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RESOURCE ALLOCATION FOR GOODPUT OPTIMIZATION IN PARALLEL SUBCHANNELS WITH ERROR CORRECTION AND SELECTIVE REPEAT ARQ

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ABSTRACT

This paper deals with the problem of allocating bits and power among a set of parallel frequency-flat subchannels. The objective is to maximize the number of information bits delivered without error to the user by unit of time, or goodput. We consider a frame-oriented transmission with convolutional coding, hard Viterbi decoding, and selective repeat automatic repeat request (ARQ) retransmission protocol. An expression for the goodput of the considered communication system is derived. Different bit and power allocations strategies are proposed and compared to one another using simulations. It turns out that the best trade-off between performance and complexity is achieved by allocating the power in such a way that the bit error rate is equal on all subchannels, and by allocating the bits by rounding the solution to the problem obtained by relaxing the constraint of integer constellation sizes.

1. INTRODUCTION

The problem of allocating resource among a set of parallel frequency-flat subchannels is often encountered in transmitter design, both in wired and wireless transmissions. For instance, two well-known communication techniques implicate the transmission over a set of parallel subchannels: the multicarrier modulation, and the use of multiple antennas (if the singular vectors of the MIMO matrix are used for pre/decoding). It has long been proved that the the mutual information of a set of parallel AWGN channels is maximized by allocating the power according to the waterfilling solution. Several algorithms were further proposed to modify the waterfilling solution in order to take into account the fact that the constellations sizes are, in practice, constrained to be integer [1, 2]. Since then, many works have treated that subject. However, most of them have focused on the optimization of uncoded quantities.

In this paper, we will treat the resource allocation problem using as criterion the goodput, defined as the number of information bits delivered without error to the user by unit of time. This system-based criterion enables to take into account the presence of error correction and frame retransmission in the communication system. In fact, the performance of a communication system can be improved if the physical layer is designed taking into consideration the error correction mechanism and retransmission protocol used in the system [3, 4]. This paper considers a frame-oriented transmission with convolutional coding, hard Viterbi decoding, and selective repeat automatic repeat request (ARQ) retransmission protocol. The paper is organized as follows. We start in section 2 by describing the communication system, while a formulation for the discrete allocation problem is given in section 3. Different power and bit allocation strategies are derived in section 4 and 5, respectively. These strategies are simulated in section 6, and finally conclusions are drawn in section 7.

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2. SYSTEM MODEL

The communication system considered in this paper is depicted in Fig. 1, where a distinction is made between the physical and data link layers. In this section, this communication system is described and modeled.

The data link layer deals with frames, where each frame contains a fixed number (N_f) of information bits. At the transmitter side, the frames which are ready to be transmitted are queued in a buffer. At the receiver side, the frames that are received without any error[1] are also buffered before being delivered in correct order to the user. However, when a received frame is detected in error, it has to be retransmitted. We consider that the transmission and retransmission of frames are controlled by an automatic repeat request (ARQ) protocol (see Fig. 1). In particular, the selective repeat ARQ protocol is considered in this paper [5].

At the transmitter side, the N_f information bits contained in a frame which has to be transmitted, are passed to the physical layer for transmission. There, the information bits u_n are first convolutionally encoded and randomly interleaved (Fig. 1). The resulting coded bits x_n are then transmitted, this operation will be described in the next paragraph. At the receiver side, first, hard-decisions are made on the received signal to produce decisions $\hat{x}_n$ on the coded bits. The bits $\hat{x}_n$ are then deinterleaved and Viterbi decoded. Finally, the detected information bits $\hat{u}_n$ are reorganized in frames of N_f bits, and passed to the data link layer. Depending on how the bits are transmitted through the channel, the bit error rate (BER) associated with the hard-decision on the coded bits might in general not be equal for all coded bits in a frame. However, thanks to the random (de)interleaver, the Viterbi decoded sees the whole channel as a binary symmetric channel with error probability given by the mean BER

[1]We suppose that the receiver is able to perfectly distinguish error-free frames from others. In other words, even though it is not really included in the system structure, this paper supposes perfect cyclic redundancy check.

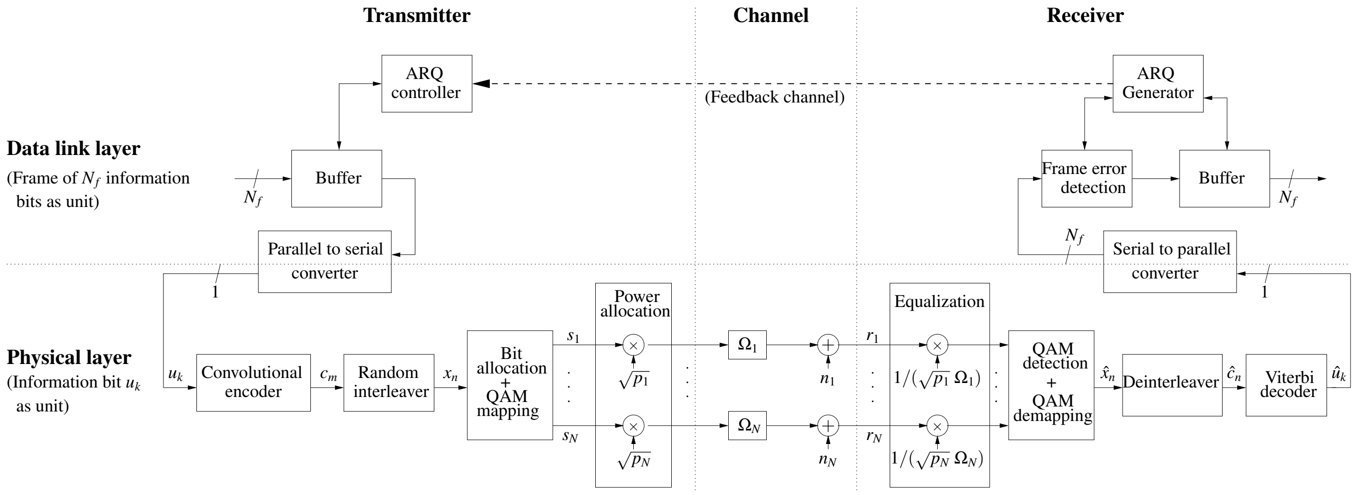

Figure 1: Structure of the communication system: physical and data link layers.

associated with the hard-decision making on the coded bits of a frame, denoted by $\overline{\rho}$. As a consequence, the probability that a frame is Viterbi decoded without any error, is a function of this mean BER $\overline{\rho}$. The following expression for the frame success rate (FSR) will be used in this paper:

$$FSR(\overline{\rho}) = d\ \exp\left(-\left(a_v\overline{\rho}^v + \cdots + a_1\overline{\rho}\right)\right) \tag{1}$$

where $d, a_v, \ldots, a_1, v$ are constants which have to be designed such that the expression (1) fits the true FSR curve. These constants depend on the convolutional code used, and on the frame size N_f.

Let us now describe the transmission in itself. The channel is composed of a set of parallel frequency-flat subchannels. As shown in Fig. 1, the power and the bits are allocated to these subchannels. This allocation is adaptive, in the sense that it depends on the the channel state on each subchannel. The coded bits x_n are spread over the set of subchannels, and mapped to constellation symbols which are then multiplied by a power allocation factor and transmitted. The allocation strategy has to determine the constellation size and the power assigned to each subchannel. Denoting by N the number of subchannels, we have the following model for the received signal on the kth subchannel:

$$r_k = \sqrt{p_k}\ \Omega_k\ s_k + n_k \qquad k = 1, \ldots, N \tag{2}$$

where p_k is the power allocated to the kth subchannel, and Ω_k is the complex channel gain on the kth subchannel.The noise samples n_k are assumed to be i.i.d. circularly symmetric complex Gaussian random variables with zero mean and variance σ_n^2. Finally, s_k is the symbol transmitted on the kth subchannel. We consider QAM symbols with unit variance. We will denote by m_k the number of bits in the constellation used on the kth subchannel. As said earlier, after equalization, hard-decision is made on the received signal, followed by QAM demapping in order to recover the coded bits. Let us denote by ρ_k the BER on the kth subchannel, associated with this hard-decision making. The approximate BER expression given in [6] for QAM constellations with Gray bit mapping will be used in this paper:

$$\rho_k \approx c_1 \exp\left(-\frac{c_2\ |\Omega_k|^2\ p_k}{(2^{m_k}-1)\sigma_n^2}\right) \tag{3}$$

with $c_1 = 0.2$, and $c_2 = 1.6$.

The transmission of the N_f information bits of a frame will typically involve the transmission over the set of subchannels during several consecutive symbol periods. We suppose that the channel remains constant over the number of consecutive symbols periods needed for transmitting a frame. In other words, the developments done in this paper are valid for static channels and for channels with slow fading. In this case, the mean BER introduced in (1) is given by the BER (3) averaged over the N subchannels, taking into account the number of bits assigned to each subchannel:

$$\overline{\rho} = \frac{1}{\sum_{i=1}^{N} m_i} \sum_{k=1}^{N} m_k\ c_1 \exp\left(-\frac{c_2\ |\Omega_k|^2\ p_k}{(2^{m_k}-1)\sigma_n^2}\right). \tag{4}$$

3. PROBLEM FORMULATION

In this section, a formulation is given for the problem treated in this paper. When evaluating the performance of the system described in section 2, the only meaningful criterion is the number of information bits delivered without error to the user by unit of time, or goodput. We will use the symbol period as unit of time. Let us denote by r the rate of the convolutional code used. We know that there are N_f information bits in a frame, and that $r\sum_{k=1}^{N} m_k$ information bits are transmitted at each symbol period through the set of subchannels. As a consequence, there are $N_f/(r\sum_{k=1}^{N} m_k)$ symbol periods needed for one frame to be transmitted. Moreover, with selective repeat ARQ, it was shown [5] that the average number of frame transmissions needed for a frame to be successfully transmitted is given by $1/FSR$. The goodput (GP) can thus be expressed as

$$GP = \frac{N_f}{\frac{N_f}{(r\sum_{k=1}^{N} m_k)}\ \frac{1}{FSR(\overline{\rho})}} = \left(r\sum_{k=1}^{N} m_k\right) FSR(\overline{\rho}). \tag{5}$$

Note that the last expression in (5) gives another interpretation for the goodput. It expresses the goodput as the number of information bits sent by symbol period, multiplied by the probability that these bits belong to an error-free frame, which makes sense. Adding constraints on the total transmitted power and on the possible constellation sizes, we end up with the following optimization problem:

$$\max_{m_k, p_k} GP = \left(r \sum_{k=1}^{N} m_k \right) FSR(\overline{\rho}) \quad (6)$$

subject to
$$\sum_{k=1}^{N} p_k \le P_T \quad (7)$$

$$m_k \in \mathscr{M}, \quad k = 1, \ldots, N \quad (8)$$

where P_T is the total power available for the set of subchannels, and with $FSR(\overline{\rho})$ and $\overline{\rho}$ respectively given by (1) and (4). The set $\mathscr{M}$ is defined as the union of the possible constellation sizes (in bits) together with 0 (no transmission). In this paper, we consider three possible constellations: 4-QAM, 16-QAM and 64-QAM. We have $\mathscr{M} = \{0, 2, 4, 6\}$.

The objective of this paper is to propose solutions for the allocation of the bits (m_k) and the power (p_k) among the subchannels in such a way that it maximizes the goodput (6) of the communication system.

4. POWER ALLOCATION

In this section, the bit allocation is assumed to be fixed. In other words, the m_k are no longer considered as variables but as given constants. The focus is set to the derivation of power allocation strategies for a given bit allocation. In particular, two different power allocation strategies are proposed.

4.1 Optimal power allocation

For a given bit allocation, i.e. for given $m_1, m_2, \ldots, m_N$, the first parenthesis in (6) is a constant. As a consequence, the optimal power allocation is such that it maximizes the frame success probability $FSR(\overline{\rho})$, and thus minimizes the mean BER $\overline{\rho}$ (since $FSR(\overline{\rho})$ is a decreasing function[2] with $\overline{\rho}$). The optimal power allocation problem comes down to the minimization of (4) subject to the power constraint (7). Using Lagrange multipliers, we find the following solution:

$$p_k = \frac{(2^{m_k} - 1)\sigma_n^2}{c_2 |\Omega_k|^2} \left[\log\left(\frac{c_1 c_2 |\Omega_k|^2 m_k}{(2^{m_k} - 1)\sigma_n^2} \right) - \log(\lambda) \right]^+ \quad (9)$$

where $[x]^+$ means $\max(x, 0)$. The Lagrange multiplier λ has to be such that (9) satisfies the power constraint (7), and has a closed-form solution. We will refer to this solution using the acronym OPA (Optimal Power Allocation).

4.2 Suboptimal power allocation

One could think that a good suboptimal strategy would be to force equal BER on all used subchannels. We are looking for the power allocation $p_1, \ldots, p_N$ such that the BER is constant over all subchannels having a non null bit allocation:

$$\rho_k = \rho, \quad \forall k \in \mathscr{K}' = \{k \in \mathbb{N} \mid 1 \le k \le N, \, m_k \ne 0\} \quad (10)$$

under the power constraint (7). This equation system has the following closed-form solution, using (3):

$$p_k = \frac{2^{m_k} - 1}{|\Omega_k|^2} \frac{P_T}{\sum_{i \in \mathscr{K}'} \frac{2^{m_i} - 1}{|\Omega_i|^2}}, \quad k \in \mathscr{K}' \quad (11)$$

The acronym EBPA (Equal BER Power Allocation) will be used to refer to this solution.

[2] The decreasing character of the expression (1) depends on the values of the constants $d, a_v, \ldots, a_1, v$. However, since the expression has to fit a true FSR curve, it is obvious that it should be a decreasing function with $\overline{\rho}$.

5. BIT ALLOCATION

In section 4, two different power allocation strategies for a given bit allocation were derived. Using these results, this section is devoted to allocating the bits among the subchannels. Several algorithms are described.

5.1 Exhaustive search

Even though very complex, a possible strategy is the exhaustive search among all possible bit allocations. In this paper, 0, 2, 4 or 6 bis can be allocated to each of the N subchannels: in total, there is 4^N possible bit allocations. The exhaustive search bit allocation (ESBA) consists in, for each of these 4^N bit allocations, computing the chosen power allocation (OPA or EBPA), deducing the mean BER (4) and the associated goodput value (6), and selecting the bit allocation with the highest goodput value. Note that the exhaustive search with the optimal power allocation (ESBA/OPA) is the optimal bit and power allocation strategy.

5.2 Greedy algorithm

In order to reduce the complexity, one alternative is to use a greedy algorithm (see [7] for details): we start with a null bit allocation on each subchannel. We then proceed iteratively. At each iteration, the allocation of two more bits on the kth subchannel is proposed, for each $k \in \{1, \ldots, N\}$. Thanks to section 4, we can associate with each of these N proposals, a new power allocation (OPA or EBPA), thus a new mean BER value (4), and finally a new goodput value (6). We choose the proposal with highest new goodput value, but only if this value is greater than the value that was reached at the previous step (otherwise the algorithm stops). The acronym GABA (Greedy Algorithm Bit Allocation) will be used to refer to this algorithm. Since it does not have to test all possible bit allocations, the GABA significantly reduces the complexity comparing to ESBA.

5.3 Relaxation of the constellation constraint

As it will be shown by simulation, the EBPA (11) is near-optimal since it barely suffers any loss comparing to the OPA (9). This section takes advantage of this result and shows that, under the hypothesis of EBPA, some analytical results can be further derived and used for developing efficient allocation strategies.

Let us consider that the power is allocated according to the EBPA (11). Inserting (11) into (4) gives

$$\overline{\rho} = c_1 \exp\left(\frac{-c_2 \, P_T}{\sigma_n^2 \sum_{i \in \mathscr{K}'} \frac{(2^{m_i} - 1)}{|\Omega_i|^2}} \right). \quad (12)$$

Suppose for a moment that the constraint (8) is relaxed, and that the variables m_k are allowed to take any positive real value. This new problem will be referred as the relaxed problem. By doing so, the goodput expression (6) can be differentiated with respect to each variable m_k. Equaling each of these derivatives to zero, we get, after calculation, that the following equality must hold

$$\frac{2^{m_k}}{|\Omega_k|^2} = \frac{\left(\sum_{i \in \mathscr{K}'} \frac{(2^{m_i} - 1)}{|\Omega_i|^2} \right)^2}{(\sum_{i \in \mathscr{K}'} m_i)(v \, a_v \, \overline{\rho}^v + \cdots + a_1 \, \overline{\rho})(\frac{c_2 P_T \ln(2)}{\sigma_n^2})} \quad (13)$$

for all $k \in \mathscr{K}'$. Since the expression on the right side of the equality (13) is independent of k, we must have that

$$\frac{2^{m_k}}{|\Omega_k|^2} = \frac{2^{m_{k'}}}{|\Omega_{k'}|^2} \qquad \forall\, k,\, k' \in \mathscr{K}'. \tag{14}$$

Using (14), the equality (13) can be rewritten as a function of m_k only:

$$\frac{2^{m_k}}{|\Omega_k|^2}\left(\sum_{i\in\mathscr{K}'}\log_2\left(\frac{|\Omega_i|^2}{|\Omega_k|^2}\right)+N'\,m_k\right)\left(v\,a_v\,\overline{\rho}^{v}+\cdots+a_1\,\overline{\rho}\right)$$
$$\left(\frac{c_2 P_T \ln(2)}{\sigma_n^2}\right)-\left(N'\frac{2^{m_k}}{|\Omega_k|^2}-\sum_{i\in\mathscr{K}'}\frac{1}{|\Omega_i|^2}\right)^2=0 \tag{15}$$

where N' denotes the number of elements in the set $\mathscr{K}'$, and $\overline{\rho}$ is given by rewriting (12) using (14). The problem of finding the bit allocation maximizing the goodput under the hypothesis of EBPA and allowing real bit allocations can then be solved by the following procedure:

1. Sort the subchannels such that $|\Omega_1|^2 \leq \cdots \leq |\Omega_N|^2$. Set $k^* = 1$.
2. Solve (15) for m_{k^*}. This is a non-linear equation which has to be solved numerically. If there is no positive solution for m_{k^*}, then $m_{k^*} = 0$, $k^* \leftarrow k^* + 1$, and go to step 2. Else, go to step 3.
3. Using (14), $\forall k \geq k^*$: $m_k = \log_2\left(\frac{|\Omega_k|^2\, 2^{m_{k^*}}}{|\Omega_{k^*}|^2}\right)$.

At this point, we are able to find the optimal real bit allocations for the relaxed goodput maximization problem, and under the assumption of EBPA. In the sequel, details are given on how to use that result to solve the unconstrained problem (i.e. with constraint (8)). In particular, three possible methods are described:

1. Rounding. Each real bit allocation m_k ($k = 1,\ldots,N$) of the solution to the relaxed problem can be rounded to the nearest element of $\mathscr{M} = \{0,2,4,6\}$. We will use the acronym RRBA (Round Relaxed Bit Allocation) to refer to this bit allocation strategy.

2. Rounding down and greedy algorithm. Each real bit allocation m_k of the solution to the relaxed problem can be rounded down to the nearest element of $\mathscr{M}$, and the greedy algorithm can be run with the result as starting bit allocation. This bit allocation will be referred as RRBA-GABA, the concatenation of the two previously defined acronyms.

3. Branch-and-bound approach. As it was explained, the ESBA consist in trying out all 4^N elements of the solution space, and has a complexity that is exponential in the number of subchannels N. However, being able to solve the relaxed problem, a branch-and-bound approach [8] can be used to find the optimal solution without exploring the whole solution space. This approach uses the following obvious property: the goodput achieved by the optimal real solution to the relaxed problem (which disregards the constraint (8)) can never be worse than the goodput associated with any integer solution (which satisfies the constraint (8)). The branch-and-bound approach is better explained using the example depicted in Fig. 2, where $N = 2$. In this example, the real solution to the relaxed problem is $[m_1,\, m_2] = [0.96,\ 2.69]$, and the associated GP is 1.6. From that, we know that the GP achieved by any integer solution will never exceed 1.6. The solution space, represented as a tree, can then be split in two branches[3] depending on if $m_2 = 2$ or 4. Solving the relaxed problems, with m_2 being fixed to 2 or 4, gives solutions with associated GP equal to 1.5 and 0.8, respectively. We thus naturally choose to further explore the left branch. The real solution achieving GP=1.5 was given by [1.35, 2]. At this point, the left branch can itself be split depending on if $m_1 = 0$ or 2, leading to two possible integer solutions [0, 2] and [2, 2]. It turns out that the second solution achieves a GP equal to 1.1 and outperforms the first one. Moreover, since the GP achieved by that solution is greater than 0.8 (which is an upper bound of what can be achieved by any solutions at the right side of the tree), we do no need to further explore the right side of the tree. Note that this the branch-and-bound approach guarantees to find the optimal solution to the constrained problem. The acronym BBBA (Branch-and-Bound Bit Allocation) will be used to refer to this approach.

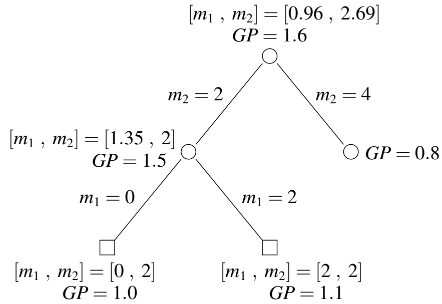

Figure 2: Illustration of the branch-and-bound approach.

6. SIMULATION RESULTS

Several bit and power allocation strategies were presented in sections 4 and 5. In this section, these strategies are simulated and compared to one another.

The described communication system will be simulated using the following simulation parameters: $N_f = 128$, and $\sigma_n^2 = 1$. The convolutional code used has memory order 2, rate $r = 1/2$, and generator polynomial [5,7] in octal notation. A random interleaver is used. Moreover, we consider an OFDM system with 7 taps long channel impulse responses. The taps are i.i.d circularly symmetric complex Gaussian random variables with zero mean and variance such that the impulse response has unitary mean energy. All curves will present the average goodput as a function of P_T/σ_n^2 and result from an average over a thousand channel realizations. The average goodput is expressed as the average number of information bits received correctly (i.e. belonging to an error-free frame) per symbol period. Moreover, in the figures we will draw the goodput normalized by the number of subchannels.

For the convolutional code and frame length used in the simulations, the constants in the expression (1) take the following values: $d = 0.999$, $v = 3$, $a_3 = 2174$, $a_2 = 50.97$, and $a_1 = -0.5740$. These values are such that the expression (1) is a good approximation for the true FSR curve, see Fig. 3.

[3] A good heuristic approach is to choose for branching the variable whose value is the closest to an element of $\mathscr{M}$.

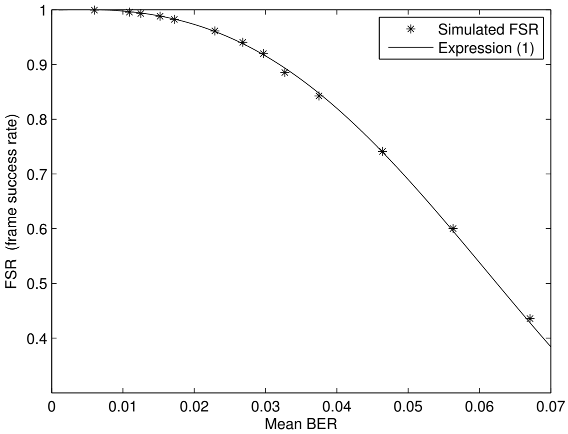

Figure 3: Comparison between the simulated and approximated FSR.

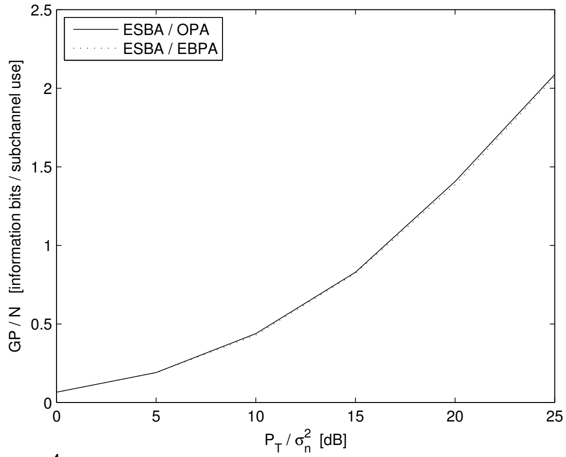

Figure 4: Goodput achieved by the ESBA with the OPA, or EBPA. $N = 8$.

We know that the optimal bit and power allocation strategy is the ESBA/OPA. The Fig. 4 analyzes the performance degradation if the suboptimal EBPA is used instead of the OPA, for $N = 8$ and with the optimal bit allocation (ESBA). It turns out that the performance degradation is very small. In other words, using the EBPA rather than the OPA has a negligible effect on the achievable goodput.

Let us now suppose that the power is allocated using the EBPA. In fact, it has just been shown that this strategy is quasi-optimal. Moreover, it was shown in section 5.3 that its relatively simple expression allowed further analytical derivations. We here compare the different proposed bit allocation strategies, supposing the EBPA. The BBBA guarantees to find the optimal bit allocation when the EBPA is used. It explains why it outperforms all the other strategies, see Fig. 5, where $N = 32$. It also shows that the GABA suffers considerable goodput loss comparing with the BBBA. However, the RRBA and RRBA-GABA strategies barely suffer any loss comparing with the BBBA. Note that the RRBA significantly reduces the complexity: the RRBA implicates only one resolution of the non-linear equation (15), while the BBBA supposes twice as many resolutions of (15) as the number of explored nodes in the tree search, and the RRBA-GABA supposes one resolution of (15) and running the greedy algorithm. We conclude that the RRBA/EBPA is the strategy achieving the best trade-off between performance and complexity.

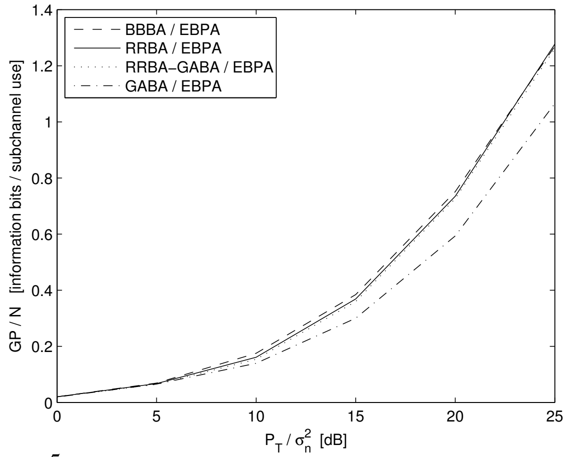

Figure 5: Goodput achieved by the different proposed bit allocation strategies, supposing EBPA. $N = 32$.

7. CONCLUSIONS

We considered the problem of allocating bits and power among a set of parallel subchannels, taking into account the presence of convolutional coding, hard Viterbi decoding, and selective repeat ARQ retransmission protocol. The objective was to maximize the number of information bits delivered without error to the user by unit of time, or goodput. We presented a formulation of the goodput, under the assumption of channel with slow fading and of perfect frame error detection. Different bit and power allocation strategies were proposed to solve that problem. The simulation results showed that the use of a greedy algorithm should be discarded since it is significantly outperformed by other allocation strategies. The best trade-off between performance and complexity was reached by the RRBA/EBPA which allocates the power in such a way that the BER is equal on all used subchannels, and allocates the bits by rounding the solution to the problem obtained by relaxing the constraint of integer constellation sizes.

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