the diagonal elements of $\mathbf{D}[l]$ are symmetric in time, the total response of the matched filter and equalizer should be identical to $\mathbf{H}^H[L_i-1-l]$.

From these observations, a tap constraint can be placed on the CMA weight vectors as follows,

$$\mathbf{w}_1[n] = \begin{bmatrix} \mathbf{c}_{11}^*[L-1-l] \\ \mathbf{c}_{12}^*[L-1-l] \\ \mathbf{c}_{21}[l] \\ \mathbf{c}_{22}[l] \end{bmatrix} \ \text{and} \ \mathbf{w}_2[n] = \begin{bmatrix} \mathbf{c}_{21}^*[L-1-l] \\ \mathbf{c}_{22}^*[L-1-l] \\ -\mathbf{c}_{11}[l] \\ -\mathbf{c}_{12}[l] \end{bmatrix}, \tag{12}$$

where L is the length of the subequalizers. A relation between $\mathbf{w}_1[n]$ and $\mathbf{w}_2[n]$ can be observed from (12),

$$\mathbf{w}_2[n] = \mathbf{P}^T\mathbf{w}_1^*[n], \tag{13}$$

where

$$\mathbf{P} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & -\tilde{I}_L & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & -\tilde{I}_L \\ \tilde{I}_L & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \tilde{I}_L & \mathbf{0} & \mathbf{0} \end{bmatrix}, \tag{14}$$

with $\tilde{I}_L$ being the reverse-identity matrix, $\tilde{I}_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

The CMA cost function for the two equalizers can be given by,

$$\begin{aligned} \xi &= \xi_1 + \xi_2 \\ &= \mathscr{E}\left\{ \left(|y_1[n]|^2 - \gamma^2 \right)^2 \right\} + \mathscr{E}\left\{ \left(|y_2[n]|^2 - \gamma^2 \right)^2 \right\}, \end{aligned} \tag{15}$$

where γ is the constant modulus of the QPSK constellation assumed at the transmitter. According to the gradient descent method,

$$\begin{aligned} \mathbf{w}_1[n+1] &= \mathbf{w}_1[n] - \mu \bigtriangledown_{\mathbf{w}_1} \hat{\xi}_n \\ &= \mathbf{w}_1[n] - \mu \left[\bigtriangledown_{\mathbf{w}_1}\hat{\xi}_1 + \bigtriangledown_{\mathbf{w}_1}\hat{\xi}_2 \right]. \end{aligned} \tag{16}$$

where $\bigtriangledown_{\mathbf{w}_i}$ denotes the gradient operator in terms of $\mathbf{w}_i$ and $\hat{\xi}_n$, is the instantaneous estimate of the cost function ξ at time n. Similar to the derivation of CMA in [5, 6], it can be verified that

$$\begin{aligned} \bigtriangledown_{\mathbf{w}_1}\hat{\xi}_1 &= \left(|y_1[n]|^2 - \gamma^2 \right) y_1^*[n]\tilde{\mathbf{x}}[n] \\ &= \varepsilon_1^*[n]\tilde{\mathbf{x}}[n] \end{aligned} \tag{17}$$

and

$$\begin{aligned} \bigtriangledown_{\mathbf{w}_1}\hat{\xi}_2 &= \left(|y_2[n]|^2 - \gamma^2 \right) y_2[n]\mathbf{P}\tilde{\mathbf{x}}^*[n] \\ &= \varepsilon_2[n]\mathbf{P}\tilde{\mathbf{x}}^*[n]. \end{aligned} \tag{18}$$

Substituting in equation (16) gives

$$\mathbf{w}_1[n+1] = \mathbf{w}_1[n] - \mu \left[\varepsilon_1^*[n]\tilde{\mathbf{x}}[n] + \varepsilon_2[n]\mathbf{P}\tilde{\mathbf{x}}^*[n] \right], \tag{19}$$

and $\mathbf{w}_2[n+1]$ is calculated from $\mathbf{w}_1[n+1]$ according to (13). This defines the update operation of the weight vectors $\mathbf{w}_1[n]$ and $\mathbf{w}_2[n]$ at the n^{th} iteration.

3. NON-BLOCK BASED APPROACH

3.1 Data Model

Consider the 2-transmit and 2-receive antenna configuration shown in figure 2. The transmitted data is encoded in space and time according to STBC, as in [1]. At times n and $n+1$, two symbols, a_1 and a_2, arrive at the encoder, which are drawn from a PSK constellation set. The transmitted symbols are calculated as,

$$\begin{bmatrix} s_1[n] & s_1[n+1] \\ s_2[n] & s_2[n+1] \end{bmatrix} = \begin{bmatrix} a_1 & -a_2^* \\ a_2 & a_1^* \end{bmatrix}. \tag{20}$$

Denote the channel from the i^{th} transmit antenna to the j^{th} receive antenna as

$$\mathbf{h}_{i,j} = [h_{i,j}[0] \ \ h_{i,j}[1] \ \cdots \ h_{i,j}[L_i-1]], \tag{21}$$

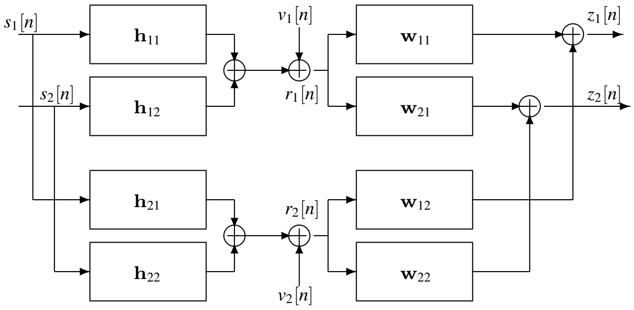

Figure 2: Channels and equalizers for a 2-by-2 MIMO system.

where the channels are assumed to be of the same length, L_i. If the length of the channels is not the same, L_i is the length of the longest channel and the other channels are appended with zeros. The receive signal at the j^{th} antenna is given by

$$r_j[n] = \mathbf{h}_{j,1}\,\mathbf{s}_{1,n} + \mathbf{h}_{j,2}\,\mathbf{s}_{2,n} + v_j[n], \tag{22}$$

where

$$\mathbf{s}_{i,n} = [s_i[n]\;\; s_i[n-1]\;\; \cdots\;\; s_i[n-L_i+1]]^T \tag{23}$$

and $v_j[n]$ is the Additive White Gaussian Noise (AWGN) at the j^{th} receive antenna. As shown in figure 2, two space-time equalizers are used, each with two subequalizers. It was proven in [8] that perfect Zero-Forcing (ZF) equalization of the p-by-m MIMO channel can be achieved if the length of the subequalizers, L satisfies

$$L \geq \underline{L} = \left\lceil \frac{L_i(p-1)}{m-p} \right\rceil, \tag{24}$$

where $\lceil\cdot\rceil$ denotes the integer part. In [8], it is assumed that $m > p$. The algorithm at hand can be trivially extended to 2-by-m.

The outputs of the two space-time equalizers are collected over consecutive symbol periods, n and $n+1$, and are given by

$$\begin{bmatrix} z_1[n] & z_1[n+1] \\ z_2[n] & z_2[n+1] \end{bmatrix} = \begin{bmatrix} \mathbf{w}_1^H[n] \\ \mathbf{w}_2^H[n] \end{bmatrix} \cdot [\mathbf{r}_n\;\; \mathbf{r}_{n+1}], \tag{25}$$

where the transversal delay line vector

$$\mathbf{r}_n = [r_1[n]\;\; \cdots\;\; r_1[n-L+1]\;\; r_2[n]\;\; \cdots\;\; r_2[n-L+1]]^T, \tag{26}$$

and the equalizer coefficients vector

$$\mathbf{w}_j[n] = \left[w_{j1}^*[0]\;\; \cdots\;\; w_{j1}^*[L-1]\;\; w_{j2}^*[0]\;\; \cdots\;\; w_{j2}^*[L-1]\right]^T, \tag{27}$$

3.2 The Algorithm

The Constant Modulus (CM) cost function for a 2x2 MIMO system can be given by

$$\xi_1 = \mathscr{E}\left\{\sum_{i=1}^{2}(|z_i[n]|^2-1)^2\right\}, \tag{28}$$

which forces the modulus of the two equalizer outputs $z_1[n]$ and $z_2[n]$ to unity. However, the same source may be identified at more than one receiver, which reduces the diversity gain of the system. In order to avoid multiple extractions of the same source, a modified cost function was used in [9, 10, 11]. In addition to the constant modulus property, the modified algorithm forces the cross-correlation of the outputs to zero. The new algorithm that is proposed here not only forces the outputs to be uncorrelated, but it forces them to have an STBC structure. Consider the following cost function

$$\xi_2 = \mathscr{E}\left\{\sum_{i=1}^{2}\sum_{\tau=0}^{1}(|z_i[n+\tau]|^2-1)^2 + \mathbf{a}_n^{\mathrm{H}}\mathbf{a}_n\right\}, \tag{29}$$

with

$$\mathbf{a} = \begin{bmatrix} z_1[n] & - & z_2^*[n+1] \\ z_2[n] & + & z_1^*[n+1] \end{bmatrix}, \tag{30}$$

where the first term of the cost function represents the CMA criterion over two consecutive symbol periods. The new term minimizes the vector $\mathbf{a}$ which forces the two outputs, $z_1[n]$ and $z_2[n]$, to have an STBC structure and consequently minimizes the cross-correlation between them.

Using the instantaneous estimate of ξ_2, the stochastic gradient descent algorithm is given by

$$\begin{aligned} \mathbf{w}_1[n+2] &= \mathbf{w}_1[n] - \mu\,\nabla_{\mathbf{w}_1}\hat{\xi}_n \\ \mathbf{w}_2[n+2] &= \mathbf{w}_2[n] - \mu\,\nabla_{\mathbf{w}_2}\hat{\xi}_n \end{aligned}, \tag{31}$$

The first components of the cost function are the standard CMA, which is straightforward to derive:

$$\frac{\partial}{\partial \mathbf{w}_i^*}(z_k[n+\tau]z_k^*[n+\tau]-1)^2 = \begin{cases} 2(z_k[n+\tau]z_k^*[n+\tau]-1)z_k^*[n+\tau]\mathbf{r}_{n+\tau} & k=i \\ \underline{0} & k\neq i \end{cases} \tag{32}$$

The second term requires closer evaluation. It can be shown that the gradient of the second term is

$$\begin{aligned} \frac{\partial}{\partial \mathbf{w}_1^*}\mathbf{a}_n^{\mathrm{H}}\mathbf{a}_n &= (z_1^*[n]-z_2[n+1])\mathbf{r}_n + (z_2[n]+z_1^*[n+1])\mathbf{r}_{n+1} \\ \frac{\partial}{\partial \mathbf{w}_2^*}\mathbf{a}_n^{\mathrm{H}}\mathbf{a}_n &= (z_2^*[n]+z_1[n+1])\mathbf{r}_n + (z_2^*[n+1]-z_1[n])\mathbf{r}_{n+1} \end{aligned} \tag{33}$$

The update equations in (31) yield

$$\begin{aligned} \mathbf{w}_1[n+2] = \mathbf{w}_1[n] &- \mu\left(2(z_1[n]z_1^*[n]-\tfrac{1}{2})z_1^*[n] - z_2[n+1]\right)\mathbf{r}_n \\ &- \mu\left(2(z_1[n+1]z_1^*[n+1]-\tfrac{1}{2})z_1^*[n+1] + z_2[n]\right)\mathbf{r}_{n+1} \\ \mathbf{w}_2[n+2] = \mathbf{w}_2[n] &- \mu\left(2(z_2[n]z_2^*[n]-\tfrac{1}{2})z_2^*[n] + z_1[n+1]\right)\mathbf{r}_n \\ &- \mu\left(2(z_2[n+1]z_2^*[n+1]-\tfrac{1}{2})z_2^*[n+1] - z_1[n]\right)\mathbf{r}_{n+1} \end{aligned} \tag{34}$$

Equation (34) describes the stochastic gradient algorithm named STBC-CMA. The new algorithm is suitable for the spatio-temporal equalization of constant modulus STBC signals. A windowed estimate of the cost function, ξ_2, can be used instead of the instantaneous estimate for better convergence. The derrived equations can be easily extended to a 2 by R receive antenna configuration for better diversity gains.

4. SIMULATION RESULTS

Computer simulation results are presented in this section to show that the new algorithm successfully decouples the two transmitted signals in an STBC system. The same MIMO model is used as in figure 2. QPSK modulation is used at the transmitter with a modulus equal to unity. The Signal-to-Noise Ratio (SNR) is set to $20dB$. Dispersive channels of length $L_p = 4$ are used. The length of the subequalizers $L = 7$, which satisfies the condition in (24). The step size $\mu = 5 \cdot 10^{-3}$. For the new algorithm, the $2Lx1$ coefficient vectors $\mathbf{w}_1$ and $\mathbf{w}_2$ are initialized having only two non-zero elements equal to unity at entries 1 and $L+1$, respectively. Experiments showed that the best position of the spike in the initialization of TRSTBC-CMA is the center.

Figure 3 shows the convergence of the two equalizers and the cross-correlation between their outputs and the transmitted signals. It can be seen from the graphs that both of the transmitted sequences, $s_1[n]$ and $s_2[n]$, are identified at the receiver.

Figure 4 shows the steady state mean Square Error (MSE) curves for the new algorithm as compared to Time-Reversal STBC. Experiments have shown that the best value of the step size for TRSTBC is $\mu = 3 \cdot 10^{-4}$. Whereas with STBC-CMA, the step size can be chosen as high as $\mu = 5 \cdot 10^{-3}$. This leads to a faster convergence as seen in the graph.

A key point when comparing two algorithms is complexity. The new algorithm uses half the number of equalizers used in TRSTBC, which makes it simpler and more suitable for real-time applications. Since the new algorithm is non-block based, it offers more flexibility when tracking fast varying channels.

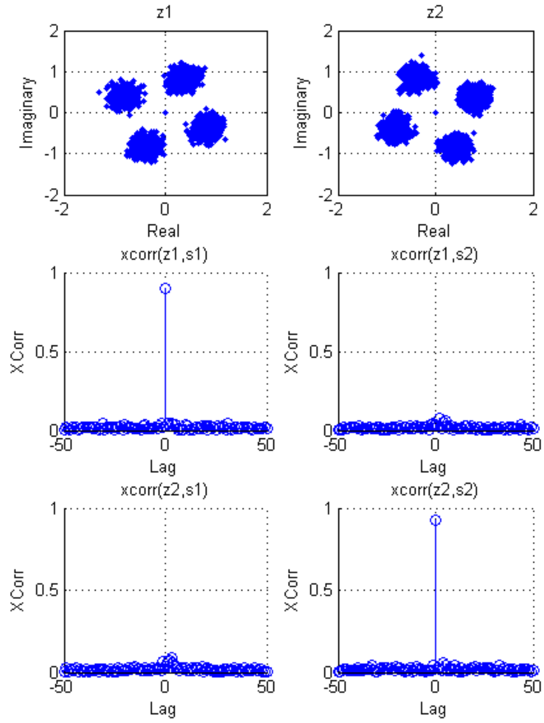

Figure 3: Equalizer outputs and cross correlation with transmitted sequences.

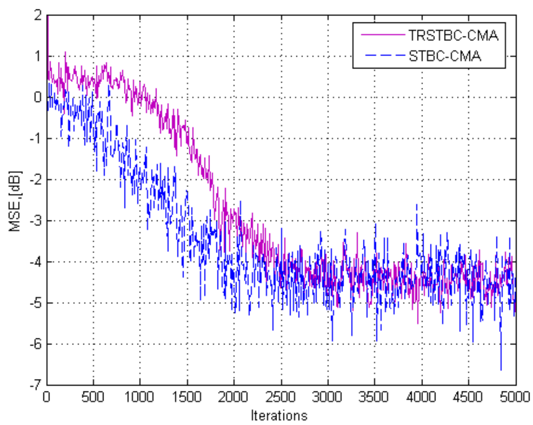

Figure 4: Mean square error curves for STBC-CMA and TRSTBC-CMA, SNR = $20dB$.

5. CONCLUSION

In this paper, a novel algorithm based on the Constant Modulus (CM) criterion has been derived for the blind equalization of STBC systems. The algorithm adds a new term to the cost function of CMA, which minimizes the cross-correlation between the outputs and forces them to have an STBC structure. Compared to TRSTBC-CMA in [4], the new algorithm achieves a better steady state MSE and allows higher values for the step size, μ, which leads to faster convergence. The algorithm, named STBC-CMA, uses half the number of equalizers as TRSTBC, which reduces the complexity of the receiver.

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